The Local Friendliness theorem

Yìlè Yīng

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Bong, et. al., arXiv:1907.05607
Cavalcanti and Wiseman, arXiv:2106.04065
Wiseman, Cavalcanti and Rieffel, arXiv:2209.08491
Haddara and Calvalcanti, arXiv:2205.12223
Yīng, et al., arXiv:2309.12987

Schmid, Yīng and Leifer, arXiv:2308.16220

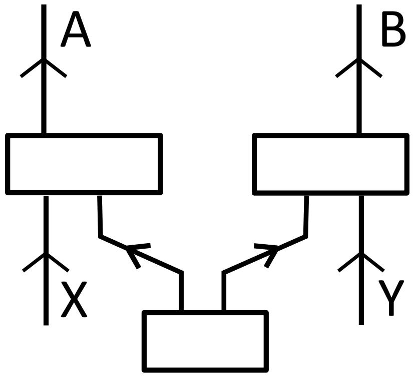
Preliminary:

Bell’s theorem (Hardy’s version)

Hardy, Phys. Rev. Lett. 71, 1665

How is it used in Frauchiger-Renner? See: arXiv:2308.16220

Bell’s theorem (Hardy’s version)



preparation $|\psi_{\text{Hardy}}\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$

mmt setting = 0 $\left\{|0\rangle\langle 0|, |1\rangle\langle 1|\right\}$ “the computational basis”

mmt setting = 1 $\left\{|+\rangle\langle +|, |-\rangle\langle -|\right\}$

Quantum Predictions

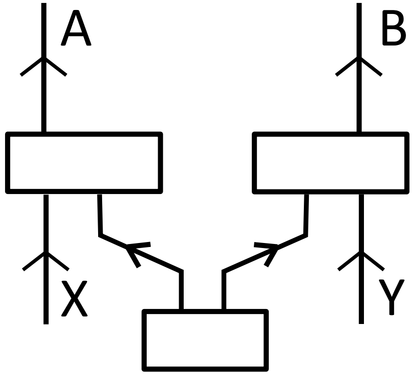
$p(a=1,b=1|x=0,y=0) = 0$

$p(a=0,b=-|x=0,y=1) = 0$

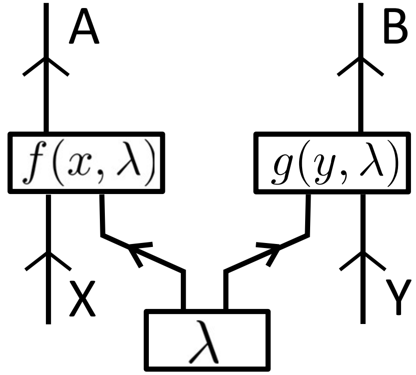
$p(a=-,b=0|x=1,y=0) = 0$

$p(a=-,b=-|x=1,y=1) \neq 0$

Bell's theorem (Hardy's version)



classical realist representations



λ specifies the outcomes of all possible mmts (performed or not)

Quantum Predictions

$p(a{=}1,b{=}1\,|\,x{=}0,y{=}0) = 0$

$p(a{=}0,b{=}-\,|\,x{=}0,y{=}1) = 0$

$p(a{=}-,b{=}0\,|\,x{=}1,y{=}0) = 0$

$p(a{=}-,b{=}-\,|\,x{=}1,y{=}1) \neq 0$

$\forall\lambda$:

$B_0{=}1 \Rightarrow A_0{=}0$

$A_0{=}0 \Rightarrow B_1{=}+$

$A_1{=}- \Rightarrow B_0{=}1$

$A_0 := f(\lambda, x{=}0)$

$A_1 := f(\lambda, x{=}1)$

$B_0 := g(\lambda, y{=}0)$

$B_1 := g(\lambda, y{=}1)$

CONTRADICTION

$A_1{=}- \Rightarrow B_1{=}+$

Relax determinism: cf. David's lecture

This argument relies on classical realism

aka the ontological models framework arXiv:0706.2661

aka hidden variables

aka the classical causal modeling framework arXiv: 1208.4119

Some interpretations reject these strong realist assumptions, notably Copenhagen(ish) interpretations

arXiv:2506.00112

Peres: “Unperformed experiments have no results.”

$\Rightarrow$ *Either* A_0 *or* A_1 is meaningful, not both!

$\Rightarrow$ *Either* B_0 *or* B_1 is meaningful, not both!

~~Counterfactuals: what outcome would Alice have obtained, if she had chosen x=1,?~~

$B_0=1 \;\Rightarrow\; A_0=0$

$A_0=0 \;\Rightarrow\; B_1=+$

$A_1=- \;\Rightarrow\; B_0=1$

$\cancel{\Rightarrow}\;\; A_1=- \;\Rightarrow\; B_1=+$

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E.g., the ones with an empiricist flavor, notably Copenhagen(ish) interpretations

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$B_0=1 \;\Rightarrow\; A_0=0$

$A_0=0 \;\Rightarrow\; B_1=+$

$A_1=- \;\Rightarrow\; B_0=1$

$\not\Rightarrow \; A_1=- \;\Rightarrow\; B_1=+$

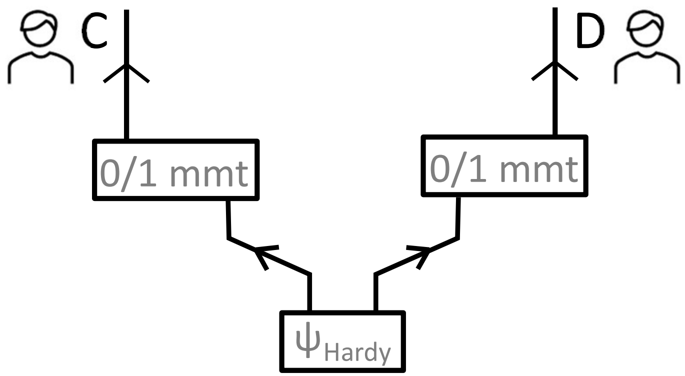
But what if all four measurements are performed in a single run?

$\Rightarrow$ Bell-Wigner mashup

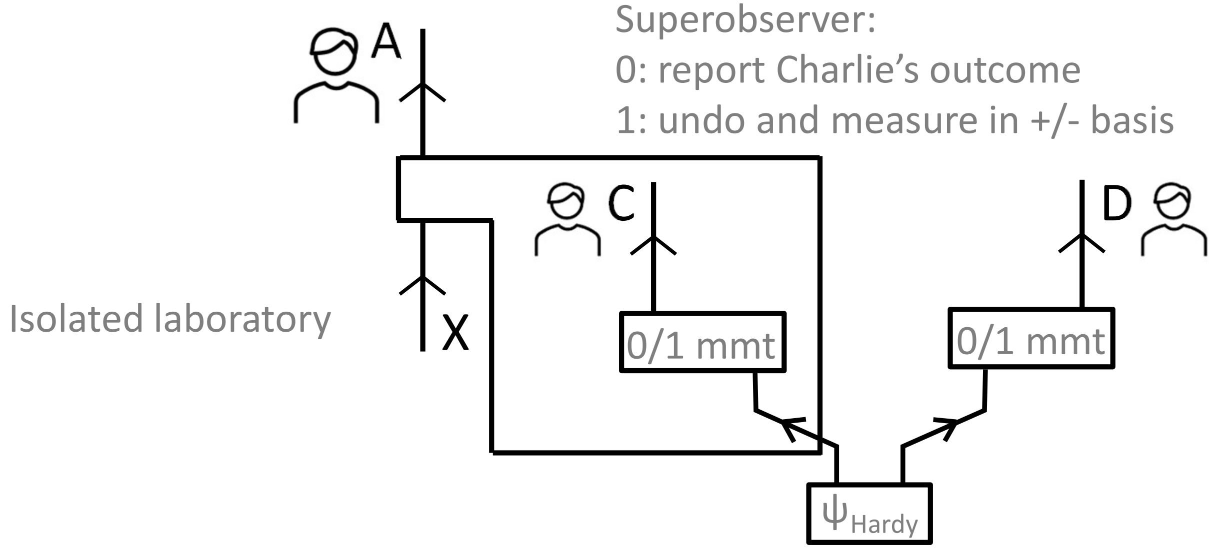
Brukner, arXiv:1507.05255

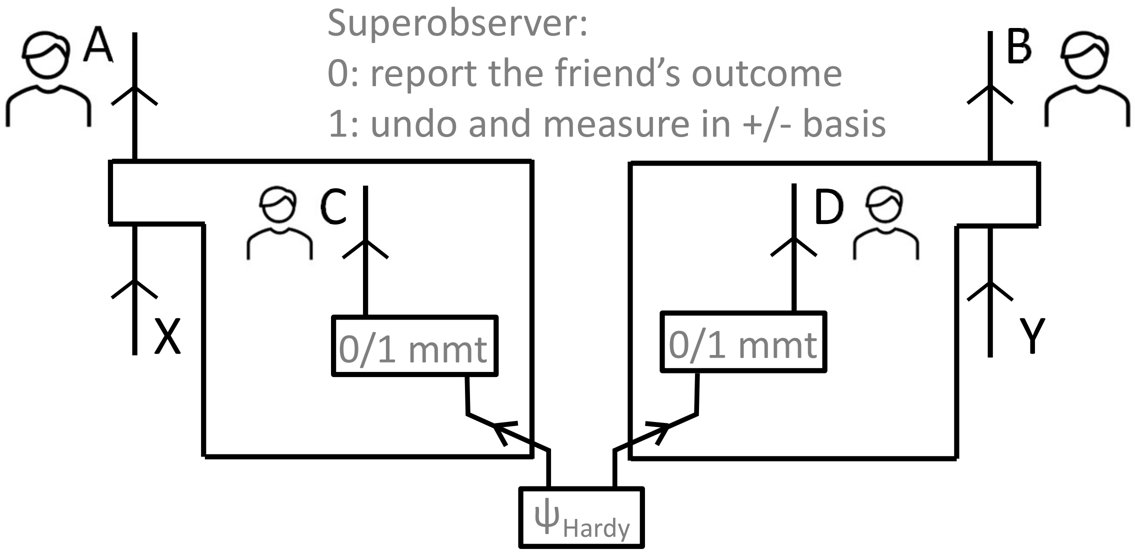
$\Rightarrow$ The Local Friendliness theorem

