

Vienna Doctoral School in Physics



Quantum Reference Frames: From Quantum Information to Spacetime

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Solstice of Foundations, June 2025









Outline

Part I: Quantum reference frames as a tool for predictions

- Gravity sourced by a mass in superposition
- Extended symmetry principles

spacetimes

- Superpositions of semi-classical spacetimes
- Symmetries and counterparts
- Quantum coordinates

Part III: Conceptual implicationsIdentification and localisation of events

- Localisation of events
- Indefinite causal order

<u>Part II:</u> Quantum reference frames for superpositions of



Part I: Quantum reference frames as a tool for predictions



























Top-down approaches in which quantum theory and general relativity are derived from a larger theory of quantum gravity.

- String Theory



Idea: Take principles of quantum theory and known theories with classical spacetime (general relativity / quantum field theory on curved spacetime) and try to push them as far as possible.



Top-Down Approach

Loop Quantum Gravity, Spin Foam Models, ...





Quantum Reference Frames

Symmetries of *known* physical theories with classical spacetime (GR; QFTCS)

Linearity of Quantum Theory

Extended Symmetry Principles

 \rightarrow Concrete predictions for applicable regimes without the need to rely on perturbative methods



Approach



Goal: Describe motion of test particle in presence of a gravitational source in superposition. ...while remaining "agnostic" about the nature of the gravitational field! ACdIH, V. Kabel, E. Castro-Ruiz, Č. Brukner, Commun Phys 6, 231 (2023).





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Covariance of dynamical laws under coordinate transformations: Physical laws retain their form under coordinate transformations.

ACdIH, V. Kabel, E. Castro-Ruiz, Č. Brukner, Commun Phys 6, 231 (2023).





Covariance of dynamical laws under quantum coordinate transformations: Physical laws retain their form under *quantum* coordinate transformations.

ACdIH, V. Kabel, E. Castro-Ruiz, Č. Brukner, Commun Phys 6, 231 (2023).



Strategy

Change into QRF in which the gravitational source is definite.



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Strategy

- Change into QRF in which the gravitational source is definite.
- Solve problem in the new reference frame.

$$\frac{d^2 x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\nu\rho} \frac{dx^{\nu}}{d\tau} \frac{dx^{\rho}}{d\tau} = 0$$

$$quantum phase$$

$$\Phi^{(i)} = \int_{A^{(i)}}^{B^{(i)}} m_S \sqrt{-g_{\mu\nu}} dx^{\mu} dx^{\nu}$$

L. Stodolsky, Matter and Light Wave Interferometry in Gravitational Fields, Gen. Rel. Grav. 11, 391-405 (1979).





Strategy

- Change into QRF in which the gravitational source is definite.
- Solve problem in the new reference frame.
- Transform back to infer the dynamics assuming that the change of QRF is a symmetry of the equations of motion.







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"extended symmetry principle"



Quantum reference frames as a tool for predictions Motion of a test particle







Quantum reference frames as a tool for predictions Application: Time dilation







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Quantum reference frames as a tool for predictions **Application: Time dilation**



Consistent with gravitational field in superposition.



ACdIH, V. Kabel, E. Castro-Ruiz, Č. Brukner, Commun Phys 6, 231 (2023).



Quantum reference frames as a tool for predictions **Comparison with other approaches**







Consistent with gravitational field in superposition.

Collapse Models





Semi-Classical Gravity

ACdIH, V. Kabel, E. Castro-Ruiz, Č. Brukner, Commun Phys 6, 231 (2023).



Implications

- Concrete predictions while staying agnostic about the quantum nature of the gravitational field.
- Coherence check for the quantum nature of the gravitational field sourced by a massive object in superposition.

Caveats

 Restricted to superposition of semiclassical position states of the gravitational source.



Extended symmetry principles Quantum reference frames as a tool



Examples

- Translations, Galilei group [1712.07207] lacksquare
- Lorentz boosts [2212.14081] Spin rotations [1811.08228, 2103.05022] •
- Euclidean group [2112,11473]



Approach



Conformal Transformations [2207.00021]

Quantum Diffeomorphisms [2211.15685] [2402.10267]

Part II: Quantum reference frames for superpositions of spacetimes





A comment on superpositions of spacetimes

Why Study Superpositions of Semi-Classical Spacetimes?

- approaches to a full theory.
- configurations

(i) Minimal, well-founded assumptions

- **Einstein's equations**
- (b) Linearity of quantum theory

cf. <u>ACdIH</u>, Quantum Reference Frames: From Quantum Information to Spacetime, Doctoral thesis (2025).

Particular regime of interest at the intersection of gravity and quantum theory, **complementary** to

Expected to arise e.g. when a massive object is placed in a spatial SP of two semi-classical

(a) Semi-classical states of the spacetime metric, peaked around classical solutions to





A comment on superpositions of spacetimes

Why Study Superpositions of Semi-Classical Spacetimes?

