Non-ideal quartum reference frames Outline 1. Ideal ORFr and how to go beyond than 2. Non-ideal QRFr in the perspective - neutral approach 2.1. Complete aRFr with coherent utate systems 2.2. Equivalance of perspectivel and AN approaches for ideal ORFS 2.3 Clautification of ron-ided frames 2.4. Example of non-ideal ORFS (S3) 1. I deal ORFs and how to go by and them Ideal QRFS quantum system carrying the regular representation (150) of the symmetry group G Hilbert space $L^2(G, dg)$ equivariant map $g \mapsto Ig \geq L^2(G)$ with perfectly durtinguisticable orthogonal frame orientation states $\langle g|g' \rangle = \delta(g,g')$ allows for quartum - controlled symmetry group transformations for $G = \mathbb{R}(+)$ (one - dim translation grap) and three systems A, B, Cwith $H_A \cong H_C \cong L^2(\mathbb{R})$ and B carrying a unitary rep U of G and Uns ading on A and C $\hat{S} (A) \rightarrow (C)$ = Pac e [Giacomini et al 2015] with $U(y)_{B} = e^{-i/\hbar y \hat{P}_{B}}$ $= \int_{\Box} dx \, A^{1-x} \, X \, X \, X_{c} \, \otimes \, \mathcal{U} \, (-x)_{B}$ [de la Hamette, Galley 2020] or for general (locally compact) symmetry groups 3(a)-(c) = S dg 15-1×gl @ U(g-1)8 L works for $\{ |g \rangle \}_{g \in G}$ forming an only of $H_A = H_C = L^2(G)$

What if we loosen the accumption that the frame orientation states are orthogonal? Theorem from I de la Hamette & Gallay 2020] The ORF transformations of the perspectival approach for general (locally compact) symmetry groups are unitary if and only if the unitary rep carried by the systems are the left and right regular reps. on states in the Hilbert space $L^2(G)$. QRF changes are unitary only if both frames are ideal frames where G adv regularly on firelf and SING)>3ge6 are persedly distinguistable. This means Convider A, B, C with A and B adding an framew. Consider a left unitary repreventation UL(k)[g) = 1kg). Proof Take two states relative to A: $|11^{1}\rangle = |2|_{A} |g_{1}\rangle_{B} |g_{2}\rangle_{C}$ $|4^{2}\rangle = |e\rangle |h_{1}\rangle_{8} |g_{2}\rangle_{c}$ as well as the linear superposition $|+^{3}\rangle = \alpha |+^{1}\rangle + \beta |+^{2}\rangle = |e\rangle_{A} (\alpha |g_{1}\rangle_{6} + \beta |h_{1}\rangle_{6}) |g_{2}\rangle_{c}$ Let us now change to the perpedive of B $|\gamma^{1}\rangle^{(A)} \stackrel{\widehat{S}}{\underset{|\gamma^{2}\rangle^{(A)}}{\overset{|\widehat{S}|}{\underset{|\widehat{S}|}}} |\varphi^{1}\rangle^{(B)}$ $|\gamma^{2}\rangle^{(A)} \stackrel{\widehat{S}}{\underset{|\gamma^{3}\rangle^{(A)}}{\overset{|\widehat{S}|}{\underset{|\widehat{S}|}}} |\varphi^{2}\rangle^{(B)}$ = $|e\rangle_{0} |g_{1}^{-1}\rangle_{A} |g_{2}g_{1}^{-1}\rangle_{C}$ = $|e\rangle_{0} |h_{1}^{-1}\rangle_{A} |g_{2}h_{1}^{-1}\rangle_{C}$ = $|e\rangle_{0} (\alpha |g_{1}\rangle_{A} |g_{2}g_{1}^{-1}\rangle_{C} + \beta |h_{1}^{-1}\rangle_{A} |g_{2}h_{1}^{-1}\rangle_{C}$ The ORF change operator $\hat{S}^{(A)\to(O)}$ all interproducts is unitary the H perever Let's consider the overlaps $\langle \gamma^{1} | \gamma^{3} \rangle = \alpha + \beta \langle g_{1} | h_{1} \rangle_{B}$

and $\langle \phi^{1} | \phi^{3} \rangle$ $\alpha + \beta < \beta_{1} | \beta_{2} \rangle$ $\alpha + \beta < g_{1}^{-1} | h_{1}^{-1} \rangle < g_{2} g_{1}^{-1} | g_{2} h_{1}^{-1} \rangle$ $= \langle g_{1}^{-1} | h_{1}^{-1} \rangle$ $\alpha + \beta (\langle g_1^{-1} | h_1^{-1} \rangle)^2$ Assuming $\langle g_n | h_n \rangle = \langle g_n^2 | h_n^2 \rangle^2$ $\iff (g_1 | h_1) = (g_1^{-1} | h_1^{-1})^2$ $(=) \langle g_{1} | U_{2} (g_{1}^{-1})^{+} U_{2} (g_{2}^{-1}) | h_{n} \rangle$ $<h_1^{-1} | U_R(h_1^{-1})^+ U_R(h_1^{-1}) | g_1^{-1})^2$ $\iff \langle e | g_{*}^{-1} h_{*} \rangle = \overline{\langle e | g_{*}^{-1} h_{*} \rangle}^{2}$ $\begin{array}{l}
\omega_{\rm H} & \int U_{\rm L}(g) |h\rangle = |gh\rangle \\
\int U_{\rm R}(g) |h\rangle = |hg^{-1}\rangle
\end{array}$ - only holds when $g_1^{-1}h_1$ orthogonal to e Ŝ unitary ⇒ <glh> = δ(g⁻¹h) ¥g,h∈ G $\langle g|h\rangle = \delta(g^{-1}h) \forall g, h \in G \implies \hat{S}^{(0+1)} \int dg |g^{-1} \times g| \otimes U(g^{-1})_{\kappa}$ unitary If we want to construct unitary frame changes between non-ideal frames, we cannot straightforwardly do so in the perspectived framework (i.e. on the kinematical lovel) Con during > one way of allowing for unitary frame changer between non-ideal frames is to first romove the gauge - induced redundancy (cf. perspective - neutral opproad) 2. Non-ideal QRFU in the perspective - neutral approach Reference: [de la Hamette, Gally, Höhn, Laveridge, Müller, 2110.13824] Quantum system carrying a (strongly continuous) unitary representation U of the symmetry group G Different frame orientations are assigned overlapping, non-orthogonal coherent states $\langle g|g' \rangle \neq \delta(g,g')$