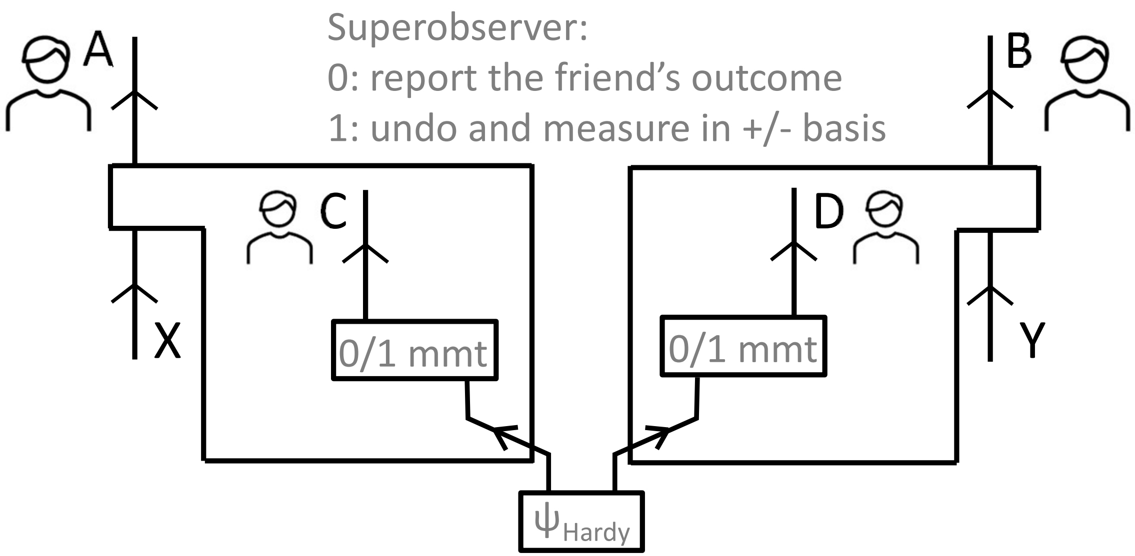
Bong, et. al., arXiv:1907.05607



Haddara and Calvalcanti, arXiv:2205.12223


A
Superobserver:
0: report Charlie's outcome
1: undo and measure in +/- basis
C
D
Isolated laboratory
X
0/1 mmt
0/1 mmt
ψ_{Hardy}

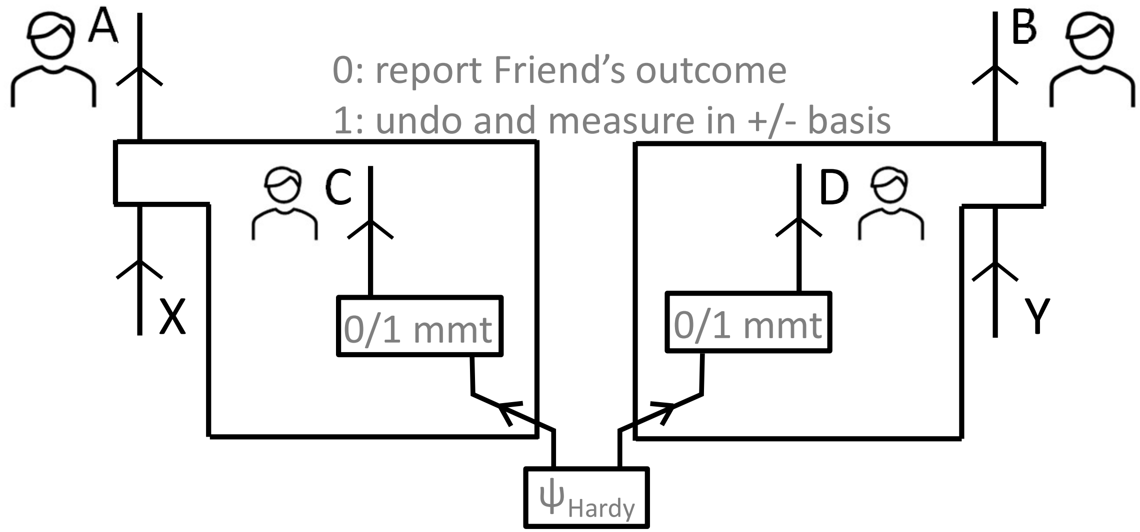

Superobserver:
0: report the friend's outcome
1: undo and measure in +/- basis
A
B
C
D
X
Y
0/1 mmt
0/1 mmt
ψHardy



The 0/1 basis measurements are **actually performed** in every run of the experiment (by Charlie and Debbie).

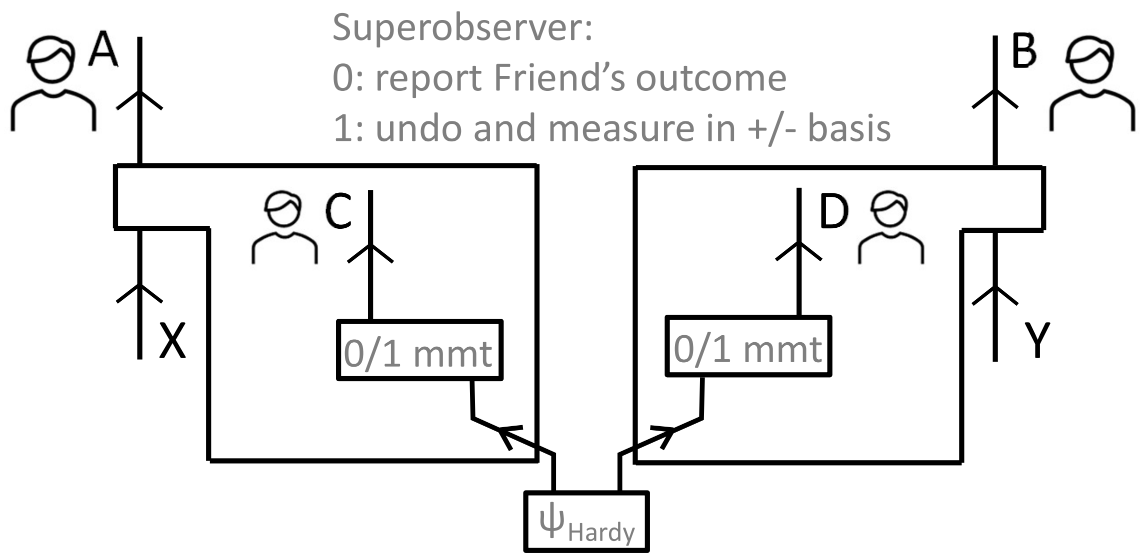
The +/- basis measurements are **actually performed** in all runs of the experiment <u>where X=1 and Y=1</u> (by Alice and Bob).

So in <u>these</u> runs, all four measurements are performed!



So we expect to derive a contradiction just like Hardy's, but *where we don't appeal to classical realism* (e.g. ontic states).

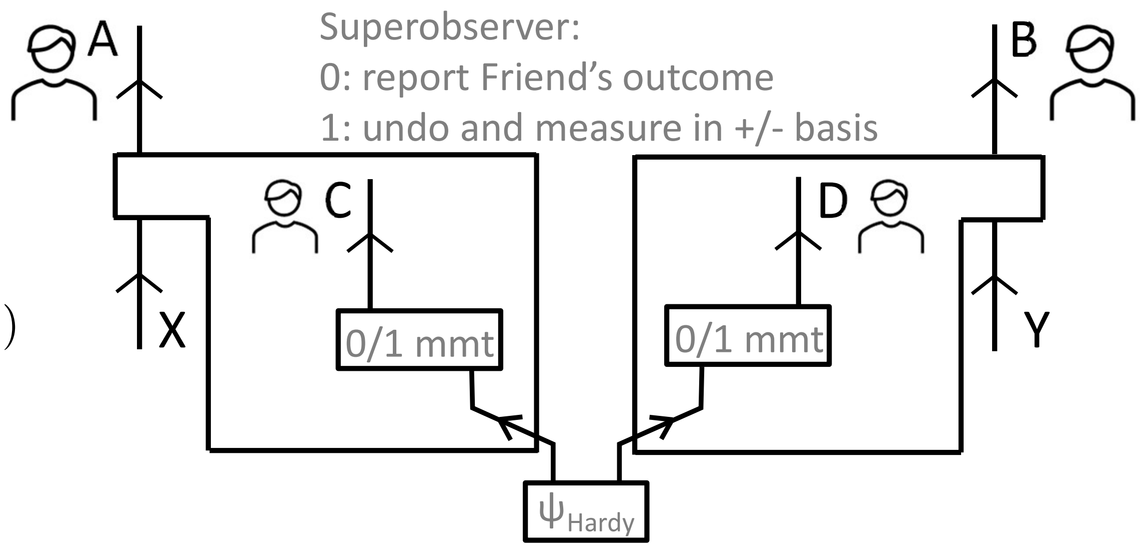
Unfortunately, Brukner still implicitly assumed classical realism.



So we expect to derive a contradiction just like Hardy's, but *where we don't appeal to classical realism* (e.g. ontic states).