- approaches to a full theory.
- configurations
- (i) Minimal, well-founded assumptions
- (ii) Agreement with predictions from linearised quantum gravity
- (iii) Testability in near-future experiments
- - Leads to quantum-controlled symmetry transformations
 - \rightarrow Identified with QRF transformations under the classical symmetry group
 - \rightarrow Enables search for QRF invariants and insight into observables in quantum gravity

cf. <u>ACdIH</u>, Quantum Reference Frames: From Quantum Information to Spacetime, Doctoral thesis (2025).

Particular regime of interest at the intersection of gravity and quantum theory, complementary to

Expected to arise e.g. when a massive object is placed in a spatial SP of two semi-classical

(iv) Extended symmetry principles from combining classical symmetries with linearity





Quantum Reference Frames for Spacetimes



What does it even mean that the mass is in a superposition of locations?





Quantum Reference Frames for Spacetimes



What does it even mean that the mass is in a superposition of locations?





Symmetries & Counterparts

- Consider a theory with symmetry group *G*.
- The space of all possible configurations (models) can be partitioned into orbits of G.
- Models on a given orbit are taken to represent the same physical state of the world.



 ${\mathscr O}_{arphi}$

 $\mathcal{O}_{\varphi'}$





Symmetries & Counterparts Example: Translation Group

- Newton's theory of a gravitating particle is invariant under the translation group.
- Two models related by a rigid translation live on the same orbit.
- Such situations are taken to be physically equivalent.



 ${\mathscr O}_{arphi}$

 ${\mathscr O}_{arphi'}$

Symmetries & Counterparts

- A section picks one representative $\sigma(\varphi)$ on each orbit \mathcal{O}_{φ} .
- The choice of section is a matter of *convention* and can be seen as a choice of **reference frame**.



Gomes (2021), Gomes & Butterfield (2023)







Symmetries & Counterparts

- A section picks one representative $\sigma(\varphi)$ on each orbit \mathcal{O}_{φ} .
- The choice of section is a matter of convention and can be seen as a choice of reference frame.
- Example: choice of origin in translationally invariant theory.



 ${\mathscr O}_{arphi}$

 ${\mathscr O}_{\varphi'}$





Symmetries & Counterparts **Counterpart Relation**



Objects are counterparts if they are similar in relevant aspects which aspects is "up to us".

Symmetries, counterparts, identification

- The choice of section determines how to **compare** different possible configurations.
- The **counterpart relation** allows us to state whether two configurations are "the same" or "different".



$$\operatorname{Counter}_{\sigma}(\varphi,\varphi') = \frac{g_{\sigma}(\varphi')^{-1}g_{\sigma}(\varphi)}{g_{\sigma}(\varphi)}$$

Gomes & Butterfield (2023), Gomes (2024)







Quantum Reference Frames Example: Translation Group

- Idea: a choice of QRF corresponds to a choice of section
- A QRF tells us what the same or **different** elements are relative to it.
- In the frame of P, the position of P is **identified** across both branches








$$|\psi\rangle_{PM}^{(P)} = |0\rangle_P \otimes (\alpha | -a\rangle_M + \beta |a\rangle_M)$$



branch of the superposition







 $|\psi\rangle_{MP}^{(M)} = |0\rangle_{M} \otimes (\alpha |a\rangle_{P} + \beta |-a\rangle_{P})$



How we identify position across the branches changes with the QRF. Superposition becomes a reference frame dependent feature.

- A model is a tuple ($\mathcal{M}, g_{ab}, \psi_{matter}$).
- Space of models Φ is the set of kinematically possible models.







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$$d \left(\begin{array}{c} \varphi_1 = (\mathcal{M}, g_{ab}, \psi_{matter}) \\ \varphi_1^d = (\mathcal{M}, d_*^{-1} g_{ab}, d_*^{-1} \psi_{matter}) \end{array} \right) \varphi_2$$



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- A model is a tuple ($\mathcal{M}, g_{ab}, \psi_{matter}$).
- Space of models Φ is the set of kinematically possible models.
- Symmetry group is G = Diff(*M*).
 Find
- Add a set of four scalar fields $\{\chi_{(A)}\}_{A=0,1,2,3}$.



Identification across a Superposition of Spacetimes The Comparison Map



 $(\mathcal{M}_2, g_{ab}^{(2)}, \chi_{(A)}^{(2)})$

use coincidences of fields to identify points across the spacetimes in superposition. Kabel, <u>ACdIH</u>, Apadula et al. (2024) cf. Westman and Sonego (2009); Hardy (2020)



Identification across a Superposition of Spacetimes The Comparison Map

$$C_{\chi} \equiv (\chi^{(2)})^{-1} \circ \chi^{(1)} : \mathcal{M}_1 \to \mathcal{M}_2$$

- ◆ If the four scalar fields define a bijective map from *M* to ℝ⁴, then fixing field values removes redundancy induced by diff-invariance.
- Bijectiveness requires
 - 1. fields inhomogeneous enough, or
 - 2. restriction to sufficiently small open subregion of \mathscr{M}



Take two models of a 3D space of models in superposition.