2.1. Complete aRFr. with coherent state systems Consider a reference syltem R and a syltem of interact S, carrying a unitary tensor product representation of a (unimodular) Lie symmetry group G, adding on RS as $g \mapsto U_{RS}(g) = U_{R}(g) \otimes U_{R}(g)$ We can adfine globally invariant (Hater (physical states) Mphys) = URS(g) Mphys) E Hphys physical Hilbert space Physical states can be written as: E htps: >= 0 (those states vanishing order the constraint) or intervers = There intun = Jdg URS(g) I trun? coherent G-twinding/ coherent group averaging -> physical states by construction highly entangled states We assume the existence of a coherent state system (i.e. system of generalised coherent states / Gilmore-Recordence coherent states) (CD) { UR, Ig>R} such that G actor transitively on the Cust $|e\rangle$ $|g\rangle = U_g |e\rangle$ Choose a normalised seed state $|e\rangle$, with e unit element of G, and set $|g\rangle := U(g)|e\rangle$. Then generate $|gg|\rangle = U(g)|g'\rangle_R$. *R*) Importantly, we <u>must</u> choose le? In such a way that we obtain a revolution of the identity of dg 19×g1 R = C 1R $|e\rangle \qquad |g\rangle = U_g |e\rangle$, dr.: dimension) of frame HS (for compact gaups: C>O, C = 1 dr Rallows for the treatment of non-ideal framer [dlH, Ludersher, Müller 2021]

The CSS defines a covariant POVM with elements $E_y = \int dg \ |g \times g| \ge 0$ (Y Bord set on the group) • normalised $E_G = \underline{M}_R$ · covariant $E_{h,y} = U_h E_y U_h^{\dagger}$ Now, we can write phyvical states as ! $|\uparrow_{dys}\rangle = \int_{G} dg' |g'\rangle_R \otimes |\uparrow_{S}^{dys}(g')\rangle$ Port - measurement state if we measure "orientation" $g \ dt R$: conditional/ relational $(f_s) = R_s(g) (f_{phys}) = (\langle g|_R \otimes 1_s) (f_{phys})$ state conditional/ relational state Generalized Page - Wootter reduction of states normalize states if pushe in fact, for compact G and $d_R < \infty$, $\Re_{\mathcal{B}}(g) = \left[d_R + \frac{1}{2} g \right]_R \otimes \underline{1}_S$ The conditional / relational chater satisfy covariance property: $U_{S}(g') | \uparrow_{S}^{phys}(g) \rangle = (\langle g|_{R} \otimes I_{S}) U_{S}(g') | \uparrow_{phys} \rangle$ = $(\langle g|_{R} \otimes \underline{A}_{S}) (\underline{U}_{R}(g') \otimes \underline{A}_{S}) | \gamma_{on_{X}} \rangle$ = < g'g | R | t pyr) = R_s (g'g) 1+ phys) = 14 shar (g'g)> The Schrödinger reduction maps are invortible, with inverse $\mathcal{R}_{S'}^{-1}(g) = \mathcal{T}_{Phys}(1g)_R \otimes \mathbb{I}_S) = \int dg' 1g'g'_R \otimes \mathcal{U}_r(g)$ [Lemma 8, [2]]

			\mathcal{R}_{S,R_1} \mathcal{R}_{S_1} $\mathcal{V}_{R_1 \to R_2}$	$\mathcal{H}_{kin} = \mathcal{H}_{R_1} \otimes \mathcal{H}_R$ $\downarrow \Pi_{phys}$ \mathcal{H}_{phys} \mathcal{H}_{ph	$\mathcal{H}_{R_2} \otimes \mathcal{H}_S$ Type $\mathcal{H}_{S,R_2}(g_2)$ $\mathcal{H}_{S,R_1,g_2}^{\text{phys}}$ $\mathcal{H}_{S,R_1,g_2}^{\text{phys}}$ $\mathcal{H}_{S,R_1,g_1}^{-1}$ $\mathcal{H}_{S,R_1}(g_1)^{-1}$ $\mathcal{H}_{S,R_1}(g_1)^{-1}$ $\mathcal{H}_{S,R_1}(g_1)^{-1}$	
	VRi- VRi-	$ = R_{3} (g_{1}) \\ = R_{3} (g_{1}) \\ = V_{R_{1}} \\ = V_{R_{1}} $	$(g_{1}, g_{2}) := \mathcal{R}_{3}^{R_{3}}$ $(g_{1}, g_{2}) := (g_{1}, g_{2}) = $	$(g_{5}) \circ \mathfrak{R}$ $(g_{5}) \rangle =$ $\int_{G} dg _{\mathcal{R}}$ $\langle g_{5} _{\mathcal{R}_{5}} \otimes$	$\frac{1}{4} \operatorname{R}_{i}^{k} (g_{i})^{-1}$ $\frac{1}{8} \operatorname{R}_{i}^{k} (g_{i})^{k} (g_{i})^{k}$ $\frac{1}{8} \operatorname{R}_{i}^{k} (g_{i})^{k} (g_{i})^$	· · · · · · · · · · · · · · · · · · ·
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22 Equivalence of perspectivel and RN approaches for seed DRE For L ¹ (6) w' regular representation, with $g = g_1 = e_1$ $ e \otimes (r+\frac{h}{h_{1,2}}(e)) - V_{R_1-R_2}(e,e) e \otimes (r+\frac{h}{h_{2,2}}(e))$ Frementier: $V_{R_1-R_2}(g_1,g_2) - \int dg_1 g_2 >_R \otimes (g^2g_3) g_1 \otimes U_2(g_2)$ $\rightarrow V_{R_1-R_2}(e,e) - \int dg_R g \times g^{-1} _{R_2} \otimes U_2(g_2)$ $= \int dg_R g^{-1} \times g _{R_2} \otimes U_2(g^{-1}) \int dg_1 g_2 g_2 + g_1 g_2 g_1 + g_2 g_2 + g_1 g_2 g_1 + g_2 g_2 + g_1 g_2 g_1 + g_1 g_2 + g_1 g_2 + g_1 g_2 + g_1 g_1 + g_1 g_2 + g_1 g_1 + g_1 + g_1 g_1 + $	
For L ¹ (6) we regular representation, with $g = g_1 = e_1$ $ c\rangle \otimes r+\frac{d_{M_{1,0}}}{d_{M_{1,0}}}(c)\rangle = V_{R_{1,-reg}}(c,e) c\rangle \otimes r+\frac{d_{M_{1,0}}}{d_{M_{1,0}}}(c)\rangle$ Frementer $V_{R_{1,-R_{2}}}(g_{1}, g_{2}) = \int_{c}^{c} dg_{1} gg_{1}\rangle_{R_{1}} \otimes (g^{-1}g_{1})g_{1} \otimes (g^{-1}g_{1})g_{1}$ $\Rightarrow V_{R_{1,-R_{2}}}(c,e) = \int_{c}^{c} dg_{R} g\chi_{g}^{-1} _{R_{2}} \otimes (U_{2}(g_{1}))f_{2}$ $= \int_{c}^{c} dg_{R} g^{-1}\chi_{g} _{R_{2}} \otimes (U_{2}(g_{1}))f_{2}$ $\left[\frac{Condution}{C} + \frac{1}{C} + \frac{1}{C} \frac$	2.2. Equivalance of perspectival and AN approaches for ideal ORFS.
