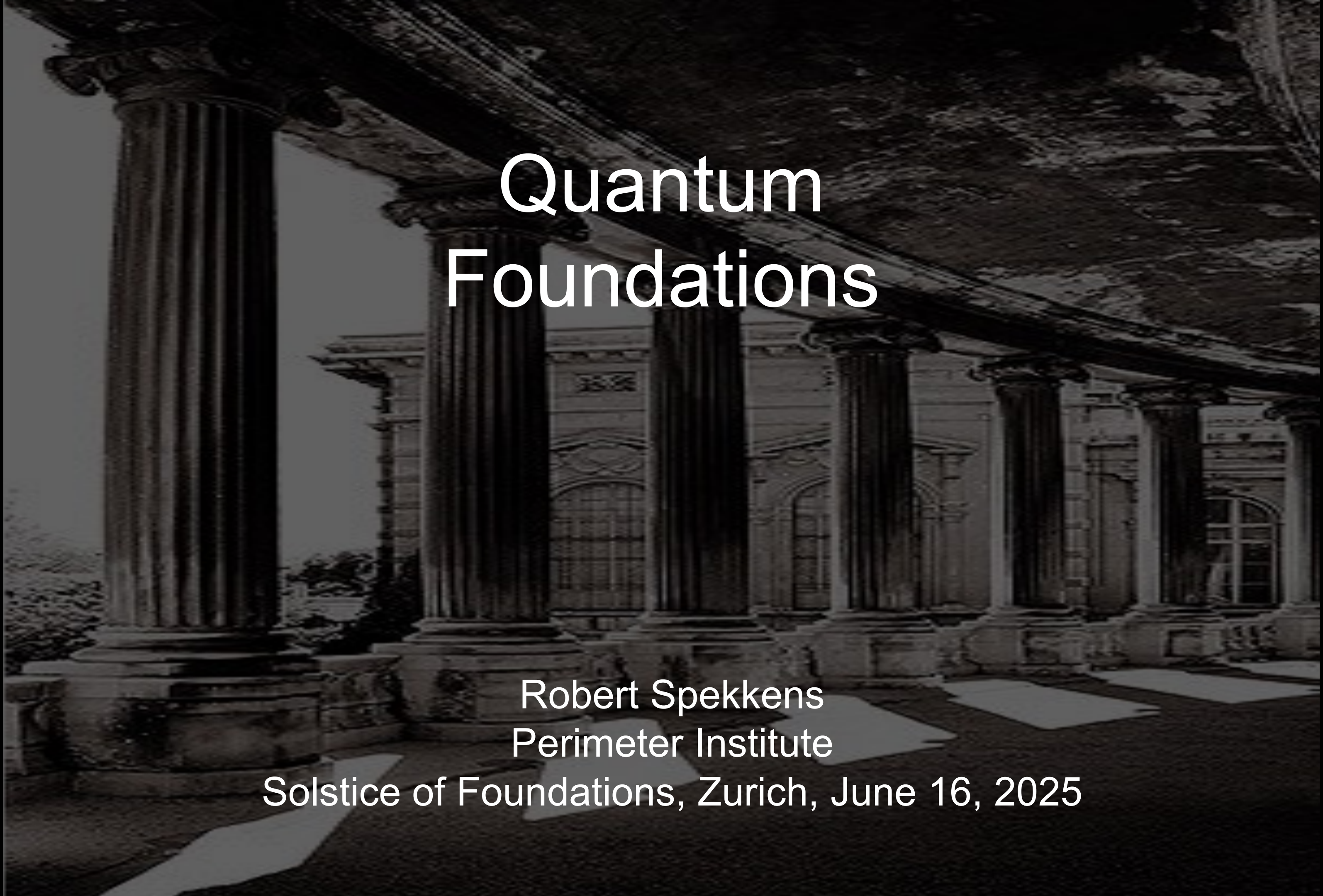


Quantum Foundations

Robert Spekkens
Perimeter Institute
Solstice of Foundations, Zurich, June 16, 2025

Scientists sometimes deceive themselves into thinking that philosophical ideas are only at best decorations or parasitic commentaries on the hard objective triumphs of science […]
But there is no such thing as philosophy-free science. There is only science whose philosophical baggage is taken on board without examination.

–Daniel C. Dennett

Interpretational commitments influence how one *applies* quantum theory and how one extends it into new domains

Quantum theory itself is evolving

---the quantum revolution is ongoing

The standard frameworks for describing quantum theory

Standard complex matrix or complex wavefunction representations

Schrodinger or Heisenberg pictures

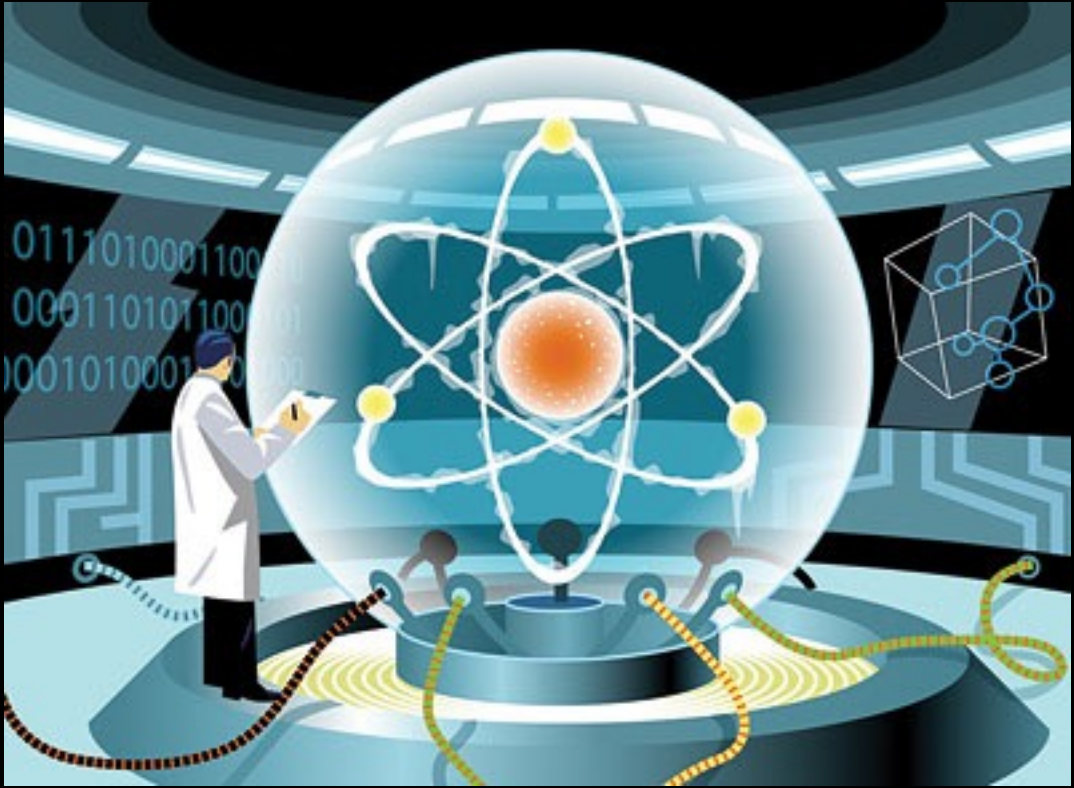
Path integral representation (of dynamics)