The Local Friendliness no-go theorem

$$\frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$$

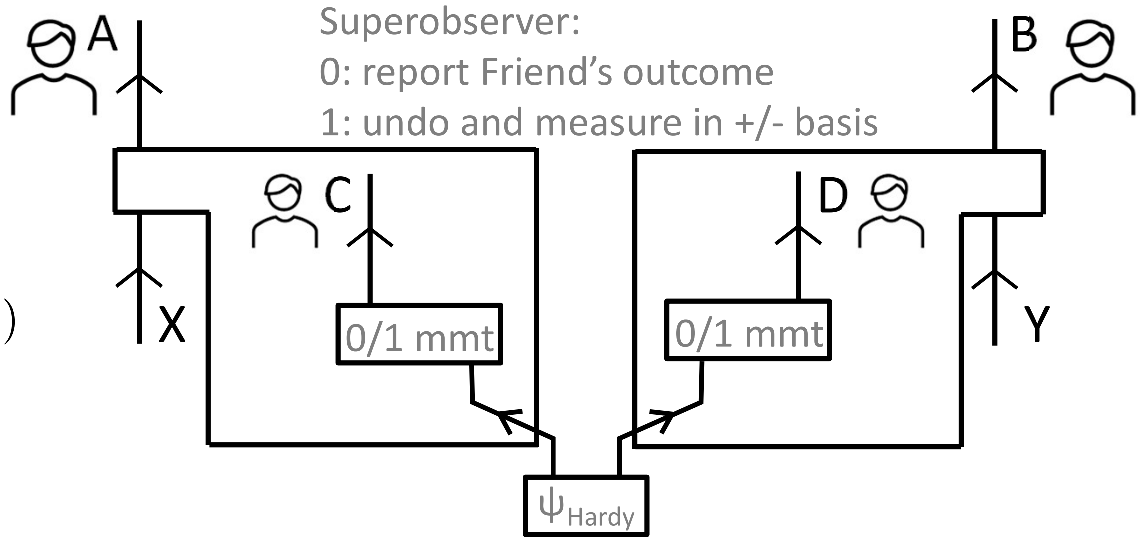


quantum predictions

x=0,y=0

no reversals

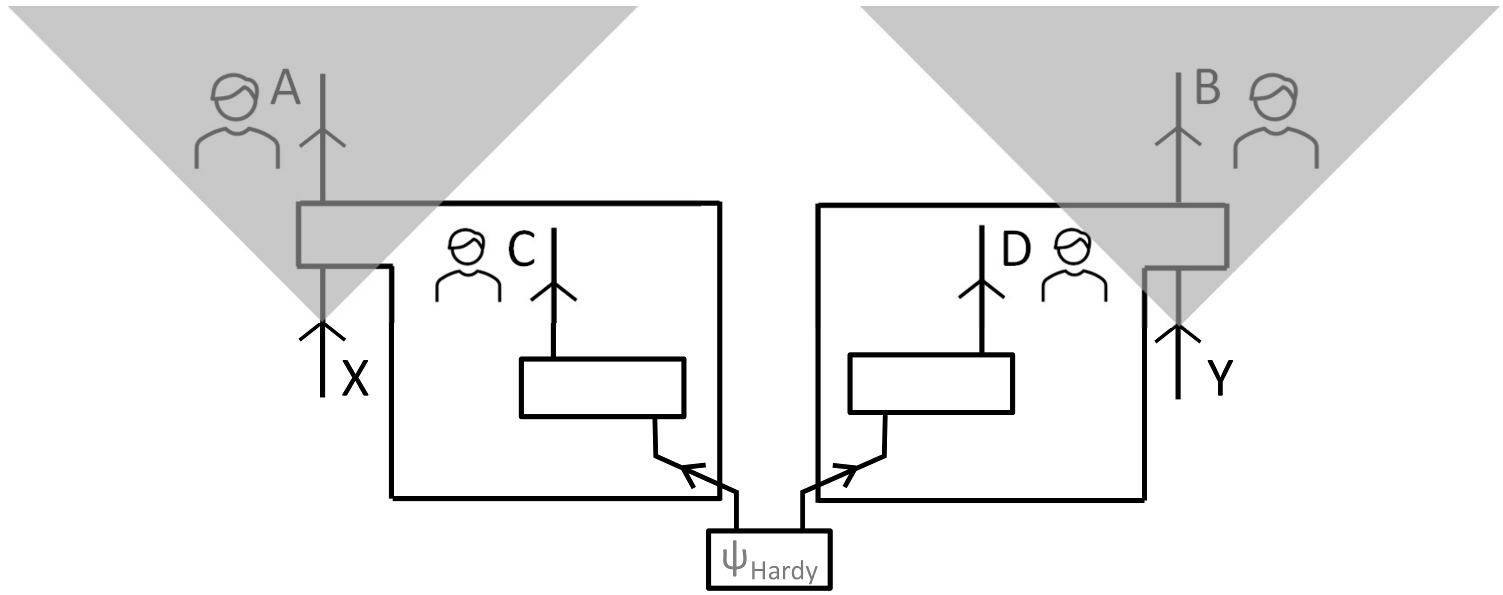
$$\frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$$



quantum predictions

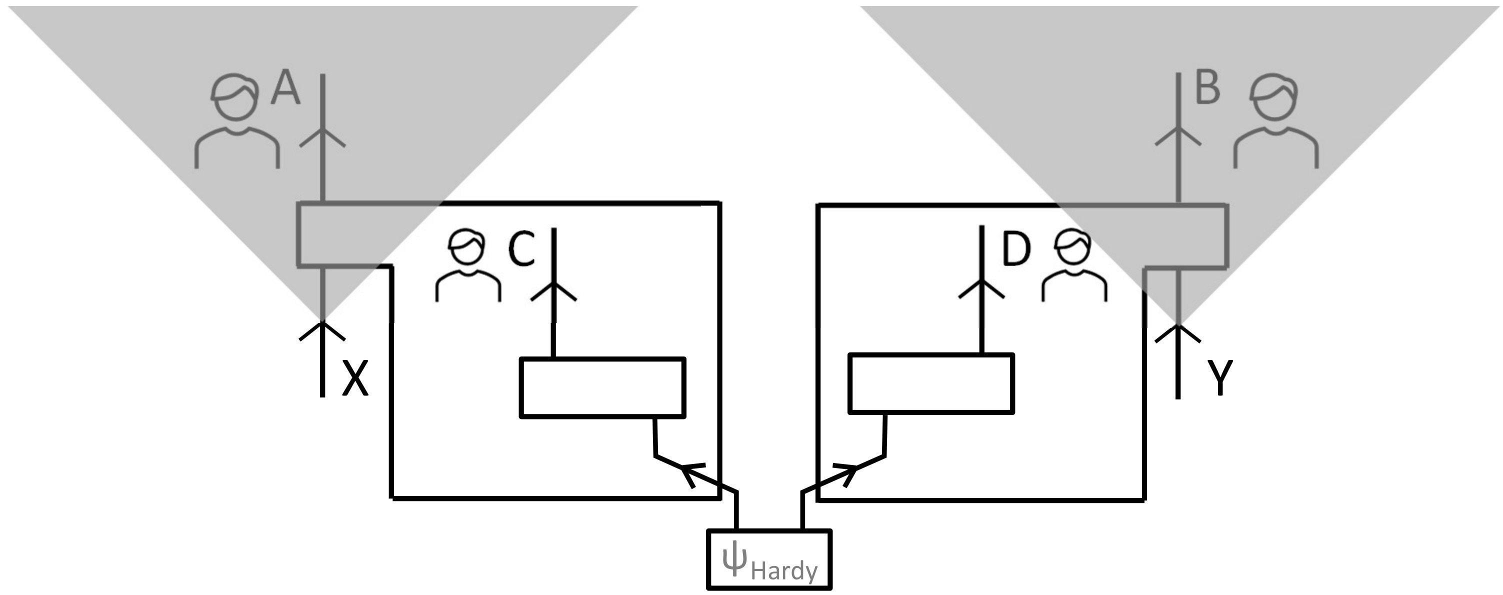
$p(c=1,d=1 \mid x=0,y=0) = 0$	no reversals
$p(c=0,b=- \mid x=0,y=1) = 0$	Bob reverses and measures +/-
$p(a=-,d=0 \mid x=1,y=0) = 0$	Alice reverses and measures +/-
$p(a=-,b=- \mid x=1,y=1) \neq 0$	both reverse and measure +/-

These are **observable** facts.
We can now convert these to facts about **only** runs where X=1, Y=1.



Local Agency: A freely chosen setting is uncorrelated with any observed events that are relevant to the phenomenon and outside the future light-cone of that setting.

x cannot be correlated with c, d, y, or b
y cannot be correlated with c, d, x, or a



quantum predictions

$$
\begin{aligned}
&p(c=1,d=1\,|\,x=0,y=0) = 0\\
&p(c=0,b=-\,|\,x=0,y=1) = 0\\
&p(a=-,d=0\,|\,x=1,y=0) = 0\\
&p(a=-,b=-\,|\,x=1,y=1) \neq 0
\end{aligned}
\quad \overset{\text{Local Agency}}{\Longrightarrow} \quad
\begin{aligned}
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&p(c=0,b=-\,|\,x=1,y=1) = 0\\
&p(a=-,d=0\,|\,x=1,y=1) = 0\\
&p(a=-,b=-\,|\,x=1,y=1) \neq 0
\end{aligned}
$$

$$a=- \;\Rightarrow\; d=1 \;\Rightarrow\; c=0 \;\Rightarrow\; b=+$$

CONTRADICTION

The Local Friendliness no-go theorem

Quantum predictions, in particular, the ones made with Universality of Unitary Dynamics (and the Born rule), are inconsistent with

- Local Agency:

Any setting is uncorrelated with any set of relevant ***observed*** *events outside its future light-cone.*

- It does not require the existence of hidden variables
- It reduces to no-signaling in the context of Bell's experiment
- Peres: "Unperf~~ormed experiments have no~~ results."

$$P(A|XY) = P(A|X)$$
$$P(B|XY) = P(B|Y)$$

- ? (an assumption arguably also implicitly used in Bell's theorem)

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$$P(A|XY) = P(A|X)$$
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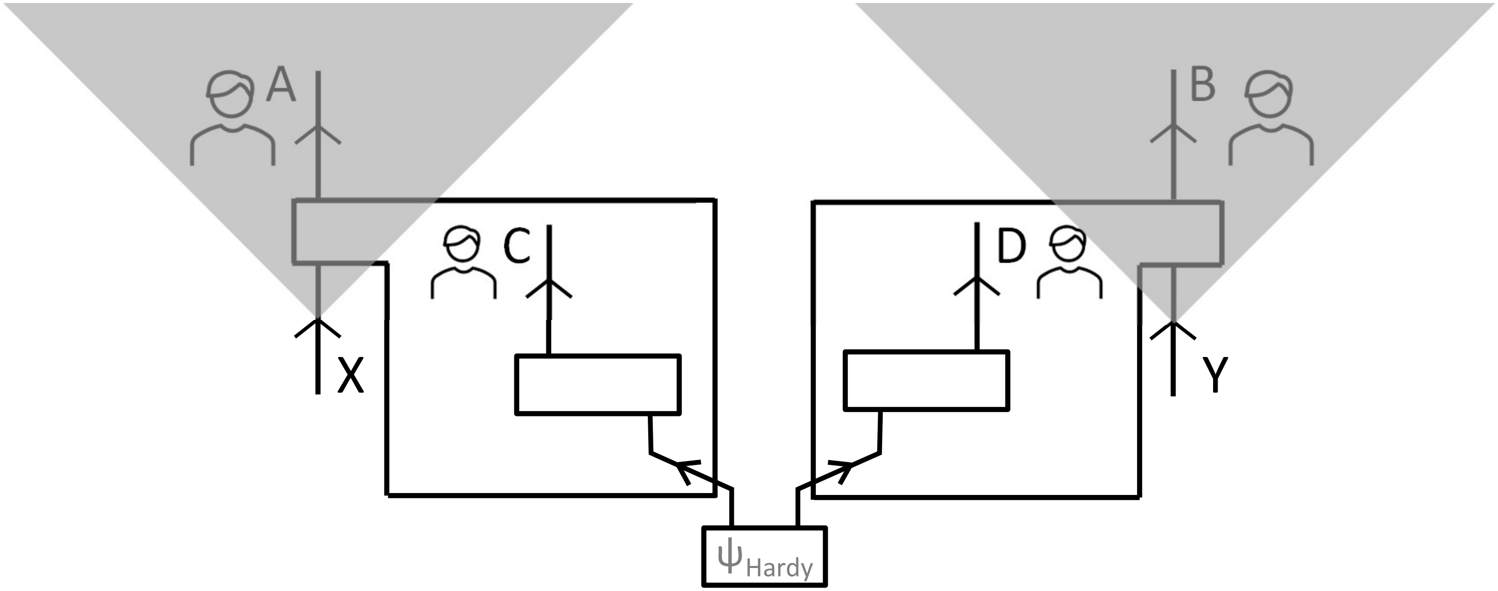
- Absoluteness of Observed Events (AOE):

Any observed event is single and absolute, not relative to anything or anyone.

- It is implicitly assumed in Bell's theorem
- It is rejected by interpretations such as Many Worlds, QBism, Relational Quantum Mechanics



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$$p(a=-,b=-\,|\,x=1,y=1) \neq 0$$

$$a=- \;\Rightarrow\; d=1 \;\Rightarrow\; c=0 \;\Rightarrow\; b=+$$

CONTRADICTION

Local Agency + Absoluteness of Observed Events

$\Rightarrow$ Local Friendliness inequalities

In general, they are less strict than Bell inequalities, since these assumptions are strictly weaker than the ones in Bell's theorem.