$ e\rangle \otimes + _{A_{1,S}}^{A_{1,Y}}(e)\rangle - V_{R_{1} \rightarrow R_{2}}(e,e) e\rangle \otimes + _{A_{1,Y}}^{A_{2,Y}}(e)\rangle$ Fremework $V_{R_{1} \rightarrow R_{2}}(g_{1}, g_{2}) = \int_{e}^{e} dg_{R} g g_{1}^{-1} R_{2} \otimes \langle g^{-1} g_{1} g \otimes U_{S}(g)$ $= \int_{e}^{e} dg_{R} g ^{-1} R_{2} \otimes U_{S}(g^{-1})$ (ordurion: The projectivel and the projective-nedral approaches are equivalent for ideal QRTS, i.e. $L^{+}(6)$ frames with othergood frame orientation states. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are not applied to the stame set of states in the two approaches. The frame change man are provided approach, the transformations are applied. To the projective - restrict approach the transformations are applied. The states in the projective of the provide system those states. The states in the projective of the provide system those states of the states. The states is the states of the provide system those formations are applied. The states is the provide of the goal of the states of the states. The states is the states of the provide system the states of the states. The states is the states of the provide system for the states of the states of the states. The states is the states of the s	For $L^{2}(G)$ we regular representation, with $g_{1} = g_{2} = e$
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$V_{R_{1} \rightarrow R_{2}}(g_{1},g_{2}) = \int_{C} dg_{1} [gg_{1} > R_{1} \otimes \langle g^{+}g_{2} _{R_{2}} \otimes \langle U_{2}(g) \rangle$ $\rightarrow V_{R_{1} \rightarrow R_{2}}(e,c) = \int_{C} dg_{1} [g \times g^{-1}]_{R_{2}} \otimes \langle U_{2}(g) \rangle$ $= \int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{2} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{1} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{2} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{2} [g^{-1} \times g]_{R_{2}} \otimes \langle U_{2}(g^{-1}) \rangle$ $\int_{C} dg_{2} [g^{-1} \otimes U_{2} \otimes U_{2}(g^{-1})]$ $\int_{C} dg_{2} [g^{-1} \otimes U_{2} \otimes U_{2$	$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i$
$V_{R_{1} \rightarrow R_{2}}(G_{1}, g_{2}) = \int_{C} dg_{1} [g_{2} > R_{2} \otimes \langle g^{-1} \rangle_{R_{2}} \otimes U_{2}(g)$ $\rightarrow V_{R_{1} \rightarrow R_{2}}(e, c) = \int_{C} dg_{R}[g \times g^{-1}]_{R_{2}} \otimes U_{2}(g)$ $= \int_{C} dg_{R}[g^{-1} \times g]_{R_{2}} \otimes U_{2}(g^{-1}) \int_{C} dg_{R_{2}}(g^{-1}) \int_{C} dg_{R_{2}}(g^{-1}$	
 → V_{Rimas} (e,c) - ∫ dg_R[g×g⁻¹]_R ⊕ Us(g). = ∫ dg_R[g⁻¹×g]_{Rj} ⊕ Us(g⁻¹) √ Condution: The perspective and the perspective-neutral approaches are equivalent for ideal ONTS, i.e. L²(6) frames with orthogonal frame orientation states. The frame change maps are not applied to the size set of states in the two approaches: ⇒ in the propertiesd approach, the transformations are applied to the kinematical states of the form le2_R ⊕ 1/2_{RJS}. → the propertiesd approach the transformations are applied to the size of the form le2_R ⊕ 1/2_{RJS}. → in the propertiesd approach the transformations are applied to states. In the projective of the project system theory of the form le2_R ⊕ th_{SJS}. → th_R ⊕ th_S = d1^{RSS}/_{RSS,S}. VgGGG only for ideal referrer former 	$V_{\mathcal{R}_i} \rightarrow R_j (g_i, g_j) = \int dg [gg_i]_{\mathcal{R}_i} \otimes \langle g^{-1}g_j _{\mathcal{R}_j} \otimes U_{\mathcal{S}_i}(g) .$
 → V_{R:-Rs} (e,e) - J dg_R[g×g⁻¹k_g] @ U₃(g) = Jdg_R[g⁻¹×g]k_g @ U₃(g⁻¹) Conduron: The perfective and the perfective -neored approacher are equivalent for ideal QRTS, i.e. L⁴(6) framer with orthogonal frame orientation states. The frame drange mpu are not applied to the same set of states in the two approaches. → in the perfectived approach, the transformations are applied to the kinematical states of the form (e)_R @ 1+2_{RS}. → in the perfectived approach, the transformations are applied to the kinematical states of the form (e)_R @ 1+2_{RS}. → in the perfectived approach, the transformations are applied to states. → in the perfective approach, the transformation are applied to states. → in the perfective approach to the form (e)_R @ 1+2_{RS}. → in the perfective approach the transformation are applied to states. → in the perfective approach the performation are applied to states. → in the perfective approach the provide system this of the states. → in the perfective construct approach the transformation are applied to states. → in the perfective construct approach the transformation are applied to states. → in the perfective construct approach the transformation are applied to states. → the perfective construct approach the perfect space. If the states of the states is a state of the states. → in the perfective construct approach the perfect space. The states of the states is a state of the state. 	