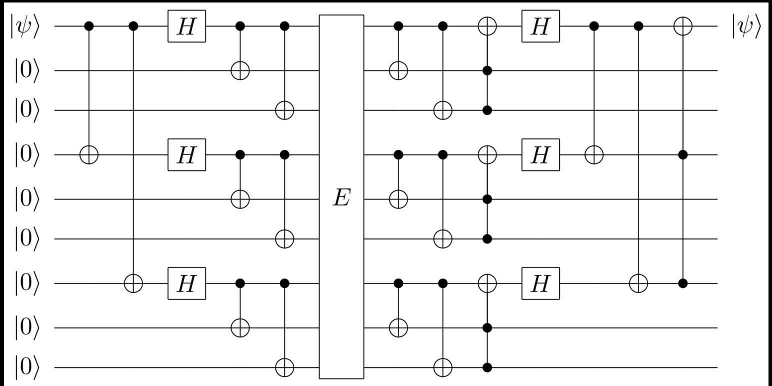
Real-valued vector representations

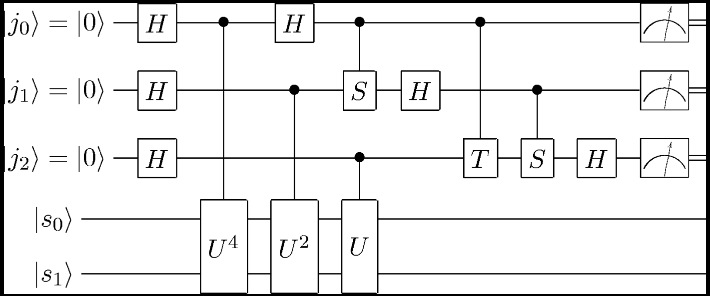
e.g., the Bloch sphere representation

Quasi-probability representations

e.g., the Wigner representation




$|\psi\rangle$
$|0\rangle$
H
E
$|\psi\rangle$


$|j_0\rangle = |0\rangle$
$|j_1\rangle = |0\rangle$
$|j_2\rangle = |0\rangle$
$|s_0\rangle$
$|s_1\rangle$
H
S
T
U^4
U^2
U

The framework of
Generalized Probabilistic Theories (GPTs)