Local Friendliness
and
Causal Models

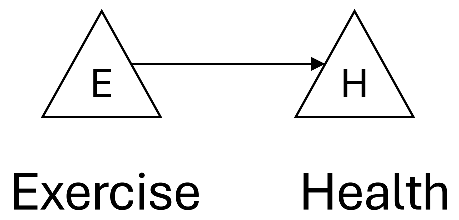
Yīng, Maciel Ansanelli, Di Biagio, Wolfe, Schmid, Cavalcanti, arXiv:2309.12987

Bell experiments
and
Classical causal models

Classical Causal Models

Causal structure:

- Directed Acyclic Graph (DAG)
  - Nodes: random variables
  - Arrows: cause-effect relations

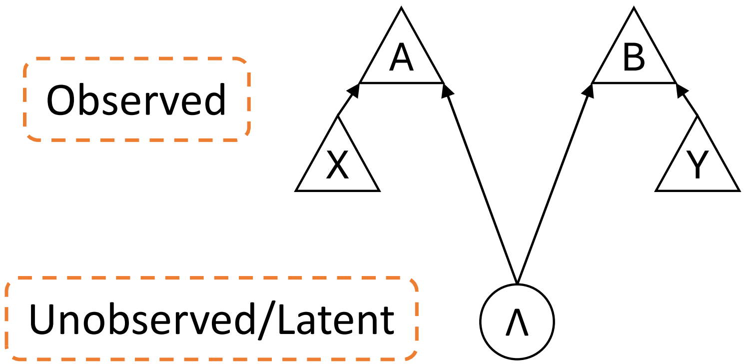


J. Pearl, *Causality: Models, Reasoning, and Inference,* Cambridge University Press, 2009

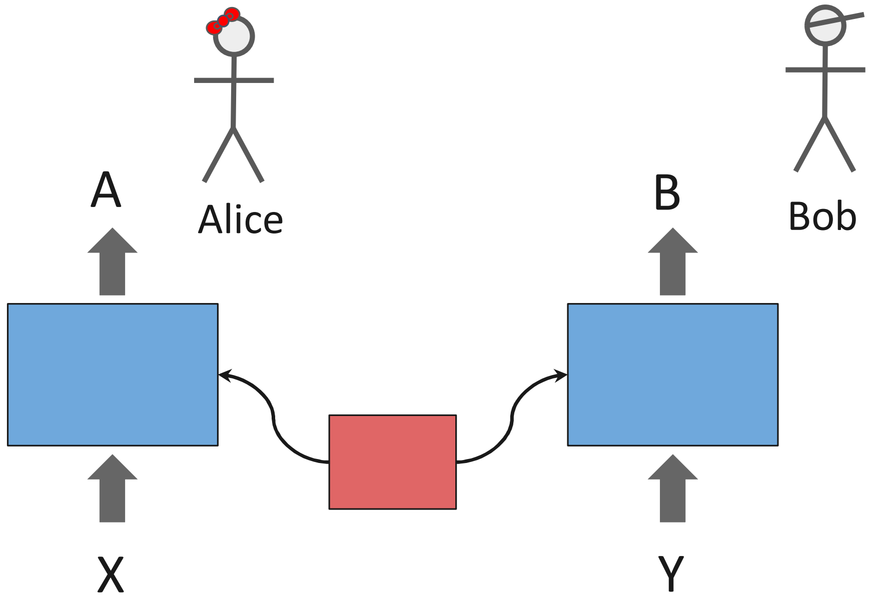
Classical Causal Models

Causal structure:

- Directed Acyclic Graph (DAG)



Classical variable



Classical Causal Models

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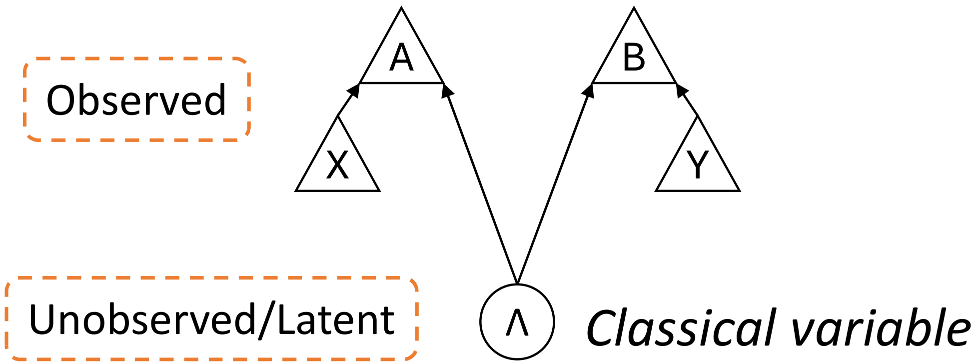
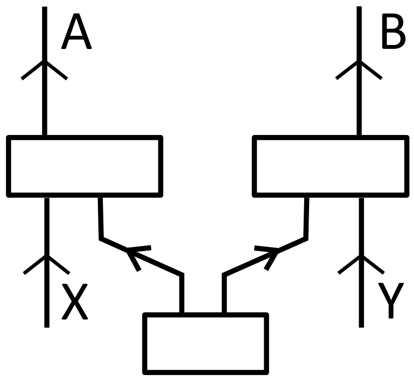


The Bell DAG



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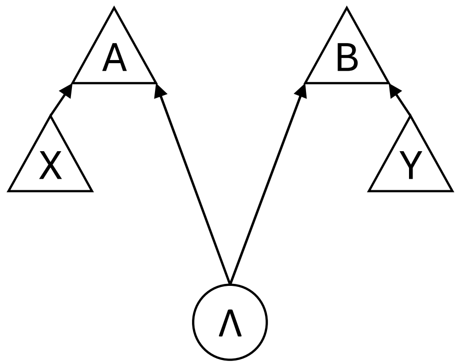


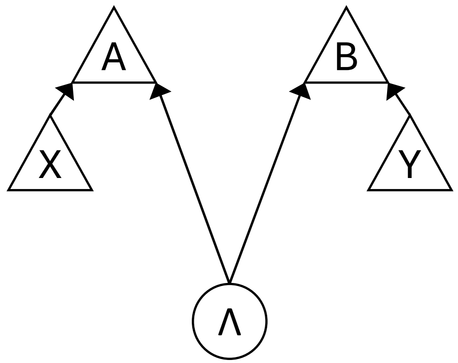
Compatible probabilities:

$$P(AB|XY) = \sum_\Lambda P(A|X\Lambda)P(B|Y\Lambda)P(\Lambda)$$

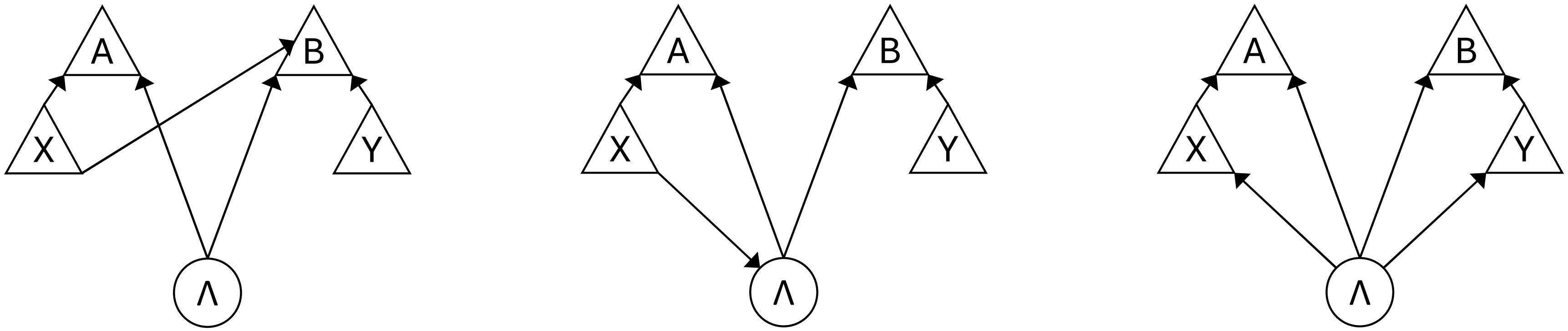
$$\left.\begin{array}{l} P(A|XY) = P(A|X) \\ P(B|XY) = P(B|Y) \end{array}\right\} \text{No signaling}$$

Bell inequalities on P(AB|XY)

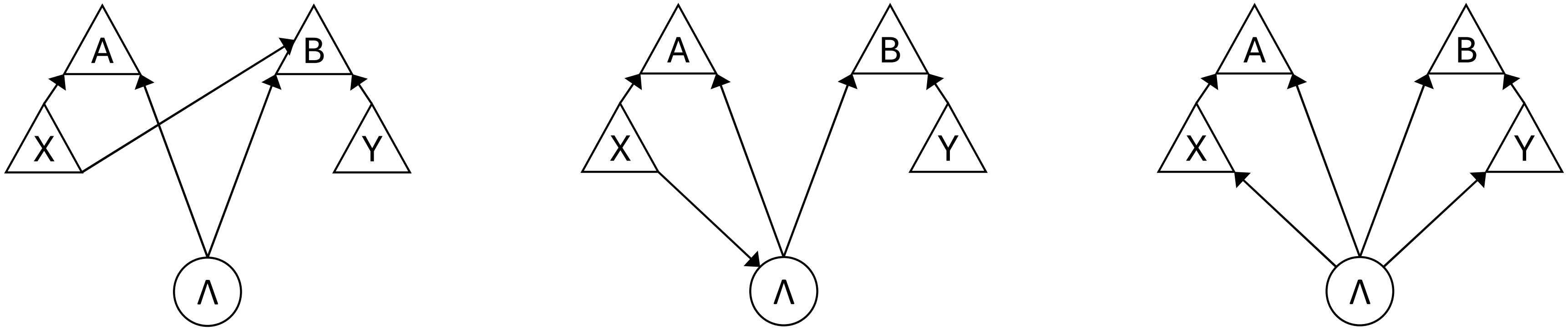
Any classical causal model with the Bell DAG cannot explain violations of Bell inequalities.