 Jdg lg 1 × glr, o Us(g-1) Condution: The peripedual and the peripedual -neutral approacher: are equivalent for ideal QRTS, i.e. L¹(6) framer with orthogonal Scare orientation states. The frame change maps are not applied to the scare set of states in the two approaches: in the peripedual approach, the transformations are applied to the knewnatical states of the form le² r₀ S lot 2 R₁s. in the peripedual approach, the transformations are applied to the knewnatical states of the form le² r₀ S lot 2 R₁s. in the peripedual approach, the transformations are applied to states. in the peripedual approach the transformations are applied to states. in the peripedual approach the transformations are applied to states. in the peripedual states of the physical states are applied to states. in the peripedual approach the transformations are applied to states. in the peripedual approach of the framework of the states. in the peripedual approach the transformations are applied to states. in the peripedual approach the physical states of the states. in the peripedual approach of the physical states. in the peripedual approach the transformations are applied to states. in the peripedual approach the transformations are applied. in the peripedual approach the physical states. in the peripedual states of the physical states. in the peripedual states. in the peripedual states of the physical states. in the peripedual st	$\longrightarrow V_{R_1 \to R_2}(e,e) = \int dg_{R_1}g \times g^{-1}_{R_2} \otimes (U_s(g)) $
 I dg R 19⁻¹ × g1R₃ e Us(g⁻¹) × Condution: The perspective and the perspective neutral approaches are equivalent for ideal ORTS, i.e. L⁴(6) frames with orthogonal frame eventation states. The frame change maps are not applied to the same set of states in the two approaches: in the peopletical approach, the transformations are applied to the kinematical states of the form 10²R₃ © 11²R₃s (called alignable states) with 14²R₃ © th₅ in the peopletical approach the transformations are applied to states. In the projective neutral approach the transformations are applied to states. In the physical system Hilbert space. H^R₃s generally a strict subspace of H_{R3} © ths. J H_{R3} © H_S = J1^{RSS} , Vg G G only for ideal reference frames 	
Londwion: The peripedius and the peripedive-neutral approaches: are equivalent for ideal OKTS, i.e. L4(6) frames with orthogonal frame eventation states. The frame change maps are not applied to the same set of states in the two approaches: in the peripedius approach, the transformations are applied to the innernatical states of the form $ c\rangle_{R,S} = 1/2_{R,S}$. (called alignable states) with $ t\rangle_{R,S} \in JH_{S,S} = J_{R,S}$. in the peripedius approach the transformations are applied to states. If $R_{S,S}^{(S)} > in$ the physical system Hilbert space $JH_{R,S}^{(S)}$ generally a strict subspace of $J_{R,S} \otimes J_{S}$. $\rightarrow JH_{R,S} \approx J_{S} = J_{R,S,S}^{(R,S)} = V_{S} \in S$ only for ideal referree frames	$= \int dg R [g^{-1} \times g R] = 0 U_{s}(g^{-1}) $
Conductor: The perspective and the perspective-needed approaches are equivalent for ideal OKTS, i.e. L4(6) frames with orthogonal have orientation states. The Rame change maps are not applied to the scare set of states in the two approaches: \rightarrow in the perspectivel approach, the transformations are applied to the konematical states of the form $[e]_R \otimes [H^*]_{RSS}$ (called alignable states) with $[H^*]_{RSS} \in H_{RS} \otimes H_{SSS}$ (called alignable states) with $[H^*]_{RSS} \in H_{RS} \otimes H_{SSS}$ in the perspectivel approach the transformations are applied to states. In the projective approach the transformations are applied to states. In the projective approach the transformations are applied to states. In the projective of $H_{RS} \otimes H_{SSS}$ generally a strict subspace of $H_{RS} \otimes H_{SSS}$ \rightarrow $H_{RS} \otimes H_{S} = H^{RSS}_{RSSS,SS}$ $VgGGS$ only for ideal reference frames	· · · · · · · · · · · · · · · · · · ·
The frame change maps are not applied to the same set of states in the two approaches: in the peoplectual approach, the transformations are applied to the karematical states of the form $ e\rangle_{R,S} \otimes t\rangle_{R,S}$. (called alignable states) with $ t\rangle_{R,S} \in J_{R,S} \otimes J_{LS}$ in the peoplecture method approach to transformations are applied to states in the projecture method approach to transformations are applied to states in the projecture method approach to transformations are applied to states in the projecture of $J_{R,S} \otimes J_{LS}$ in the peopleture method approach to transformations are applied to states in the projecture of $J_{R,S} \otimes J_{LS}$ \longrightarrow $J_{LR} \otimes J_{LS} = J_{R,S} \otimes J_{S} \otimes J_{S} \otimes J_{S}$	Conducion: The perspective and the perspective-neotral approaches are a second
The frame change maps are not applied to the same set of states in the two approaches: \rightarrow in the perpetitud approach, the transformations are applied to the kinematical states of the form $ e\rangle_{R,S} \otimes h\rangle_{R,S}$. (called alignable states) with $ +\rangle_{R,S} \in H_{R,S} \otimes H_{S}$ \rightarrow in the perpetitie-restrict approach the transformation are applied to states: $ h _{R,S}^{PS}$ in the physical system Hilbert space $H_{R,S}^{PW}$ generally a strict subspace of $H_{R,S} \otimes H_{S}$ \rightarrow $H_{R,S} \otimes H_{S} = H_{R,S,S}^{PW} \otimes V_{G} \in G$ only for ideal referree frames	