Cases where there is controversy about how to apply quantum theory

Gravitational phenomena

Superselection rules

Indefinite causal structure

Models of computation

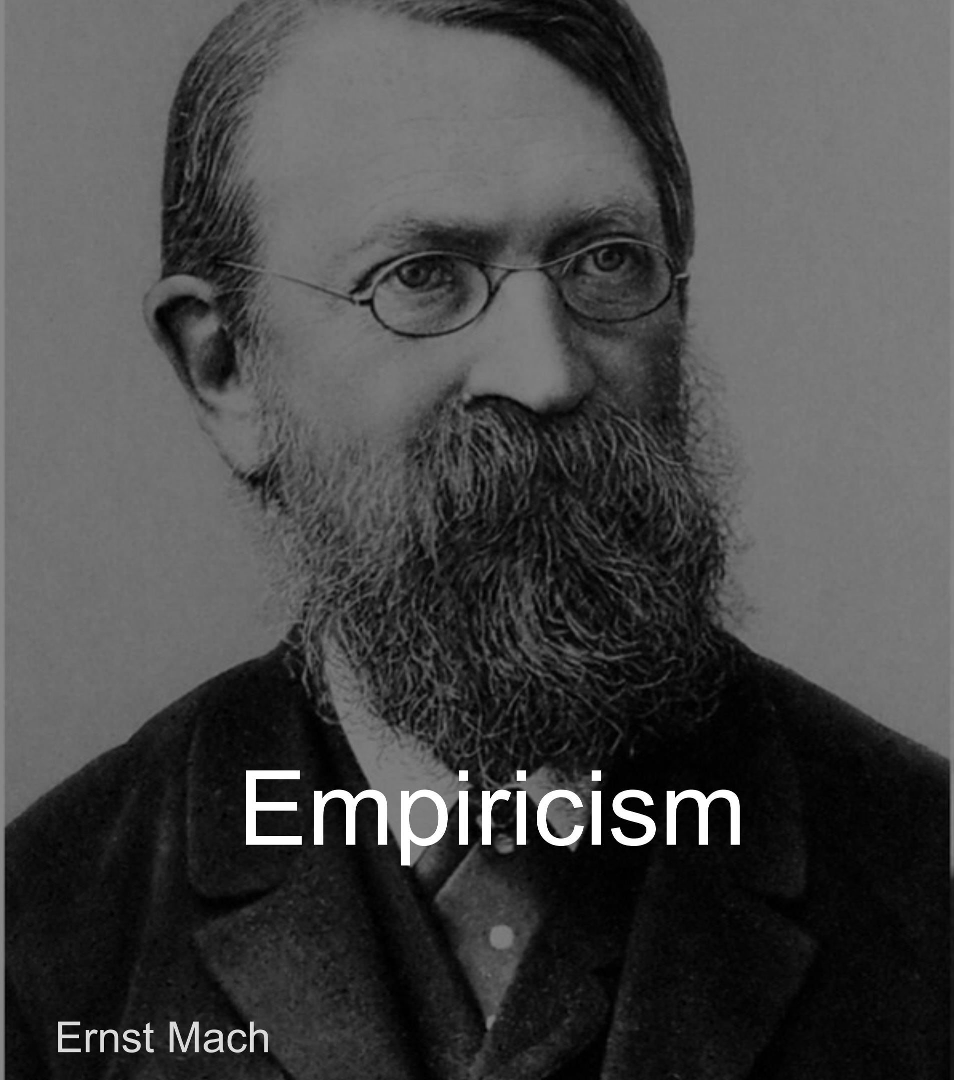
Theory of causal inference

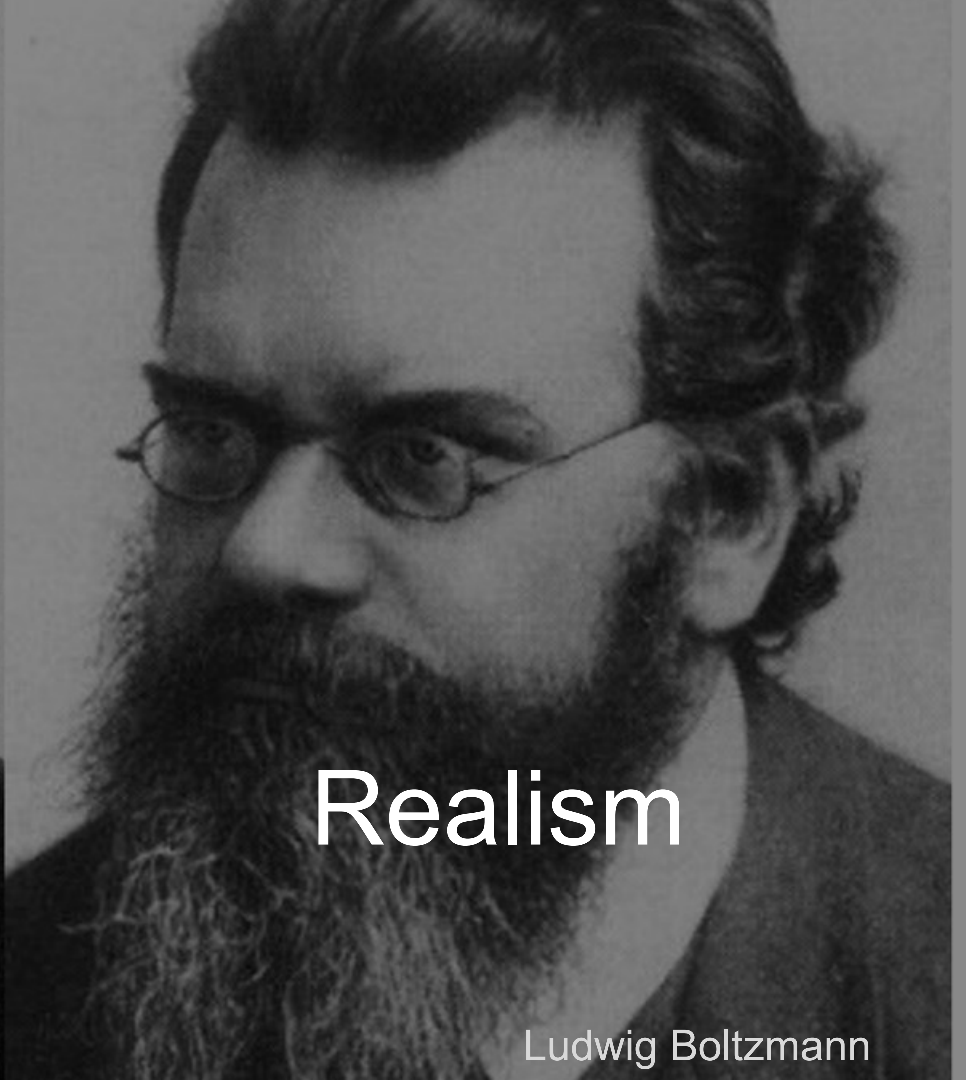
Algorithmic information theory

Machine learning

In particular, no agreement about how to define nonclassicality

A debate over foundational matters can often come from a disagreement about philosophical commitments


Empiricism
Ernst Mach


Realism
Ludwig Boltzmann

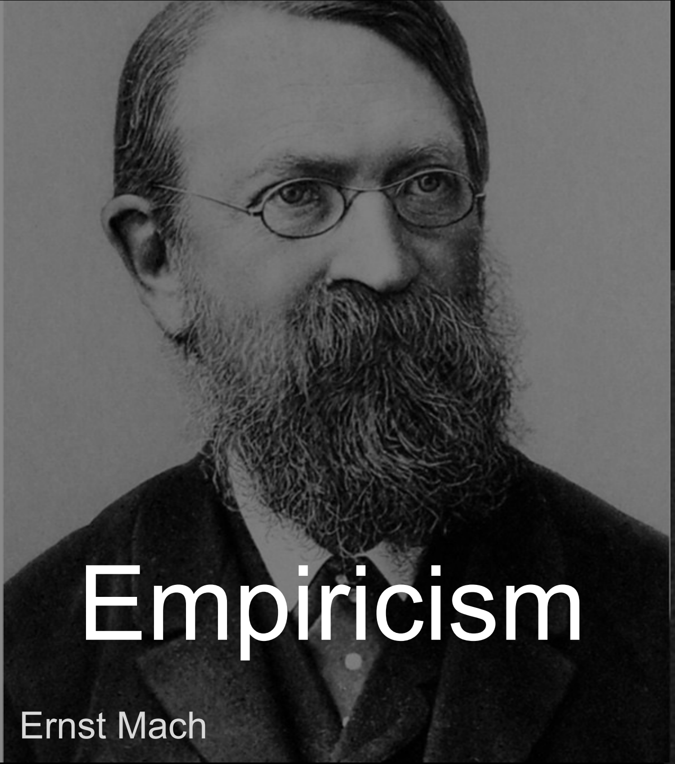
What does a scientific theory aim to do?

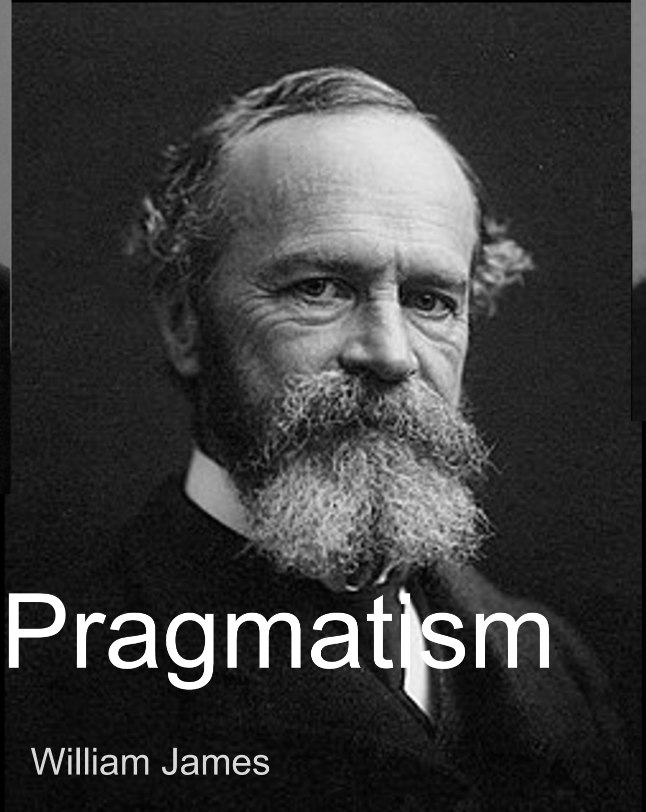
Realism

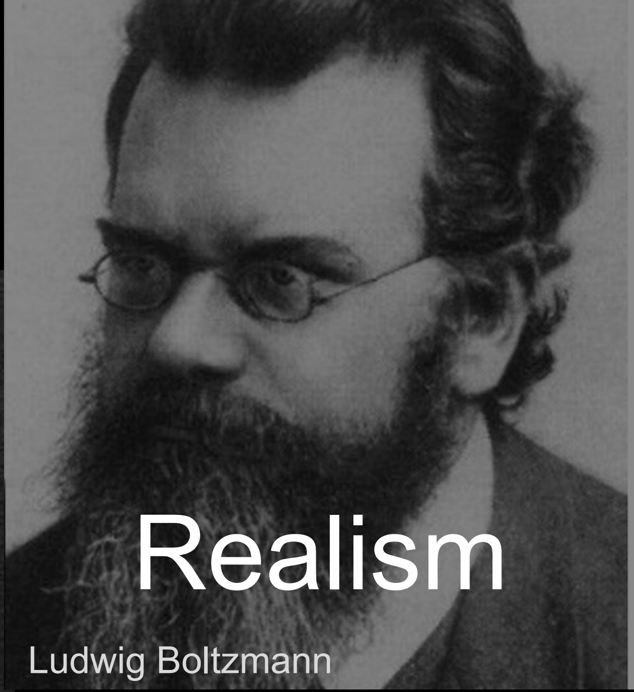
It aims at a true description of physical objects and their attributes, and it aims to provide successively better approximations to the truth over time.

Empiricism

It aims at an efficient summary of our experience. The empiricist seeks to avoid false belief by building on top of what we cannot be mistaken about, such as statements about what we've observed directly.