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- Consider three $\{\chi_{(A)}\}_{A=1,2,3}$ -fields that return the **three RGB values** at any **point** on the manifold.





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- Consider three $\{\chi_{(A)}\}_{A=1,2,3}$ -fields that return the **three RGB values** at any **point** on the manifold.
- If $\chi^{(1)}(p) = (193, 140, 143) = \chi^{(2)}(q)$, then *p* and *q* are **identified** relative to these χ -fields.
- Taking $\chi^{(1)}$ and $\chi^{(2)}$ to be different configurations of the same physical fields, we find a natural strategy to identify points across manifolds.



Identification across a Superposition of Spacetimes Quantum coordinate fields

Three options for modelling scalar **reference** fields:

- idealised or coordinate fields
- 2. dynamical fields without back-reaction
- 3. dynamical fields with back-reaction







Rovelli (1991); Bamonti (2023)



Identification across a Superposition of Spacetimes **Changes of QRF**

- Choice of QRF ~ choice of scalar fields and corresponding identification
- Change of QRF $\chi \to \tilde{\chi}$
 - (i) apply **quantum-controlled** diffeomorphism to make the reference fields definite
 - (ii) change the comparison map from C_{χ} to $C_{\tilde{\chi}}$







Identification across a Superposition of Spacetimes **Changes of QRF**

- + Choice of QRF ~ choice of scalar fields and corresponding identification
- + Change of QRF $\chi \to \tilde{\chi}$
 - (i) apply quantum-controlled diffeomorphism to make the reference fields definite
 - change the comparison map (II)from C_{χ} to $C_{\tilde{\chi}}$







Part III: Conceptual implications



Localisation of events













 $d^{(2)}(q) = d^{(2)} \circ C_{\chi}(p) = d^{(2)} \circ C_{\chi} \circ (d^{(1)})^{-1} \circ d^{(1)}(p) = C'_{\chi}(d^{(1)}(p))$









Quantum diffeomorphisms and quantum coordinates A concrete toy example

 χ -fields: (Riem² – Weyl², $\Box R$, Ric², \Box Weyl²)





Quantum diffeomorphisms and quantum coordinates A concrete toy example

 χ -fields: (Riem² – Weyl², $\Box R$, Ric², \Box Weyl²)







Identification of spacetime points and localisation of events are frame-dependent and have no absolute physical meaning.

 $d^{(1)}$



Indefinite causal order



Indefinite Causal Structures



Hardy, A Framework for Probabilistic Theories with Non-Fixed Causal Structure (2006)











We want to describe the probabilities for outcomes a,b:

P(a,b)

 $\mathcal{H}^{A_{out}}$



 $\mathscr{H}^{A_{in}}$



 $\mathcal{H}^{B_{in}}$



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 $\mathcal{H}^{A_{in}}$

 $\mathcal{H}^{B_{out}}$

Oreshkov, Costa, Brukner (2013).



Processes with indefinite causal order The Quantum SWITCH







Chiribella, D'Ariano, Perinotti, Valiron (2013); Oreshkov (2019)



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Processes with indefinite causal order



- Indefinite Causal Order (ICO): no classical mixture of $A \prec B$ and $B \prec A$
- Process with ICO : the quantum SWITCH

$$|\Psi\rangle_{CT} = \frac{1}{\sqrt{2}} \left(|0\rangle_{C} U_{B} U_{A} + |1\rangle_{C} U_{A} U_{B}\right) |\Psi^{in}\rangle_{T}$$

Different implementations of this abstract process.

Hardy, J. Phys. A 40, 3081(2007). Oreshkov, Costa, Brukner, Nat Commun 3, 1092 (2012). Chiribella, D'Ariano, Perinotti, Valiron, Phys. Rev. A 88, 022318 (2013). Zych, Costa, Brukner, Nat Commun 10, 3772 (2019).

- Two experimenters Alice and Bob.
- Each of them
 - receives a physical system
 - performs an operation U_i on it, and
 - sends it out of their laboratory.
- If the system moves in a superposition of paths, the operations are performed in a superposition of orders.

Indefinite causal order through superposition of paths.



Chiribella, D'Ariano, Perinotti, Valiron (2013); Oreshkov (2019)



Has been realised in **optical experiments**:

- Path as control
- Polarisation as control
- Debated whether this counts as genuine realisation of ICO.