The frame change maps are not applied to the same set of states in the two approaches: \rightarrow in the perspectivel approach, the transformations are applied to the knownatical states of the form $ e\rangle_{R,S} H\rangle_{R,S}$ (called algorithe states) with $ T\rangle_{R,S} \in H_{R,S} \oplus H_{S,S}$ (called algorithe states) with $ T\rangle_{R,S} \in H_{R,S} \oplus H_{S,S}$ \rightarrow in the perspective method approach the transformations are applied to states $ T _{R,S} >$ in the physical system thibert space $ T _{R,S,S}$ generally a states $v \oplus space of H_{R,S} \oplus H_{S}$ \rightarrow $H_{R,S} \oplus H_{S} = H_{R,S,S} \oplus H_{S} \oplus G$ only for ideal reference frames	· · · · · · · · · · · · · · · · · · ·
The frame change maps are not applied to the same set of states in the two approaches: \rightarrow in the perpetitual approach, the transformations are applied to the kinematical states of the form $ c\rangle_{R, \odot}$ $ t\rangle_{R_{3}s}$ $(called alignable states) with t\rangle_{Rss} \in Jt_{R, \odot} Jt_{S}in the perpetitue mestred approach the transformations are appliedto states t _{R_{3}s} > 1 in the physical system. Hilbert space Jt_{R_{3}s}generally a strict subspace of Jt_{R_{3}} \odot Jt_{S}\rightarrow Jt_{R, \odot} Jt_{S} = Jt_{R_{3}s, \odot} ~ Vg \in G only for ideal reference frames$	
the two approaches: \rightarrow in the perspectivel approach, the transformations are applied to the kinematical states of the form $ e\rangle_{R, \otimes} t\rangle_{R, s}$ (called alignable states) with $ t\rangle_{R, s} \in J_{R, s} \supset J_{s}$ \rightarrow in the perspective neutral approach the transformations are applied to states $ t _{R, s} \supset$ in the physical system Hilbert space $H_{R, s}^{physics}$ generally a strict subspace of $H_{R, s} \supset J_{s}$ \rightarrow $H_{R, \otimes} \supset J_{s} = J_{R, s} \supset J_{g} \in G$ only for ideal reference frames	The frame change maps are not applied to the same set of states in
The properties approach, the transformations are applied to the kanenatical states of the form $ e\rangle_{R, \otimes} rt\rangle_{R_{3}, S}$. (called alignable states) with $ t\rangle_{R,S} \in H_{R_{3}, \otimes} H_{S}$ to the properties restrict approach the transformation are applied to states $ rt _{R_{3}, S}^{PV}$ in the physical system Hilbert space $H_{R_{3}, S}^{PV}$ generally a strict subspace of $H_{R_{3}, \otimes}$ Hs $H_{R_{3}, \otimes} H_{S} = H_{R_{3}, S}^{PV}$ $\forall g \in G$ only for ideal reference frames	
to the kinematical states of the form $(E^{2}_{R} \otimes 14^{2}_{R})_{RSS}$ (called alignable states) with $(1+)_{RSS} \in H_{RS} \otimes H_{S}$ in the peopleture - restrict approach the transformations are applied to states $(1+)_{RSS}^{ress}$ in the physical system Hilbert space H_{RSS}^{ress} generally a strict subspace of $H_{RS} \otimes H_{S}$ $\rightarrow H_{R} \otimes H_{S} = H_{RSS,S}^{ress} + M_{SSS}^{ress}$ deal reference frames	
The perspective neared approach the transformation are applied to states. In the physical system Hilbert space $H_{BJS,S}^{\mu\mu\nu}$ generally a strict subspace of $H_{BJ} \otimes JJ_S$ $\rightarrow JJ_R, \otimes JJ_S = JJ_{RJS,S}^{\mu\mu\nu}, Jg \in G$ only for ideal reference frames	-> in the perspectival approach, the transformations are applied
in the pappedule - restrict approach the transformation are applied to states. In R_{SS} in the physical system. Hilbert space $H_{R_{SS}}$ generally a strict subspace of $H_{R_{S}} \otimes H_{S}$. $H_{R_{S}} \otimes H_{S} = H_{R_{SS}} \otimes H_{S} \otimes H_{S} \otimes H_{S}$	-> in the perspectivel approach the transformations are applied to the kinematical states of the form (E)R @ (+)RJS (called alignable states) with (+)RJS (= HR: @ HS
$= H_R \otimes H_S = H_{RSS,S}^{POS} \forall g \in G or y Rec ideal reference Pramer$	-> in the perspectival approach the transformations are applied to the kinematical states of the form lerr @ Intra- (called alignable states) with Intras E Hrz @ Hz
\rightarrow $H_{R_{3}} \otimes H_{S} = H_{R_{3}S, S}$ $\forall g \in G$ only for ideal reference frame.	→ in the perspectivel approach the transformations are applied to the kinematical states of the form ler ∞ 1+7 R55 (called alignable states) with 1+7 R55 ∈ HR5∞ H5 → in the perspective - neutral approach the transformations are applied to states 1+ Physical states the transformations are applied
HR. & HS = HRS. J. YGEG only for ideal reference frames	→ in the perspectivel approach the transformations are applied to the kinematical states of the form le? R @ 1+> R_55 (called alignable states) with 1+> R_55 ∈ HR; @ H_5 → in the perspective - nestral approach the transformation are applied to states 1+ Rys > in the physical system Hilbert space HR; B generally a strict subspace of HR; @ Hs
	 in the perspectivel approach the transformations are applied to the kinematical states of the form le>R @ 1+>Rss (called alignable states) with (+>Rss ∈ HR; @ Hs in the perspective - nestral approach the transformations are applied to states in the physical system Hilbert space HR; generally a strict subspace of HR; @ Hs