the Bell DAG



Superluminality Retrocausality Superdeterminism

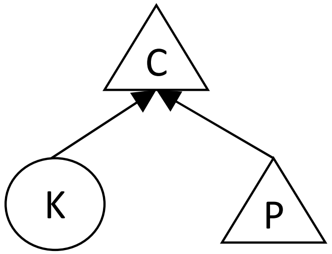


Need **fine-tuning** to explain no-signaling

$$P(A|XY) = P(A|X)$$

$$P(B|XY) = P(B|Y)$$

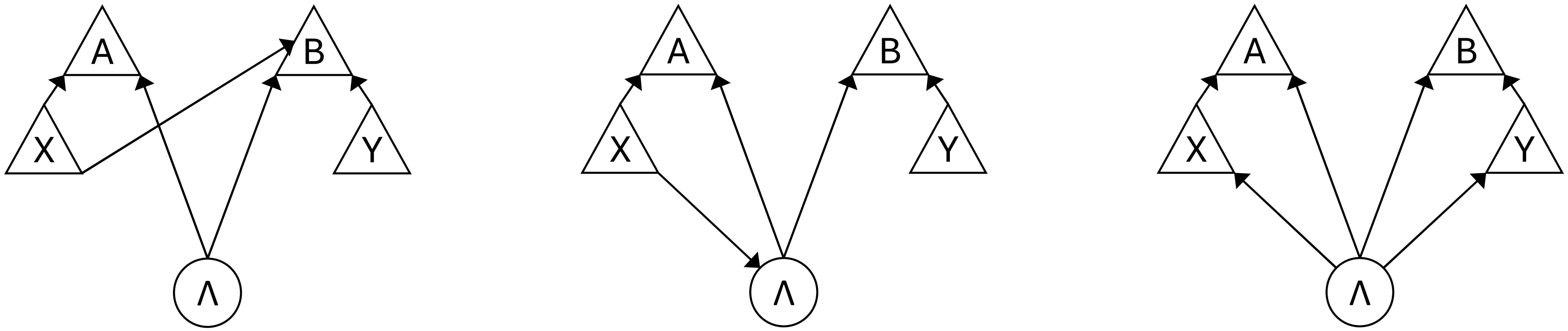
Fine-Tuning



P: Plain text
C: Cypher text
K: Key

P = 0 or 1
K = 0 or 1

$$C = K \oplus P \pmod{2}$$



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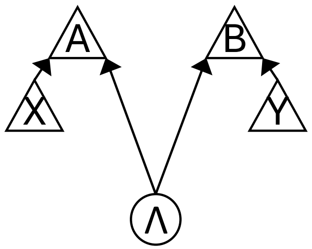
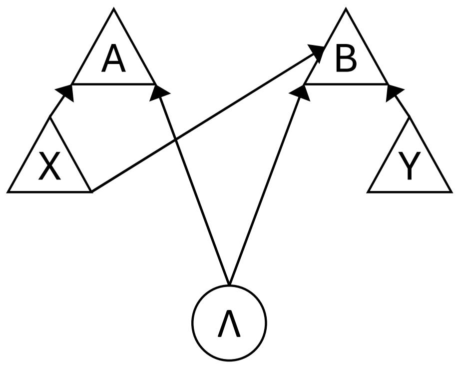


$$P(A|XY) = P(A|X)$$

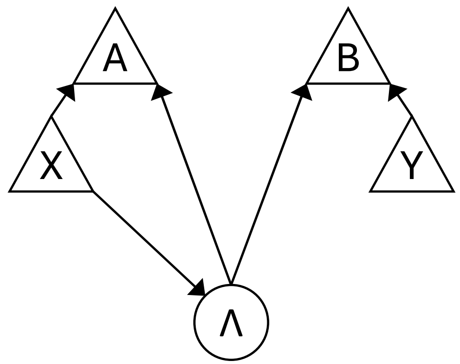
$$P(B|XY) = P(B|Y)$$

C. J. Wood and R. W. Spekkens, arXiv:1208.4119
E. G. Cavalcanti, arXiv:1705.05961

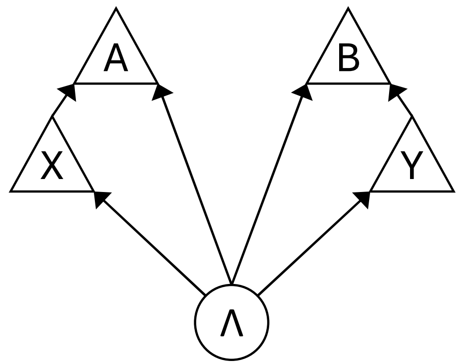
Problems with classical causal explanations



Superluminality



Retrocausality



Superdeterminism

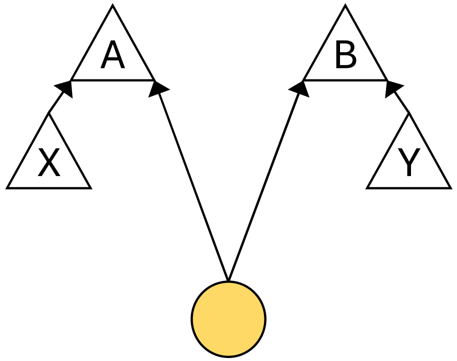
Need **fine-tuning** to explain no-signaling

Nonclassical causal models

Keep the causal structure intact
Generalize the notion of causality

Nonclassical Causal Models

Causal structure:



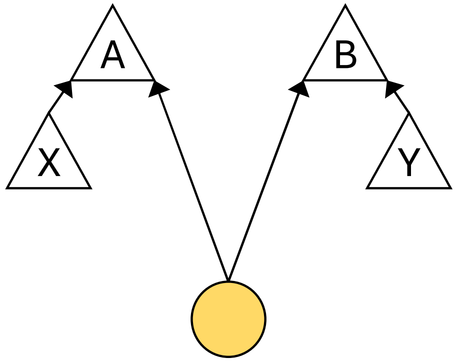
Compatible probabilities

$$P(AB|XY) = \sum_{\Lambda} \cancel{P}(A|X\Lambda)P(B|Y\Lambda)P(\Lambda)$$

Bell ineq~~u~~alities on P(AB|XY)

Quantum Causal Models

Causal structure:



Compatible probabilities

$$P(AB|XY) = \sum_\Lambda \cancel{P(A|X\Lambda)}P(B|Y\Lambda)P(\Lambda)$$

$$P(AB|XY) = \mathrm{Tr}\left[\rho_{A|X\Lambda}\,\rho_{B|Y\Lambda}\,\rho_\Lambda\right]$$

arXiv:1609.09487

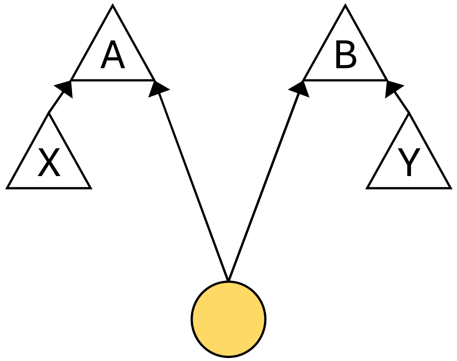
Quantum bound on P(AB|XY)

(e.g., Tsirelson's bound for CHSH)

Generalized Probabilistic Theory (can be classical, quantum, or beyond-quantum)

GPT Causal Models

Causal structure:



Compatible probabilities

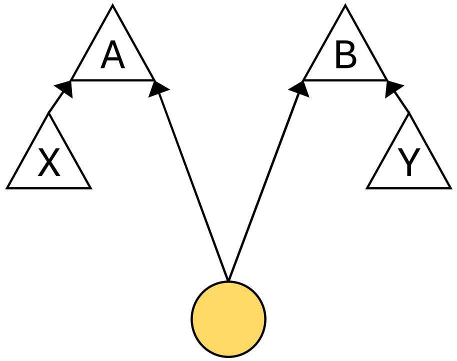
$$P(AB|XY) = \sum_\Lambda P(A|X\Lambda)P(B|Y\Lambda)P(\Lambda)$$

No-signaling bound on P(AB|XY)

Generalized Probabilistic Theory (can be classical, quantum, or beyond-quantum)

GPT Causal Models

Causal structure:



Compatible probabilities

Independence constraints on observed nodes

$$P(A|XY) = P(A|X)$$

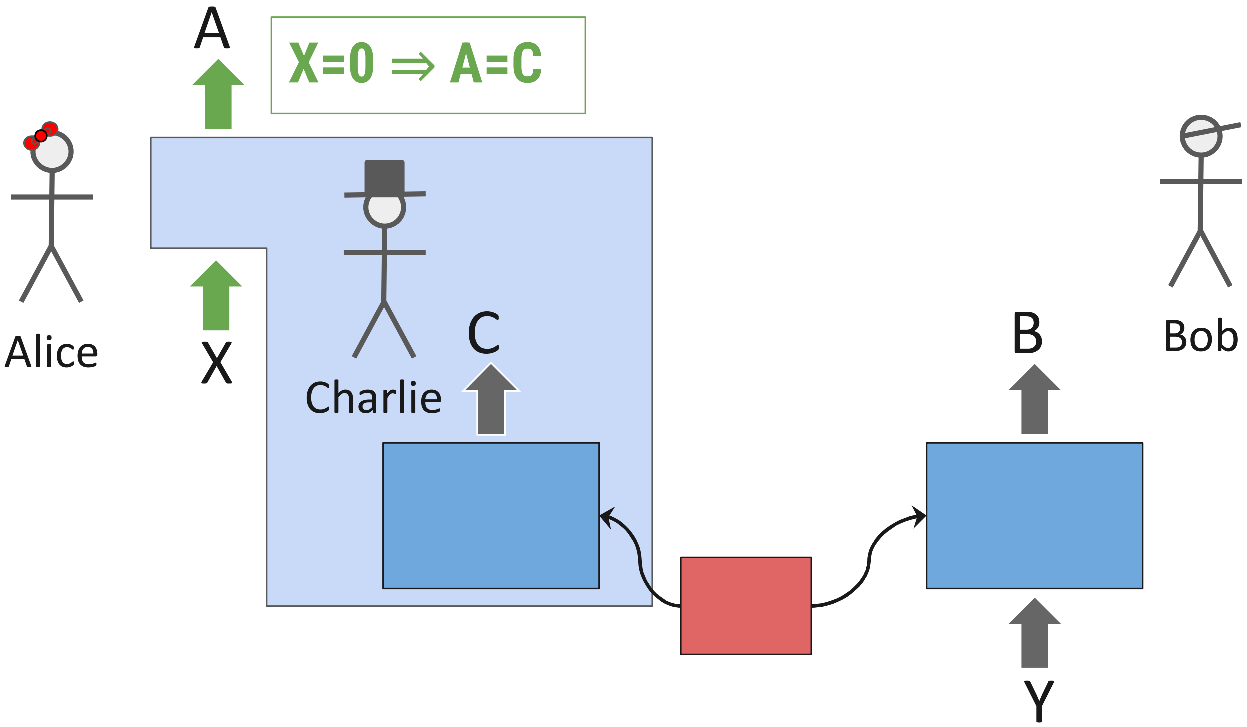
$$P(B|XY) = P(B|Y)$$

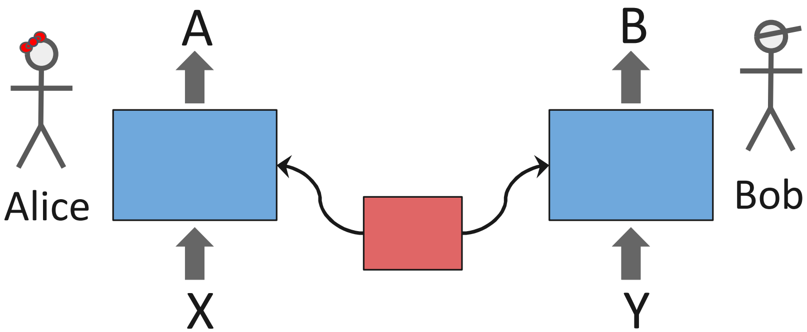
No-signaling bound on P(AB|XY)