Empiricism
Ernst Mach

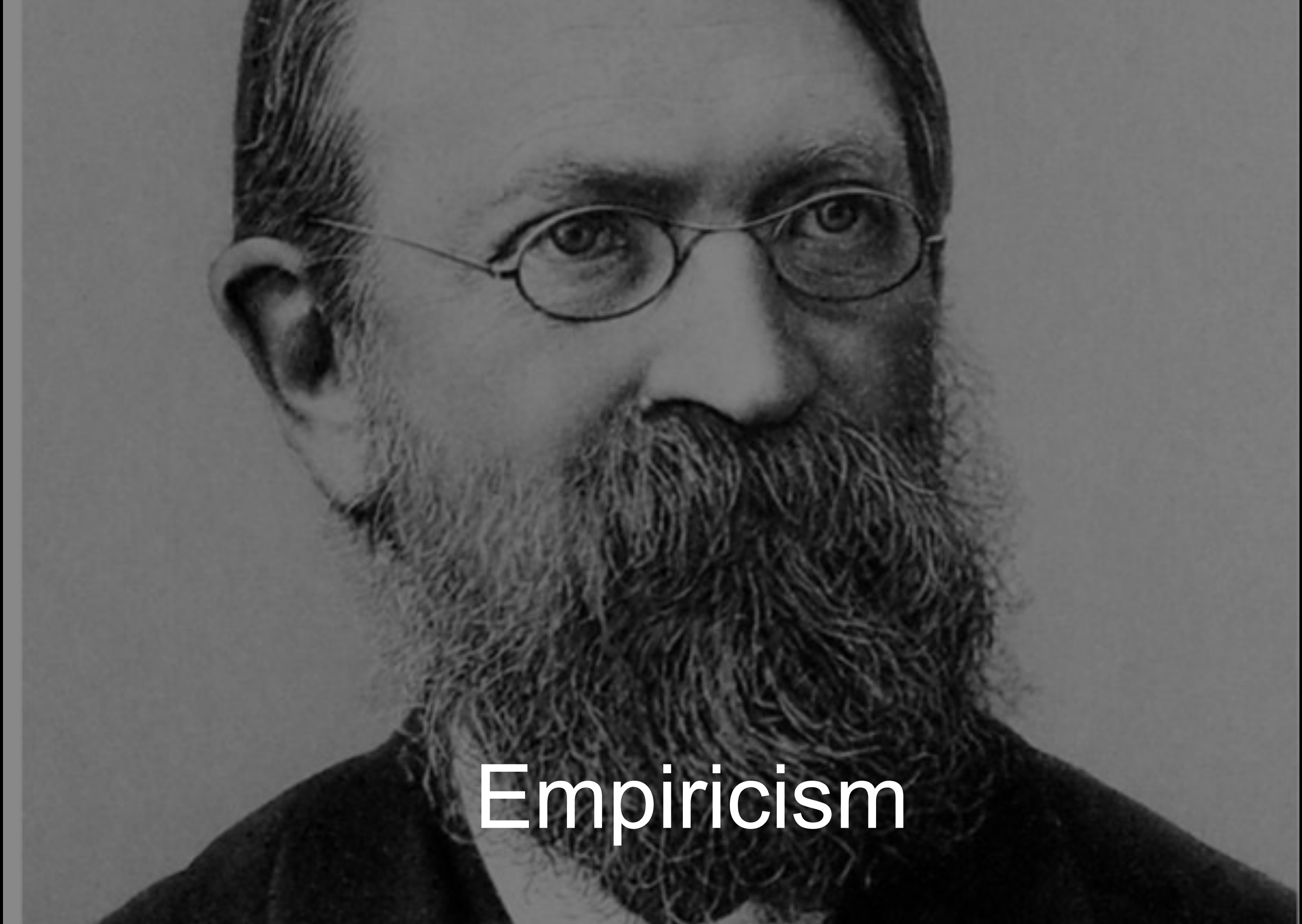

Pragmatism
William James


Realism
Ludwig Boltzmann

What does a scientific theory aim to do?

Pragmatism

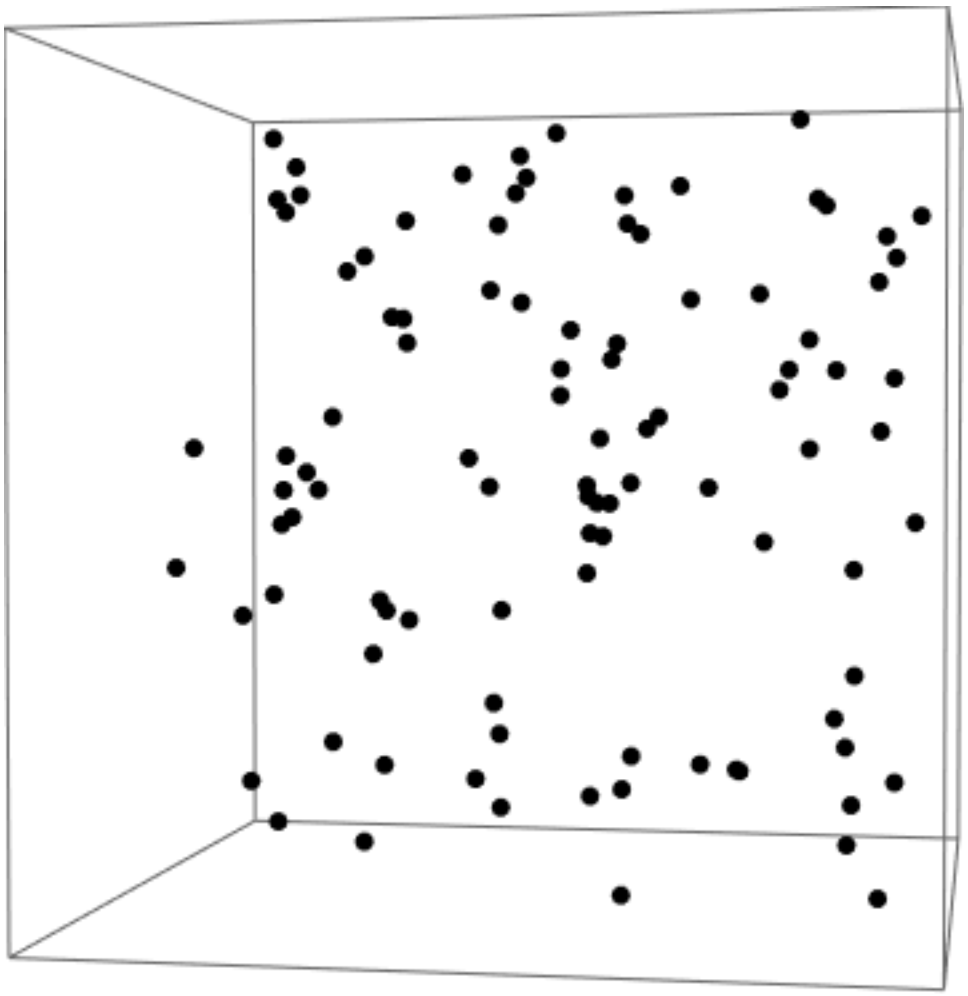
While realists and operationalists are generally committed to a correspondence theory of truth, the pragmatist drops this notion of truth altogether and suggests that a scientific theory aims only to be useful to us in achieving various goals.

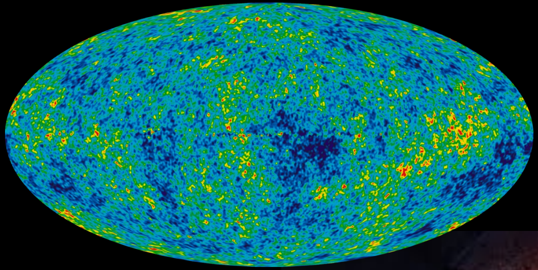


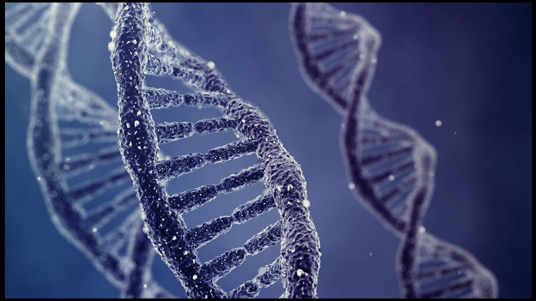
Empiricism

“In a strict sense, quantum theory is a set of rules allowing the computation of probabilities for the outcomes of tests which follow specified preparations.”