1 1	→ in the perspectivel approach the transformations are applied to the kinematical states of the form le? _R ⊗ 1+> _{R5} s (called alignable states) with 1+> _{R5} s ∈ H _{R5} ⊗ H ₅ → in the perspective neutral approach the transformations are applied to states 1+ ^{Phys} > in the physical system Hilbert space H _{R5} s, generally a strict subspace of H _{R5} ⊗ H ₅
	→ in the perspectivel approach the transformations are applied to the kinematical states of the form le? _R ⊗ 1+> _{Rs} s (called alignable states) with 1+> _{Bis} ∈ H _{Bi} ⊗ H _s in the perspective -nestral approach the transformation are applied to states 1+ ^{phys} > in the physical system Hilbert space H ^{phys} _{Riss} generally a strict subspace of H _{Ri} ⊗ H _s +H _{Ri} ⊗ H _s = H ^{phys} _{Riss} , yg∈G only for ideal reference frames
1 1	The first approach the transformations are applied to the kinematical states of the form $ e\rangle_{R, \otimes} H\rangle_{R, S}$ (called alignable states) with $ H\rangle_{R, S} \in H_{R, \otimes} M_{S}$ in the passpective -restrict approach the transformations are applied to states $ H _{R, S} > in the physical system Hilbert space H_{R, S}^{Phys}generally a strict subspace of H_{R, \otimes} H_{S} and H_{S, S, S}\rightarrow H_{R, \otimes} H_{S} = H_{R, S, S}^{Phys} = H_{R, S, S}$
. .	The two uppedial approach the transformations are applied to the kinematical states of the form $ e\rangle_{R, \otimes} t\rangle_{R_{3}, S}$ (called alignable states) with $ t\rangle_{R, S} \in H_{R, \otimes} D_{S}$ (called alignable states) with $ t\rangle_{R, S} \in H_{R, S} \oplus H_{S}$ in the prospective neutral approach the transformations are applied to states $ t _{R, S} >$ in the physical system Hilbert space $H_{R, S, S}$ generally a strict subspace of $H_{R, S} \otimes H_{S}$ $\rightarrow H_{R, \otimes} H_{S} = H_{R, S, S} \forall g \in G$ only for ideal reference frames
. .	 in the perspectivel approach, the transformations are applied to the kinematical states of the form 16>_R @ 1+>_{Rs}s. (called alignable states) with 1+>_{Rs}s ∈ H_R: @ H_s in the pospedive neutral approach the transformations are applied to states. In the physical system thibert space H_{Rs}s, generally a strict subspace of H_{Rs} @ H_s. H_R: @ H_s = H_{Rs}s, general vgeG only for ideal reference frames
· · · · · · · · · · · · · · · · · · ·	 → in the perpetitual approach, the transformations are applied to the kanematical states of the form le>_R ⊗ lnt>_{R₃}s. (called alignable states) with lnt>_{R₃}s ∈ the states → in the perpetitue - netral approach the transformations are applied to states. Int Physical system Hilbert space the s
· · · · · · · · · · · · · · · · · · ·	$ \rightarrow \text{ in the perpetitud approach, the transformations are applied to the kinematical states of the form [e]_{R} \otimes [H]_{RSS}.(called alignable states) with [H]_{RSS} \in H_{RS} \otimes H_{S}. \rightarrow \text{ in the perspective -nested approach the transformations are applied to states. If \frac{e}{RSS} in the physical system Hilbert space H_{RSSS}^{HSS}.generally a strict subspace of H_{RS} \otimes H_{S}. \rightarrow H_{RS} \otimes H_{S} = H_{RSSSS}^{HSS} = M_{RSSSS} = M_{RSSSS}^{HSS}$
	→ in the perspectivel approach, the transformations are applied to the kinematical states of the form le? R, @ 1+> R_55 (called alignable states) with 1+> R_55 ∈ HR; @ H_5 (called alignable states) with 1+> R_55 ∈ HR; @ H_5 to states 1+ R_55 > in the physical system Hilbert space HR; generally a strict subspace of HR; @ H_5 HR; @ H_5 = HR; S, JGEG only for ideal reference frames
	The perspectivel approach, the transformations are applied to the knematical states of the form $ C _{R_1} \otimes T _{R_1} s_1$ (called alignable states) with $ T _{R_2} \in J_{R_2} \otimes J_{S_1}$ to the perspective neutral approach the transformations are applied to states $ T _{R_2}^{R_2}$ in the physical system (Hilbert space $J_{R_2}^{R_2}$) generally a strict subspace of $J_{R_2} \otimes J_{S_2}$ $\rightarrow J_{R_2} \otimes J_{S_2} = J_{R_2}^{R_2} \otimes J_{S_2} \otimes J_{S_2}$

2.3 (2007h when it are ited haves) (non - when the
complete ORTE with ideant state systems
* G adds requirery on the CSS (require advan, but to be contract
with require representation)
register advan = free and transitive advan
where free advan :
$$g : x = x$$
 for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
transitive advan : $y : x = x$ for some $x \in x$ implies $g = c$
the consolution is the G orbits in the states on x used to
completely parametrize the G orbits in the states on x used to
complete ORTE : non - trivial stabilister begraps
H = $\{g \in G \in x : |g \times g| = |c \times c|\}$

$$\int_{a}^{b} f_{a}^{a} = \int_{a}^{b} f_{a} x |_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g}|_{g,g$$

2.4. Example for non-ideal (complete) QRFs: the grap S3
Ework with Stefan Ludercher and Markur Müller
S3 has 6 domonts [e, d, d, s1, s2, s3] and is the smallast non-Abelian
. Group up to isomorphismus.
Caryley table $e d d^2 s_1 s_2 s_3$ $e e d d^2 s_1 s_2 s_3$ $d d d^2 s_1 s_2 s_3$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
3 conjugacy darrer $[e] = [e], [d] = [d, d^2], [S_1] = [S_1, S_2, S_3]$
(1d)
· the sign representation TT2 (1d)
the standard representation T(z (20)
Coherent date system
Our Hibert space of interart is $J = C^3$ (notice that dim $(H) = 3 < 6 = S_3 $)
for one system and
$\mathcal{H}^{N} = (\mathbb{C}^{3})^{\otimes N}$ for N oyutoms.