Rubino et al. (2016)

Procopio et al. (2014), Rubino et al. (2016), Goswami et al. (2018)



- Two experimenters Alice and Bob.
- Each of them performs an operation U_i at fixed proper time τ_* .
- Gravitational field sourced by a massive object in superposition of locations.





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 - 1. Bob experiences stronger time dilation: $A \prec B$





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 - 2. Alice experiences stronger time dilation: $\prec A$





- Two experimenters Alice and Bob.
- \bullet Each of them performs an operation U_i at fixed proper time τ_* .
- Gravitational field sourced by a massive object in superposition of locations.
 - 1. Bob experiences stronger time dilation: $A \prec B$
 - 2. Alice experiences stronger time dilation: $B \prec A$

Indefinite causal order through superposition of gravitational fields.







Gravitational Quantum Switch

 Ongoing debate regarding the implementation of the "quantum switch" exhibiting indefinite causal order.

> Oreshkov (2019), Quantum 3, 206 (2019). Paunkovic & Vojinovic, Quantum 4, 275 (2020). Ormrod, Vanrietvelde, Barrett, Quantum 7, 1028 (2023).

Indefinite causal order **Simulation vs Realisation**



From a foundational point of view, our experiment can be seen as the first realization of a "superposition of causal orders", which represents an instance of an indefinite causal structure [25].

Procopio, Moquanaki, Araujo, et. al. (2014)





We show that the current quantum switch experimental implementations do not feature superpositions of causal orders between spacetime events, and that these superpositions can only occur in the context of superposed gravitational fields. Paunković, Vojinović (2020)



Indefinite causal order **Simulation vs Realisation**



However, the interpretation of such experiments as realizations of a process with indefinite causal structure as opposed to some form of simulation of such a process has remained controversial.





Oreshkov (2019)

Indefinite causal order Optical vs gravitational implementations



Indefinite causal order through superposition of paths.



Indefinite causal order through superposition of gravitational fields.



- Ongoing debate regarding the implementation of the "quantum switch" exhibiting indefinite causal order.
- Core of the debate: **how many events** are there?
 - "Spatiotemporalists": 4 events (spacetime points)
 - "Operationalists": 2 events (application of operation)
- **Insight:** change of QRF can change the number of spacetime points.





Four different spacetime points. Fixed spacetime.



Two different spacetime points. Superposition of spacetimes.

<u>ACdIH</u>, Kabel, Christodoulou, Brukner (2022); Kabel, <u>ACdIH</u>, Apadula et al. (2024).



Whether a process displays ICO 'due' to delocalised events or a superposition of spacetime metrics can change under quantum diffeomorphisms.

1. ICO 'due' to delocalised events in fixed spacetime.



Delocalised events

Localised events

ICO due to delocalised events in fixed spacetime ICO due to superposition of spacetime metrics at localised events

Whether a process displays ICO 'due' to delocalised events or a superposition of spacetime metrics can change under quantum diffeomorphisms.

2. ICO 'due' to a superposition of spacetime metrics.



QRFs for indefinite causal order



Whether ICO is due to delocalised events or a superposition of metrics depends on the choice of quantum coordinates.

A no-go theorem

There is no quantum coordinate system in which the following three statements hold:

- 1. Both events \mathscr{C}_A and \mathscr{C}_R are localised.
- 2. The spacetime **metric** is **definite**.
- 3. The **causal order** between \mathscr{C}_A and \mathscr{C}_B is **indefinite**.



*consistent with Cor. 8.2 in Vilasini & Renner PRA 110, 022227 (2024). <u>ACdIH</u>, Kabel, Christodoulou, Brukner (2022); Kabel, <u>ACdIH</u>, Apadula et al. (2024).



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- When formulated in an invariant manner, optical and gravitational QS exhibit same type of ICO.
- Should not take spacetime location of event as relevant property.



Summary

Part I: Quantum reference frames as a tool for predictions

- Gravity sourced by a mass in superposition
- Extended symmetry principles

spacetimes

- Superpositions of semi-classical spacetimes
- Symmetries and counterparts
- Quantum coordinates

Part III: Conceptual implicationsIdentification and localisation of events

- Localisation of events
- Indefinite causal order

<u>Part II:</u> Quantum reference frames for superpositions of





Outlook (non-exhaustive)

- Go beyond
 - semi-classical spacetimes in superposition (model genuine) spacetime **fluctuations**)
 - semi-classical reference field configurations in superposition
- Quantum equivalence principle for general non-classical spacetime
- Model **non-ideal** frames for non-classical spacetimes
 - no perfect spacetime localisation
- Model measurement in QRFs (see work by Fewster & Verch for construction in AQFT)
- Clarify how different QRF approaches relate to each other (perspectival, perspective-neutral, operational, quantum) information and its extensions) and which situations each is most suited for

Thank you for your attention!