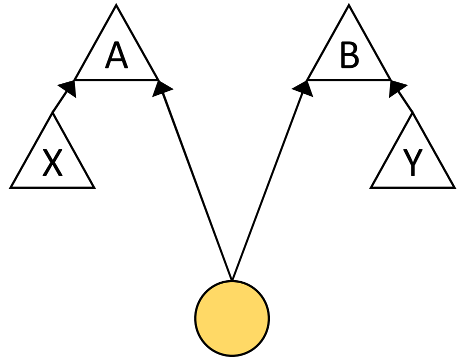
J. Henson, R. Lal, and M. F. Pusey [arXiv:1405.2572]

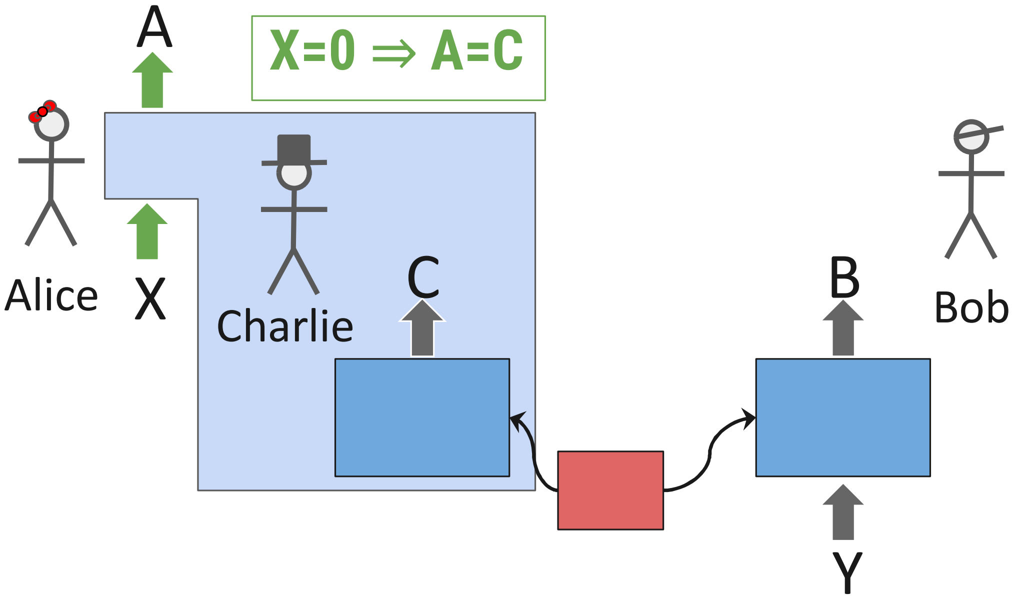
However…here comes Local Friendliness

Local Friendliness (LF) experiment
(the minimal version)

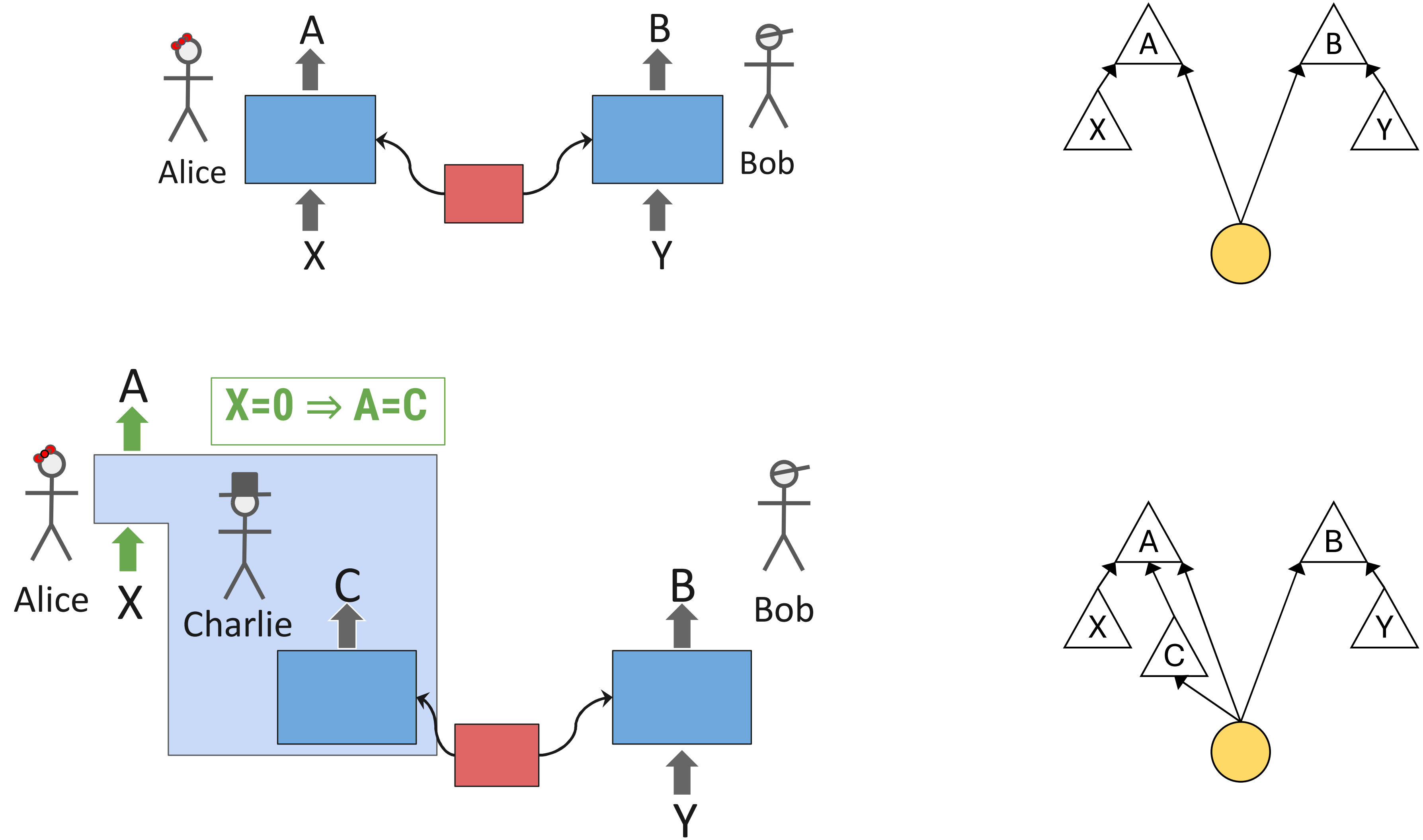



A
B
Alice
Bob
X
Y


A
B
X
Y


A
X=0 ⇒ A=C
Alice
X
Charlie
C
B
Bob
Y

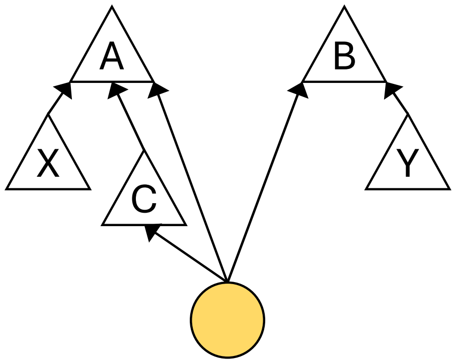

?



Alice
A
X
B
Y
Bob
A
B
X
Y
A
X=0 ⇒ A=C
Alice
X
Charlie
C
B
Bob
Y
A
B
X
C
Y

GPT Causal Models

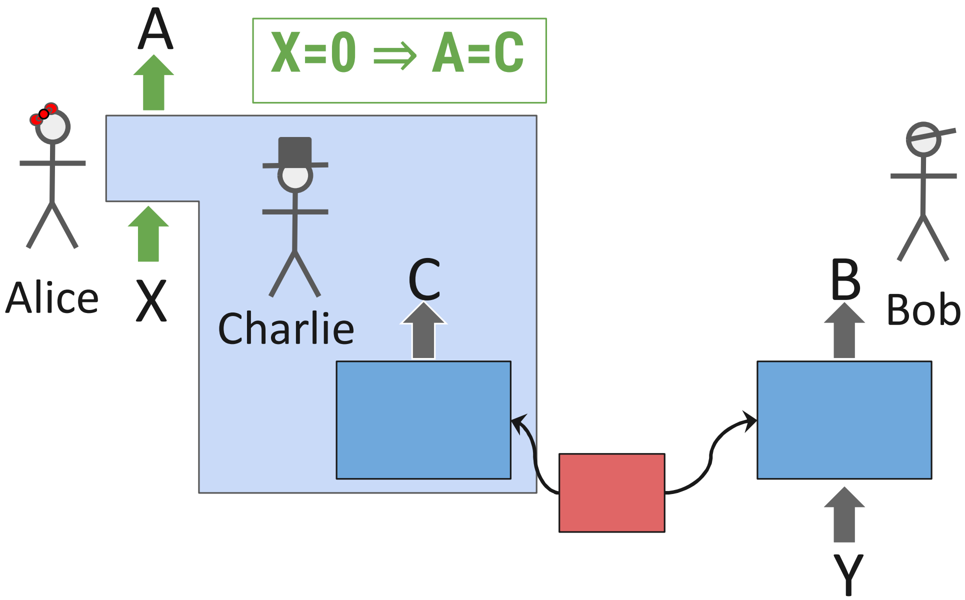
Causal structure:



Compatible probabilities

Monogamy relations on P(ABC|XY)!

P(AB|XY) and P(AC|XY) constrain each other

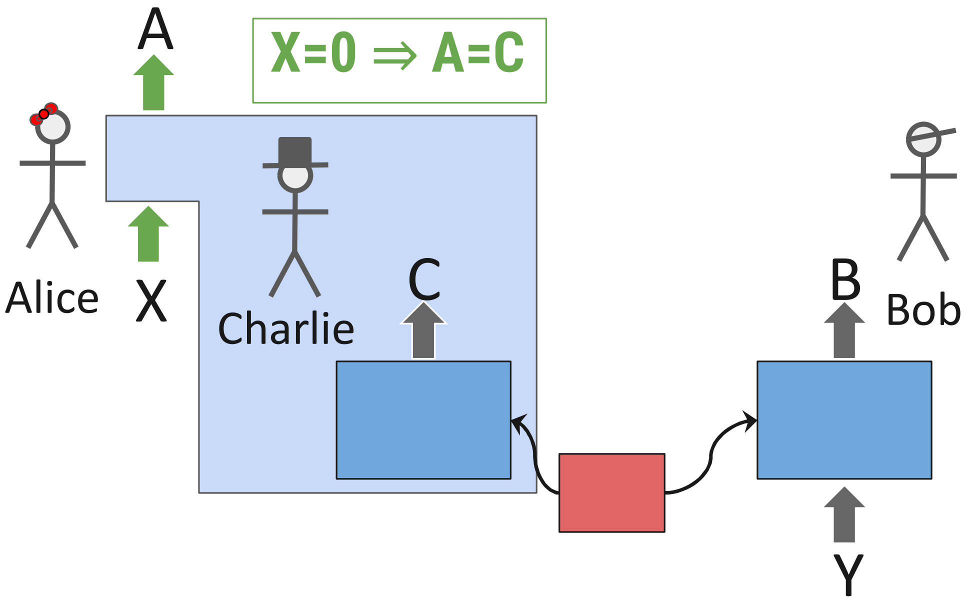


Monogamy relations!