$(\lambda_{2}, \dots, \lambda_{n}) = (\lambda_{n}, \lambda_{n}) + (\lambda_{n}, \dots, \lambda_{n}) + (\lambda_{n}, \dots, \lambda_{n}) = (\lambda_{n}, \dots, \lambda_{n})$
as permutations of the three basis redout $\{102, 112, 122\}$. Write $U(g) = Ug$ for every $g \in S_3$.
as permutations of the three basis vectors $\{10\}, 11\}, 12\}$. Write $U(g) = U_g$ for every $g \in S_3$.
We consider the (initiary) representation of 0_3 on (C) where 0_3 is represented as permutations of the three basis vectors {10>, 11>, 12>}. Write $U(g) = U_3$ for every $g \in S_3$. In contrast to working with thibert space $L^2(S_3)$, it is not passible to find a set of vectors {1g>}_{g \in S_3} s.t. $(g g) = S_{g_3}$. It is however, purifie to find a set of vectors $S(g) = C_3$.
We consider the (Unitary) representation of 03 on (C) where 03 is represented as permutations of the three basis redown {10>, 11>, 12>}. Write $U(g) = U_g$ for every $g \in S_3$. In contrast to working with thibert space $L^2(S_3)$, it is not passible to find a set of redown {1g>}_{g \in S_3} s.t. $\langle g g \rangle = \delta_{gs}$ It is however, possible to find a set of redown $S[g] \rangle_{g \in S_3} s.t.$
We conclude the (unitary) representation of 03 on L where 03 is represented as permutation of the three bars redorn (10>, 11>, 12>). Write $U(g) = U_g$ for every $g \in S_3$. In contrast to working with thibert space $L^2(S_3)$, it is not particle to find a set of redorn $\{I_g\}_{g \in S_3}$ s.t. $\{gI_g\}_{g \in S_3}$ s.t. $I = S_{gg}$. It is not particle to find a set of redorn $\{I_g\}_{g \in S_3}$ s.t. $\{gI_g\}_{g \in S_3}$ s.t. $I = I_g$. I = U(g) I = (1)
We consider the contrary) representation of S, on L. Where S, is represented as permutations of the three basis vector $\{10, 11, 12\}$. Write $U(g) = U_g$ for every $g \in S_3$. In contrast to working with thibert space $L^2(S_3)$, it is not particle to find a set of vectors $\{1g\}_{g \in S_3}$ s.t. $\{g g \} = S_{gg}$, $S g\}_{g \in S_3}$ s.t. It is however, possible to find a set of vectors $\{1g\}_{g \in S_3}$ s.t. (A) $(g) \neq [g']$ for $g \neq g'$ (2)
the contrary the contrary) representation of S, on C where S, is represented as permutations of the three bars redoor (10>, 11>, 12>). Write $U(g) = U_{5}$ for overy $g \in S_{3}$. In contrast to working with thibert space $L^{2}(S_{3})$, it is <u>net</u> particle to find a set of redoor $\{I_{3}\}_{g \in S_{3}}$ set. $\langle g_{1}g_{1} \rangle = \delta_{g3}$ It is <u>net</u> particle to find a set of redoor $\{I_{3}\}_{g \in S_{3}}$ set. $\langle g_{1}g_{1} \rangle = \delta_{g3}$ It is <u>net</u> particle to find I = U(g)Ie > (1) Ig > = U(g)Ie > (1) $Ig > \neq Ig' > For g \neq gI$ (2) $\sum_{g \in S_{3}} [g \times g_{1}] = k \cdot A$ ($k \in C$) (3)
We conclude the contrary, representation of G, on L. Where G, is impresented as permutation of the three barn vector $\{10\}, 11\}, 12\}$. Write $U(g) = U_g$ for overy $g \in S_3$. In contrast to working with thibert space $L^2(S_3)$, it is not particle to find a set of vector $\{1g\}\}_{g \in S_3}$ is to $\{S_3[5]\} = \delta_{g_3}$. It is not for vector $\{1g\}\}_{g \in S_3}$ is to $\{S_3[5]\} = \delta_{g_3}$. It is not of vector $\{1g\}\}_{g \in S_3}$ is to $\{S_3[5]\} = \delta_{g_3}$. It is not of vector $\{1g\}\}_{g \in S_3}$ is to $\{1g\}\}_{g \in S_3}$ is the field of vector $\{1g\}\}_{g \in S_3}$ is to $\{1g\}\}_{g \in S_3}$ is to $\{1g\}\}_{g \in S_3}$ is the field of vector $\{1g\}\}_{g \in S_3}$ is the field of ve
We contrary the contrary representation of $(3, 5n \in C)$ where $(3, 5)$ represented as permutations of the three bars redor $\{102, 112, 122\}$. Write $U(g) = U_g$ for every $g \in S_3$. In contrast to working with thibert space $L^2(S_3)$, it is <u>not</u> particle to find a sol of vectors $\{1g\}_{g \in S_3}$ set $(3g)_2 = S_{23}$. It is not develop for $(1g)_{g \in S_3}$ set $(1g)_2 = U(g)_2 = S_{23}$. $(1g)_3 = U(g)_2 = S_{23}$ (1) $(1g)_3 = U(g)_2 = S_{23}$ (2) $(1g)_3 = U(g)_3 = k \cdot A (k \in C)$ (3) Write the seed state led in the baris $\{102, 143, 122\}$ as
We contrart the contrary representation of G, on L. Where G, is inpresentation as permutations of the three bars redorn {10?, 13, 12?}. Write U(g) = Ug for every $g \in G_3$. In contrast to working with thibert space $L^2(S_3)$, it is not particle to find a set of redorn {19?} $g \in S_3$. It is not for every $g \in S_3$. It is not for every $g \in S_3$. It is not for every $g \in S_3$. It is not for $S(g)_{g \in S_3}$ is to $S(g)_{g \in S_3}$ is to $1g? = U(g) e^{2}$. $1g? = U(g) e^{2}$. $1g? = U(g) e^{2}$. $2g \in S_3$. Write the seed state le? In the bars $f(o?, 1A), 12?$ as $1e^{2} = \alpha (o? + B(A) + g + 2?)$.