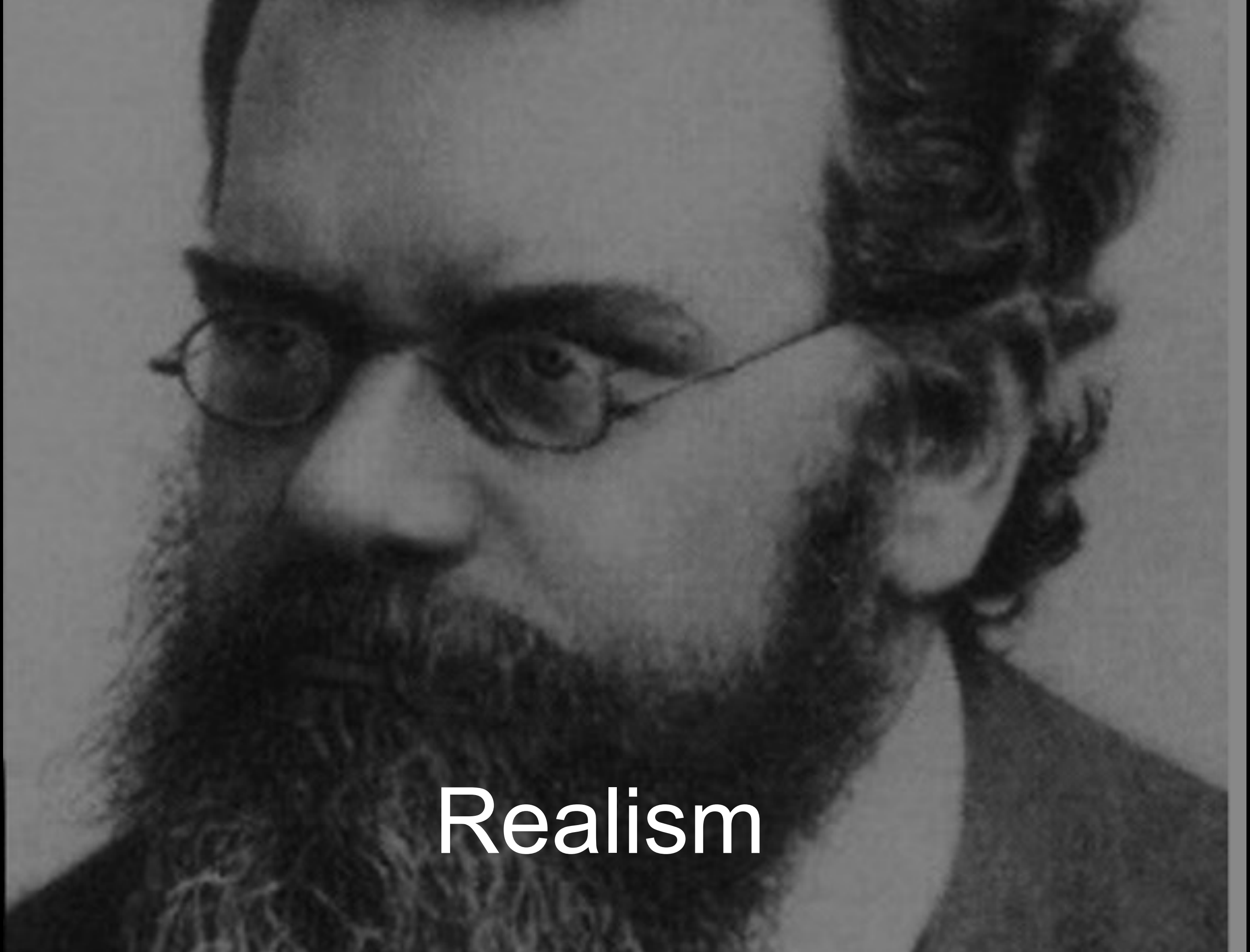
- Asher Peres







The Duhem-Quine thesis:
All observations are theory-laden

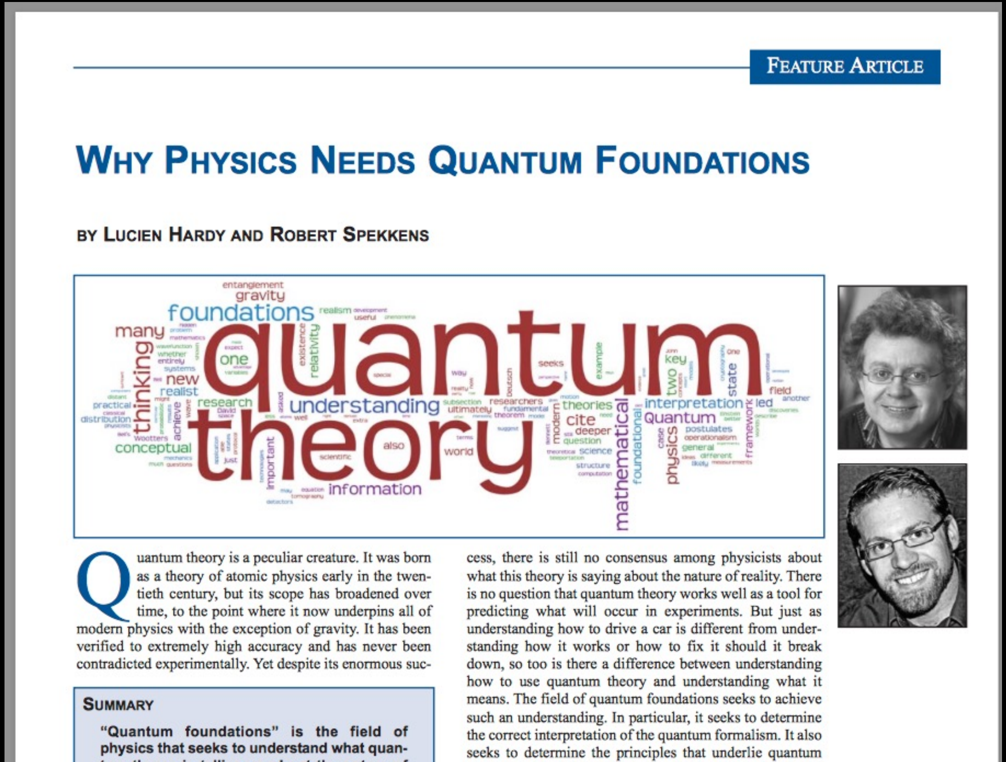

Realism

“It would seem that the theory is exclusively concerned about ’results of measurement’, and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? O r did it have to wait a little longer, for some better qualified system . . . with a PhD?”

- John Bell

Bohm → Bell → Ekert
→ device-independent key distribution (Barrett-Hardy-Kent)