P(AB|XY) and P(AC|XY) constrain each other

Local Friendliness (LF) inequalities on P(AB|XY)

weaker than Bell inequalities in general



Monogamy relations on P(ABC|XY)!

P(AB|XY) and P(AC|XY) constrain each other

Local Friendliness (LF) inequalities on P(AB|XY)

Monogamy relations

Example: binary settings and outcomes

$$\overset{\leq 1}{\mathrm{P(A+}}\ \frac{1}{2}P(A = \overset{\leq 1/2}{C|X} = 0) \leq \frac{5}{4}$$

$$P(A \oplus B = XY|XY) + \tfrac{1}{2}P(A = C|X = 0) \leq \tfrac{5}{4}$$

$$\tfrac{1}{4}\left[\textstyle\sum_{A=B} P(AB|00) + \sum_{A=B} P(AB|01) + \sum_{A=B} P(AB|10) + \sum_{A\neq B} P(AB|11)\right]$$

$$\boxed{\textbf{X=0} \Rightarrow \textbf{A=C} \Longrightarrow CHSH \leq \frac{3}{4}}$$

Local Friendliness (LF) inequality

In more general scenarios, LF inequalities are strictly weaker than Bell inequalities.

Any violations of LF inequalities are also violations of Bell inequalities, so are also nonclassical in the sense of contextuality (cf. David's lecture)

Monogamy relations

Example: binary settings and outcomes

$$\frac{1}{2}\overset{\leq 1/2}{P(A=C|X=0)} + \overset{\leq 1}{P(A \oplus B = XY)} \leq \frac{5}{4}$$

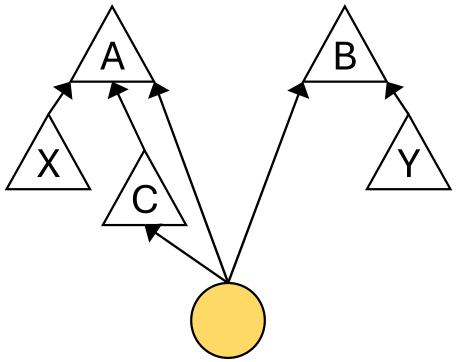
$$\boxed{X=0 \Rightarrow A=C} \quad \Rightarrow \quad P(A \oplus B = XY) \leq \frac{3}{4}$$

a Local Friendliness (LF) inequality

In more general scenarios, LF inequalities are strictly weaker than Bell inequalities, i.e., a LF polytopes can be a strict superset of a Bell polytope.

Any violations of LF inequalities are also violations of Bell inequalities, and are therefore Bell nonclassical as well as nonclassical in the sense of contextuality. (cf. David's lecture)

GPT Causal Models



Compatible probabilities

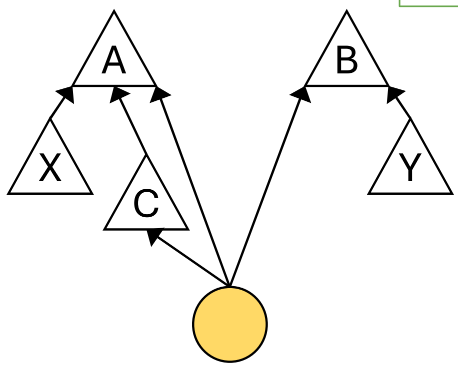
Monogamy relations on P(ABC|XY)!

$\boxed{X=0 \Rightarrow A=C}$

$\Rightarrow$ Local Friendliness (LF) inequalities on P(AB|XY)

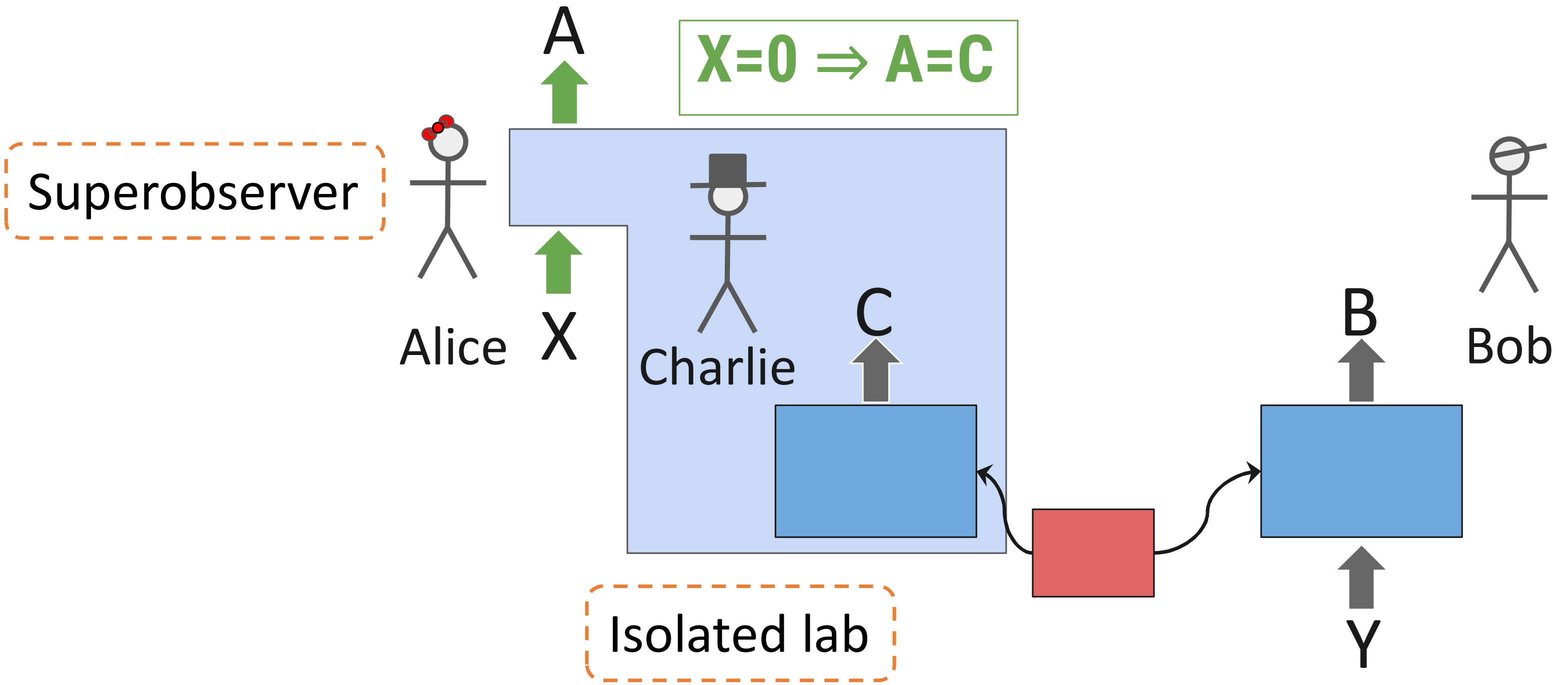
Any GPT causal model with the LF DAG cannot explain violations of Local Friendliness inequalities!

$\boxed{X=0 \Rightarrow A=C}$

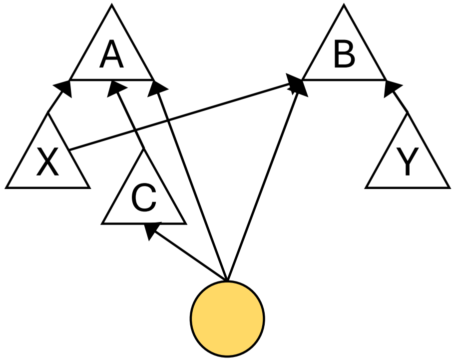


the LF DAG

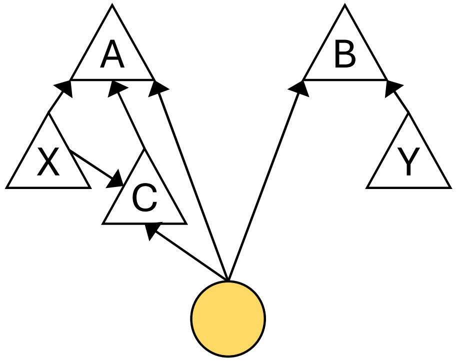
Quantum violations of LF inequalities



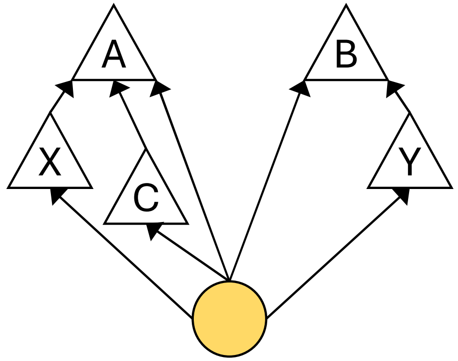
Wiseman, Cavalcanti and Rieffel, [arXiv:2209.08491]



Superluminality

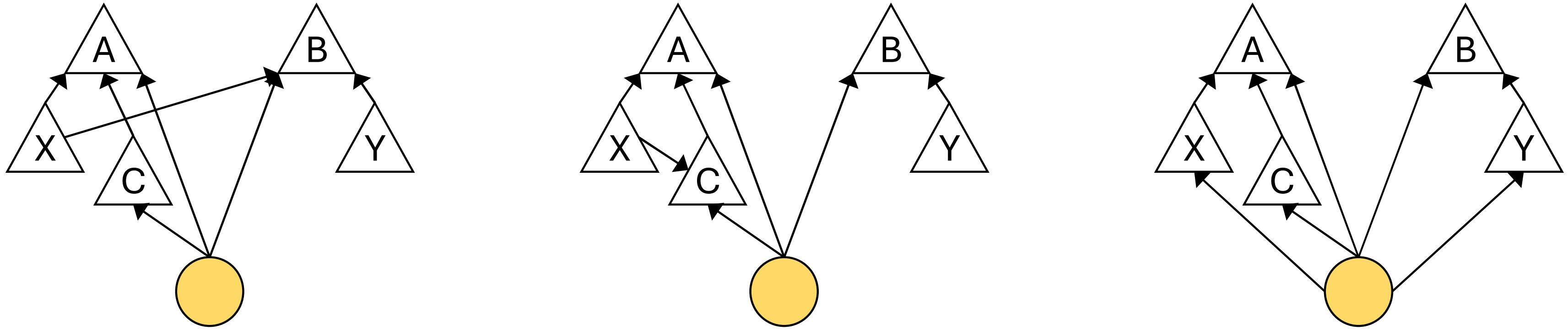


Retrocausality



Superdeterminism

Problems with GPT causal explanations



Need **fine-tuning** to explain no-superluminal/retro-signaling!

$$P(A|XY) = P(A|X)$$

$$P(B|XY) = P(B|Y)$$

$$P(C|XY) = P(C)$$

arXiv:2309.12987

We are sent back to the conundrum we had earlier!