We consider the contrary) representation of G_{3} on C where G_{3} is represented as permutations of the three bars vectors $\{10\}, 11, 12\}$. Write $U(g) = U_{3}$ for only $g \in S_{3}$. In contrast to working with thibert space $L^{2}(S_{3})$, it is not possible to find a set of vectors $\{1g\}_{g \in S_{3}}$ set. $\langle g g^{1}\rangle = \mathcal{S}_{3}$; $S g^{2}\rangle_{g \in S_{3}}$ set. $\cdot g\rangle = U(g) e\rangle$ (A) $\cdot g\rangle = U(g) e\rangle$ (A) $\cdot g\rangle \neq g^{1}\rangle$ for $g \neq g^{1}$ (2) $\cdot \sum_{j \in S_{3}} g \times g = k \cdot AL$ ($k \in C$) (3) Write the seed state $ e\rangle$ in the basis $\{10\}, 11, 12\rangle$ as $ e\rangle = \alpha o\rangle + \beta A\rangle + \beta 2\rangle + \beta o\rangle$
$ \begin{array}{l} \text{ (brinder the Unitary) representation of G, on C where G, is inpresented as permutations of the three bars redor. {10>, 11>, 12>}. \\ \text{Write } (U(g) = U_{3}, \text{for overy } g \in S_{3}. \\ \text{Write } (U(g) = U_{3}, \text{for overy } g \in S_{3}. \\ \text{Write } (U(g) = U_{3}, \text{for overy } g \in S_{3}. \\ \text{(a) solved of vector } \{g\}\}_{g \in S_{3}}, \text{ (b) } \{g\}\}_{g \in S_{3}}, \text{ (c) } \{g\}\}_{g \in S_{3}}, $
$ \begin{array}{llllllllllllllllllllllllllllllllllll$

$(2) \stackrel{=}{=} \alpha \neq \beta, \alpha \neq \gamma, \beta \neq \gamma$
Write $[g] = \alpha [a] + \beta [b] + \gamma [c]$ where $\alpha, b, c \in Sij_{i=0}$, $\alpha \neq b, b \neq c, \alpha \neq c$
$(8) \Rightarrow \left\{ \alpha ^2 + \beta ^2 + \beta ^2 = \frac{k}{2} \right\}$
$\left(\alpha\beta^{*} + \alpha^{*}\beta + \alpha^{*}\delta + \alpha\gamma^{*} + \beta\delta^{*} + \beta^{*}\delta = 0\right)$
Example of a valid choice $ e\rangle = \frac{1}{2}(0) + \frac{1}{2}(1)(+0(12))$ $(\rightarrow k=2)$
The phyurcal Hilbert space
$\mathcal{H}_{phyr} = \{ \wp \rangle \in \mathcal{H}^{N} \mid U(\varsigma)^{\otimes N} \wp \rangle = \wp \rangle \forall g \in \mathcal{S}_{3} \}$
Note that every 10) defines a subspace $H_{10} = span S(10))$ on which H^{00} acts trivially
- have to find the decomposition of Ug into impar, and then we can autociate the copies of the trivial representation with Hyper.
$\overline{N} = \overline{1} \qquad \overline{1}\rangle = \frac{1}{\sqrt{2}} (10) + 1(\sqrt{2})$
$ \pi_{3}, \mp\rangle = \frac{4}{3} (10) + e^{\frac{1}{2}\frac{2\pi i}{3}} \lambda\rangle + e^{\frac{1}{4}\frac{2\pi i}{3}} 2\rangle$
$U_{\alpha} = 1 \oplus \overline{\Pi}_{2} \in \mathbb{C}$
Standard representation trivial representation
$4 \text{ for } N = 1 \text{Hense = span } \left\{ \frac{1}{11} \right\} = \text{span } \left\{ \frac{1}{13} (10) + 1(1) + 1(2) \right\}$
Lemma For N systems,
$U_{g}^{\otimes N} = \frac{3^{N-1}+1}{2} \int \oplus \frac{3^{N-1}-1}{2} T_{2}(g) \oplus 3^{N-1} T_{3}(g)$
Proof by induction. (Hint: prove that $\Pi_2 \otimes \Pi_3 = \Pi_3$
$ \dots \dots$
Corollary dim $(H_{phys}) = \frac{3^{N-1}+1}{2}$
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. 6	$ \psi_{ m ph} $	$_{\rm vs}\rangle_{C}$	A D =	= 1	1	(<i>i</i> 0	$\left \begin{array}{c} 02 \end{array} \right\rangle$	+ 0	$ 12\rangle$	-1	022	$2\rangle +$	i 11	10> -	+ 1	$ 20\rangle$	- 1	00>	+i	$221\rangle$	+	$201\rangle$	- :	$211\rangle$,		•		•	•	
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\mathcal{R}_{s}	$\mathbf{s}_{,C}(e)$ $\mathbf{s}_{,A}(e)$	$ \psi_{ m ph} $	$ _{\rm hys}\rangle_C$	AB = AB = AB	$\frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{6}$	$\frac{1}{2}(i 0 $	$0\rangle + 01\rangle$	<i>i</i> 01 + (-	.⟩+ -1 -	10 angle - $i) 2$	$\rangle - $ $20\rangle$	11⟩ + (1	+2 1+a	12 angle $i) 21$	-i $ \rangle +$	20 angle $2 12$	+ 21 $2\rangle)_{CE}$		(i —	1) 2	$2 angle)_A$	в		S	et set	gy oper	con ty	varia	in ce	•	•
\mathcal{R}_{s}	$\mathbf{s}_{,C}(e)$ $\mathbf{s}_{,A}(e)$	$ \psi_{\mathrm{ph}} $	$ _{\rm hys}\rangle_{C}$	$_{AB} =$ $_{AB} =$	$= \frac{\sqrt{3}}{6}$ $= \frac{\sqrt{3}}{6}$	$\frac{1}{3}(2i)$	$\left. 0 \right\rangle + \left. 01 \right\rangle$	<i>i</i> 01 + (-	-1 -	10 angle - $i) 2$	· 20> ·	11> + (1	+2 1+i	12 angle $i) 21$	-i $ \rangle +$	$ 20\rangle$ 2 12	+ 21 2)) _{CE}	.)+ 3.	(i —	1) 2	$2\rangle)_A$	B·		5	et sut pr	by oper	601 ty	kina	en ce	•	•
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