Everett → Deutsch → quantum computation



FEATURE ARTICLE

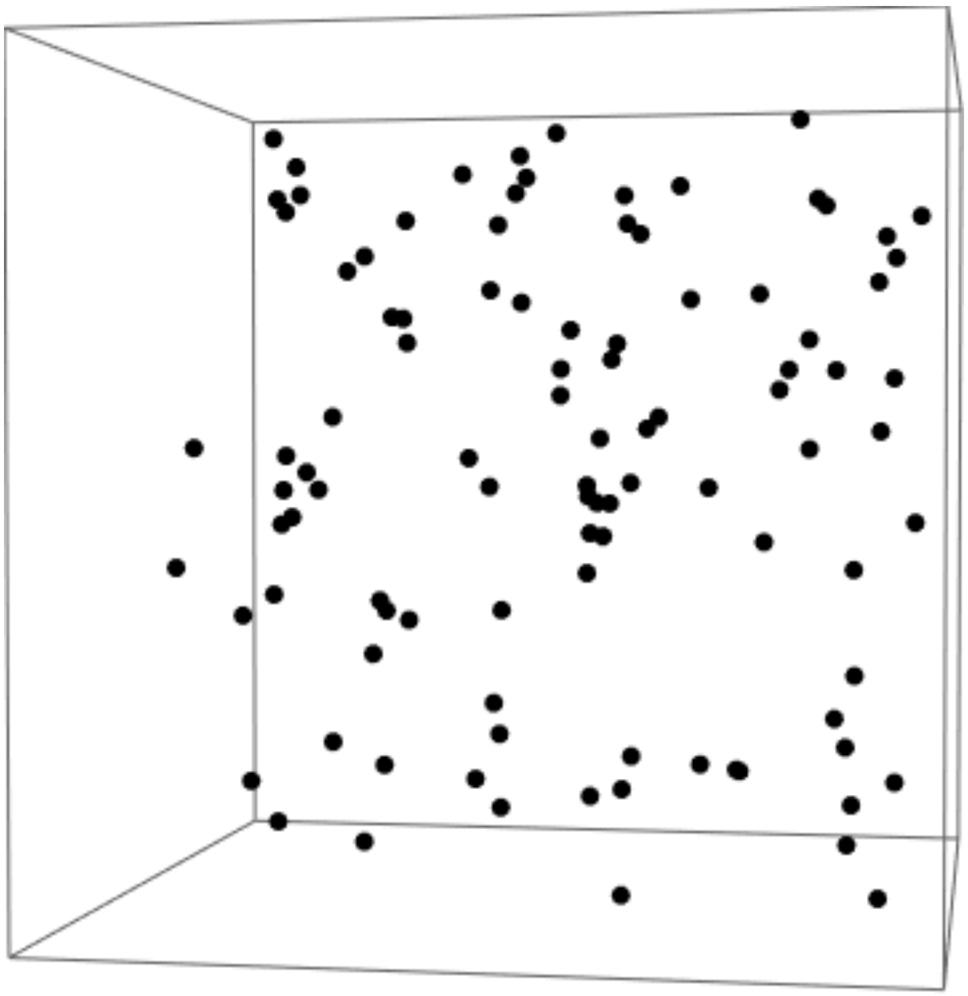
WHY PHYSICS NEEDS QUANTUM FOUNDATIONS

BY LUCIEN HARDY AND ROBERT SPEKKENS

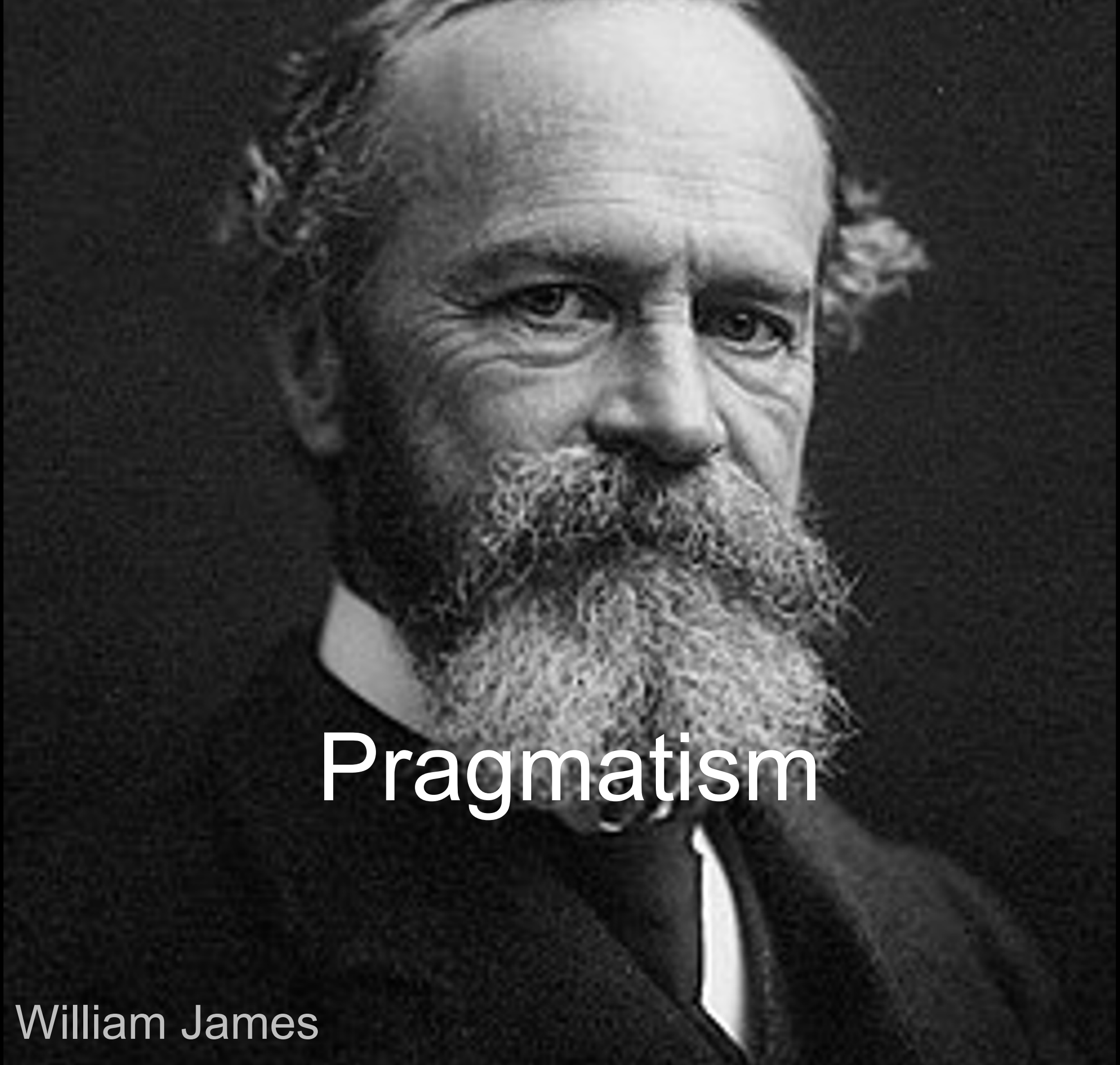
Quantum theory is a peculiar creature. It was born as a theory of atomic physics early in the twentieth century, but its scope has broadened over time, to the point where it now underpins all of modern physics with the exception of gravity. It has been verified to extremely high accuracy and has never been contradicted experimentally. Yet despite its enormous success, there is still no consensus among physicists about what this theory is saying about the nature of reality. There is no question that quantum theory works well as a tool for predicting what will occur in experiments. But just as understanding how to drive a car is different from understanding how it works or how to fix it should it break down, so too is there a difference between understanding how to use quantum theory and understanding what it means. The field of quantum foundations seeks to achieve such an understanding. In particular, it seeks to determine the correct interpretation of the quantum formalism. It also seeks to determine the principles that underlie quantum

SUMMARY

"Quantum foundations" is the field of physics that seeks to understand what quan-