We can explain Bell inequality violations by invoking a quantum common cause in the Bell DAG.

But ***NO*** quantum or GPT causal models with the LF DAG can explain any violation of LF inequalities.

Explaining by modifying the LF DAG ***always*** leads to contradictions with crucial causal principles.

The Local Friendliness experiment poses a much ***stronger*** *challenge to causal modeling than Bell's experiment.*

We are sent back to the conundrum we had earlier!

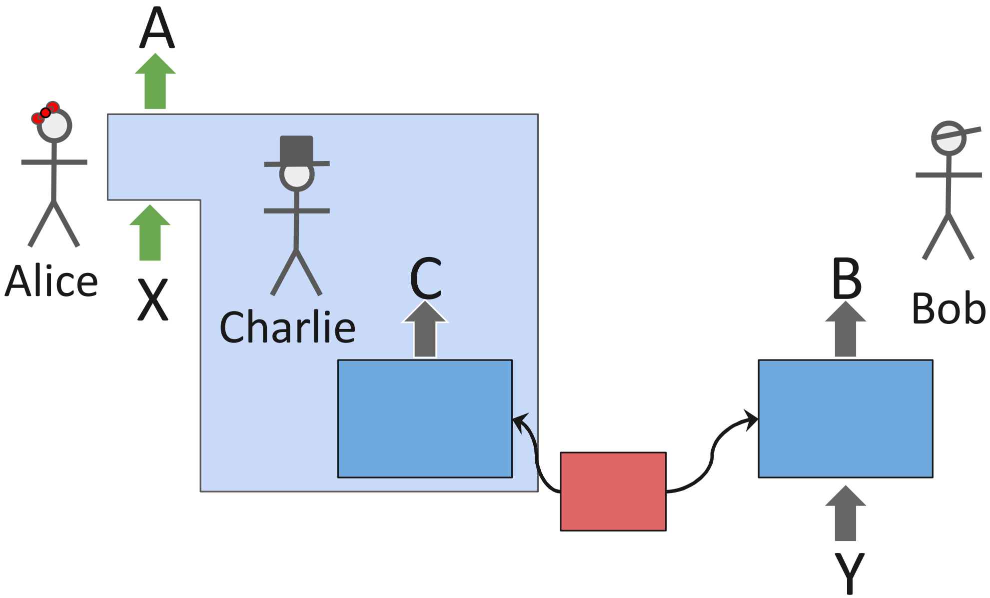
We can explain Bell inequality violations by invoking a quantum common cause in the Bell DAG.

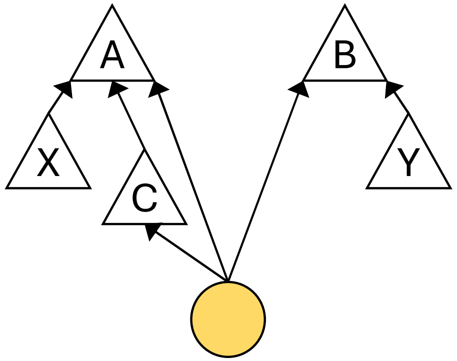
But ***NO*** quantum, GPT (or certain even-more-exotic) causal models with the LF DAG can explain any violation of LF inequalities.

Explaining by modifying the LF DAG (including making it cyclic) ***always*** leads to ***fine-tuning*** and **contradictions** with crucial causal principles.

*The Local Friendliness experiment poses a much **stronger** challenge to causal modeling than Bell's experiment.*

The existence of the joint distribution P(ABCXY)





the LF DAG

The existence of the joint distribution P(ABCXY)

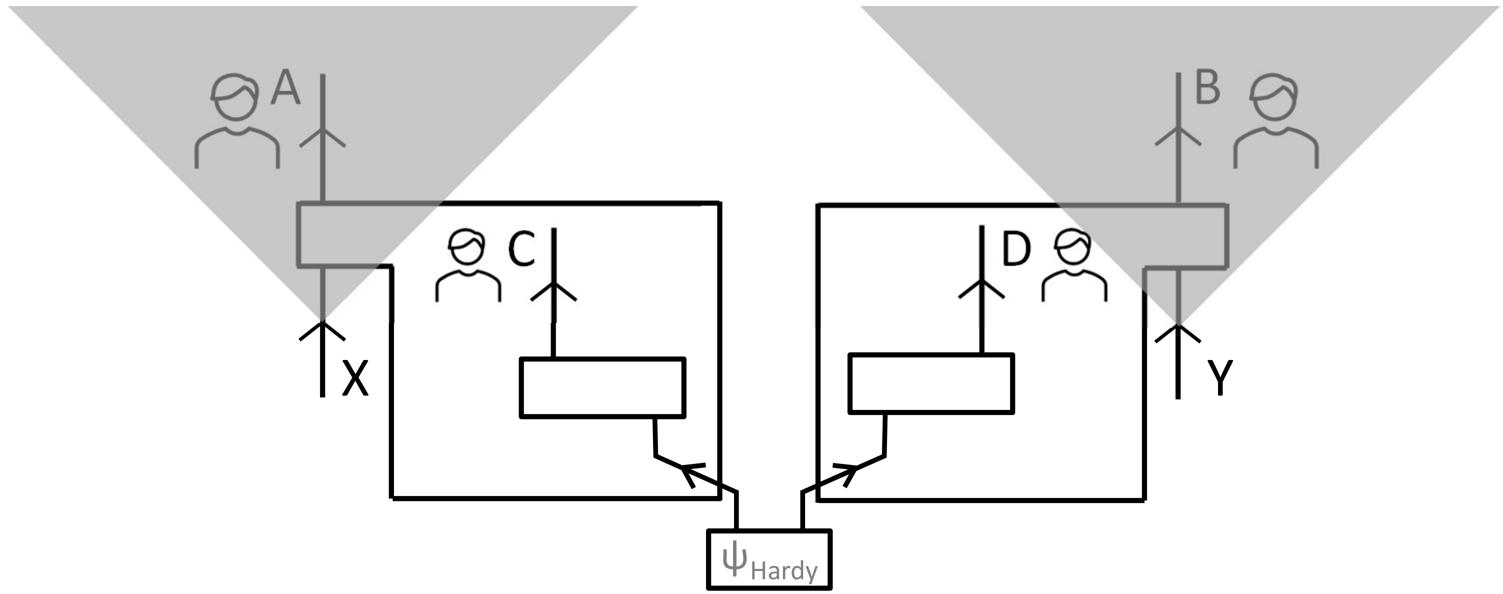
Absoluteness of Observed Events (AOE):

Any observed event is single and absolute; it is not relative to anything or anyone.

- Violated by Many-worlds, Qbism, Relational Quantum Mechanics, etc.

- Arguably implicitly assumed in Bell's and other theorems in physics, chemistry, psychology, biology, etc.

AOE ⇒ a random variable for Charlie's **outcome** (even if its **record** may be erased)



quantum predictions

$$p(c=1,d=1|x=0,y=0) = 0$$
$$p(c=0,b=-|x=0,y=1) = 0$$
$$p(a=-,d=0|x=1,y=0) = 0$$
$$p(a=-,b=-|x=1,y=1) \neq 0$$

Local Agency ====⇒

$$p(c=1,d=1|x=1,y=1) = 0$$
$$p(c=0,b=-|x=1,y=1) = 0$$
$$p(a=-,d=0|x=1,y=1) = 0$$
$$p(a=-,b=-|x=1,y=1) \neq 0$$

$$a=- \;\Rightarrow\; d=1 \;\Rightarrow\; c=0 \;\Rightarrow\; b=+$$

CONTRADICTION

The existence of the joint distribution P(ABCXY)

Absoluteness of Observed Events (AOE):

Any observed event is singleand absolute; it is not relative to anything or anyone.

→ a random variable for Charlie's **outcome** (even if its **record** may be erased)

- Arguably implicitly assumed in Bell's and other theorems in physics/biology…
- AOE **can** coexist with the assumption of universality unitary dynamics

  Unlike the collapse postulate, AOE is **not** about dynamics

  e.g., in the de Broglie-Bohm (pilot-wave) theory

  \- A key innovation of ***extended*** Wigner's friend theorems: no collapse postulate
- Rejected by Many-worlds, Relational Quantum Mechanics, QBism, etc.

Local Friendliness no-go theorems

Quantum predictions, in particular, the ones made with Universality of Unitary Dynamics (and the Born rule), are inconsistent with

(Experimental statistics are inconsistent with)

1. AOE + Local Agency, or

2. Nonclassical causal modeling framework + the LF DAG

no superluminal-causation

no retro-causation

no superdeterminism

(No Fine-Tuning)

arXiv:2309.12987

Thanks! :)

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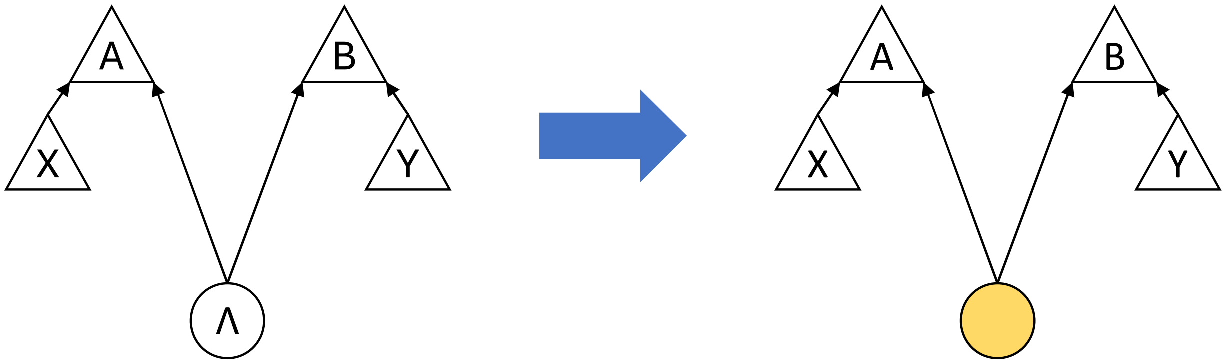
My thoughts?

Keep the causal structure intact
Update the notion of causality



My thoughts?

Keep the causal structure intact
Update the notion of causality and inference

Stay tuned ;)

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D. Schmid, J. H. Selby, and R. W. Spekkens, [arXiv:2009.03297]