Pragmatism

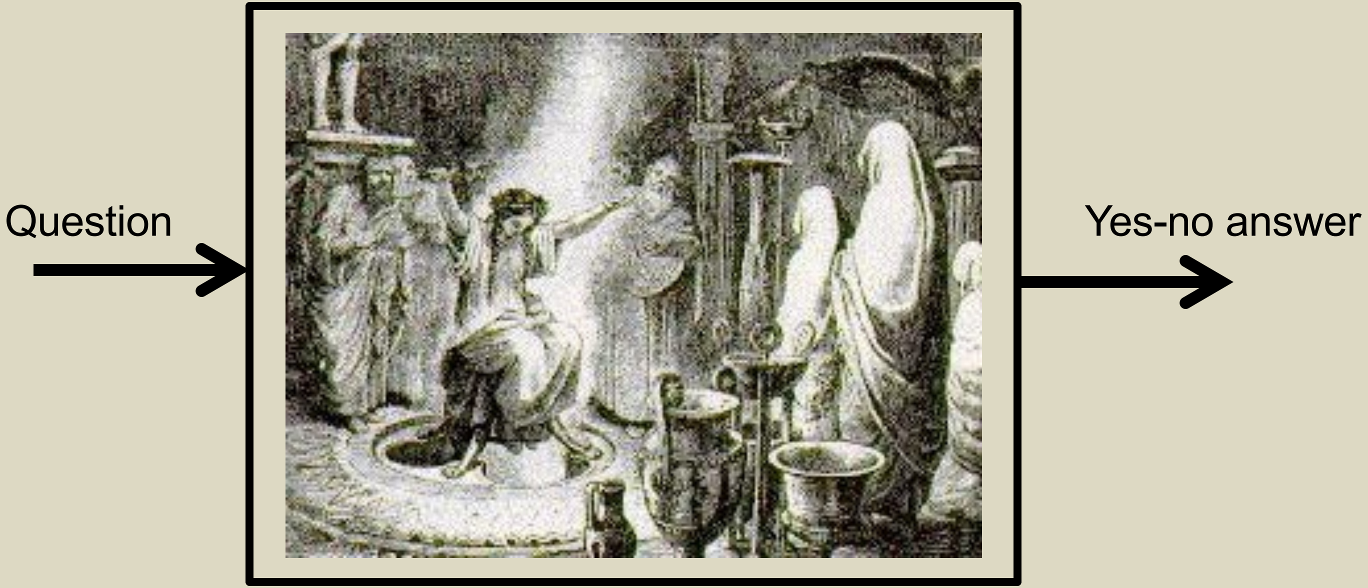


William James

Empiricist vs pragmatist traditions in physics

Empiricist: the physicist's job is to make predictions about what will be observed in well-described experimental scenarios.

Pragmatist: we want more than prediction, we want to be able to achieve our goals


Question
Yes-no answer

Realist vs pragmatist traditions in physics

Realist: the physicist’s job is to describe the natural dynamical behaviour of a system, without reference to agents or their purposes

Pragmatist: the laws of physics can be characterized in terms of the extent to which agents can achieve various goals within a universe obeying these laws

Cases where there is controversy about how to apply quantum theory

Gravitational phenomena

Superselection rules

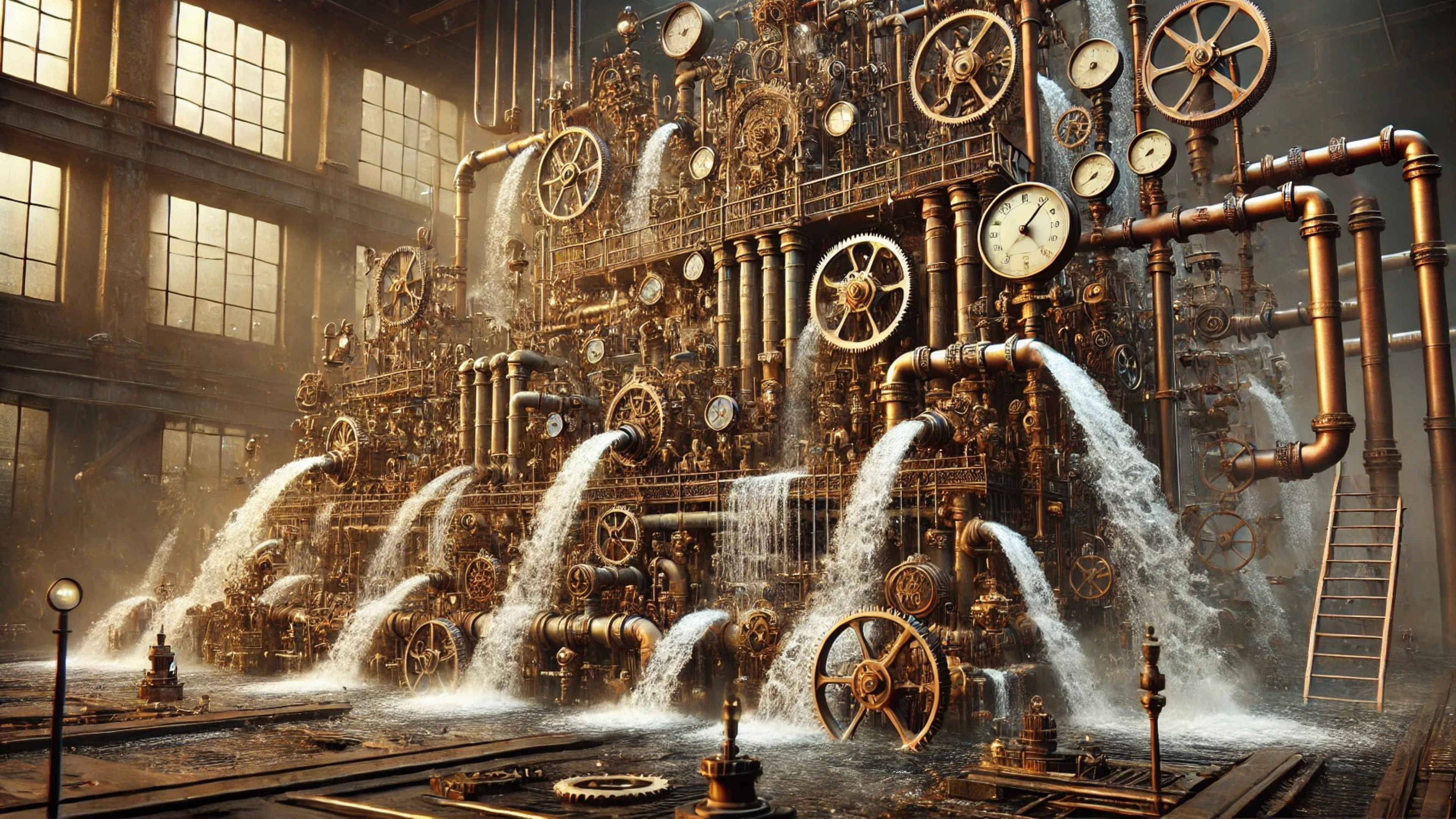
Indefinite causal structure

Models of computation

Theory of causal inference

Algorithmic information theory

Machine learning



The Kelvin-Planck statement of the second law of thermodynamics

It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work

Cases where there is controversy about how to apply quantum theory

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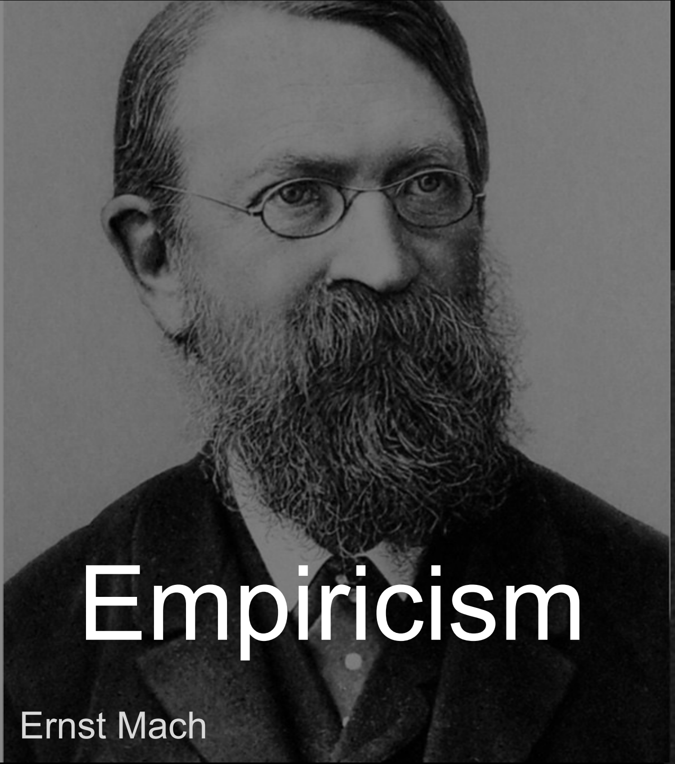
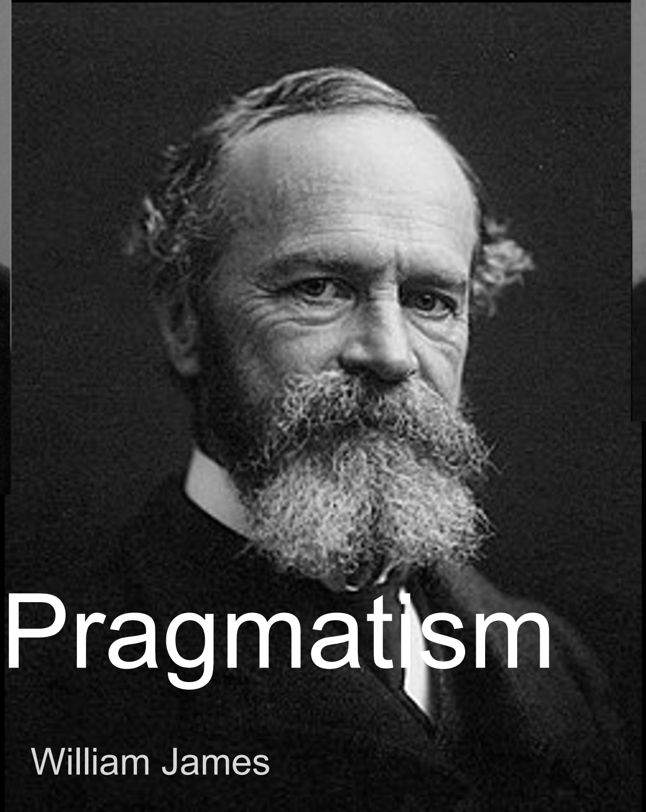
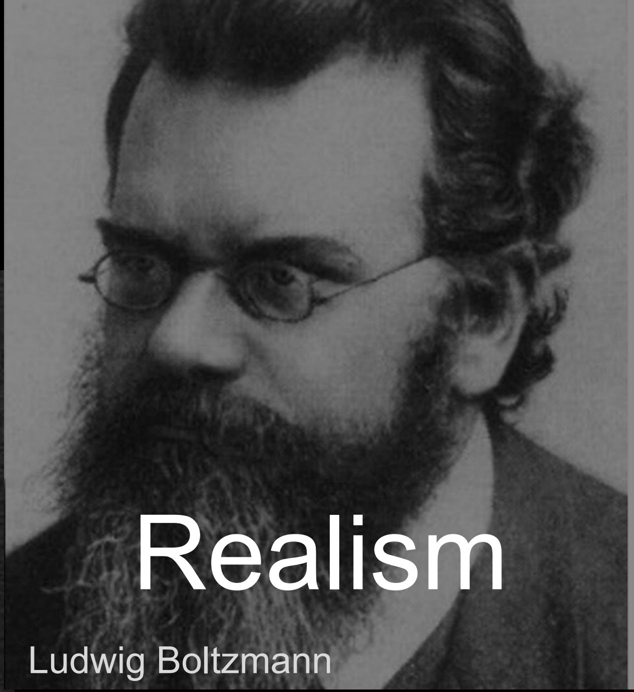
Machine learning

What about truth?


Empiricism
Ernst Mach


Pragmatism
William James


Realism
Ludwig Boltzmann

Realist

Ontological
models

GPTs w/
symmetries

Process
theories

Interventionist
Causal models

Empiricist

Generalized
probabilistic
theories

Theory of
Bayesian
inference

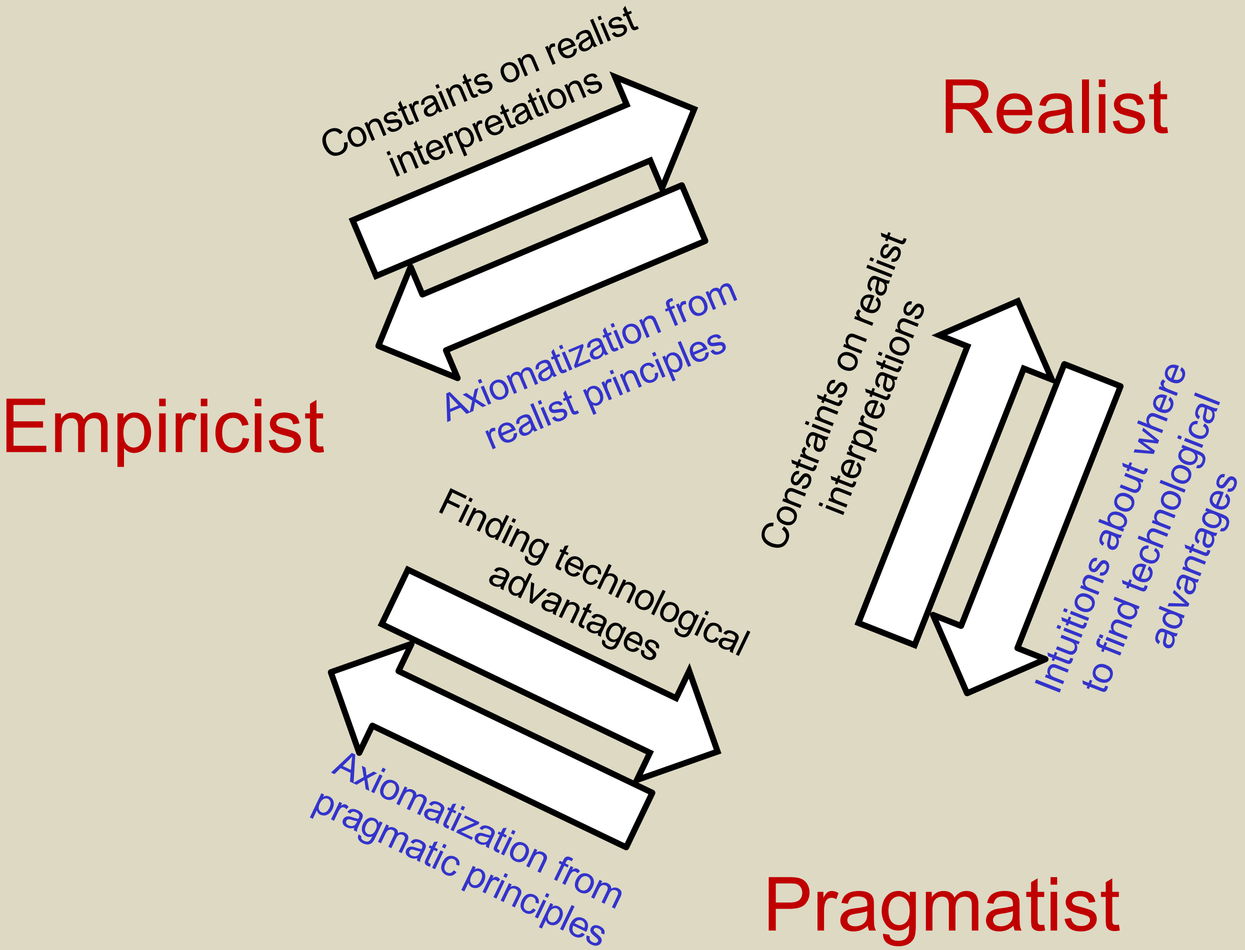
Device-
independent
paradigm

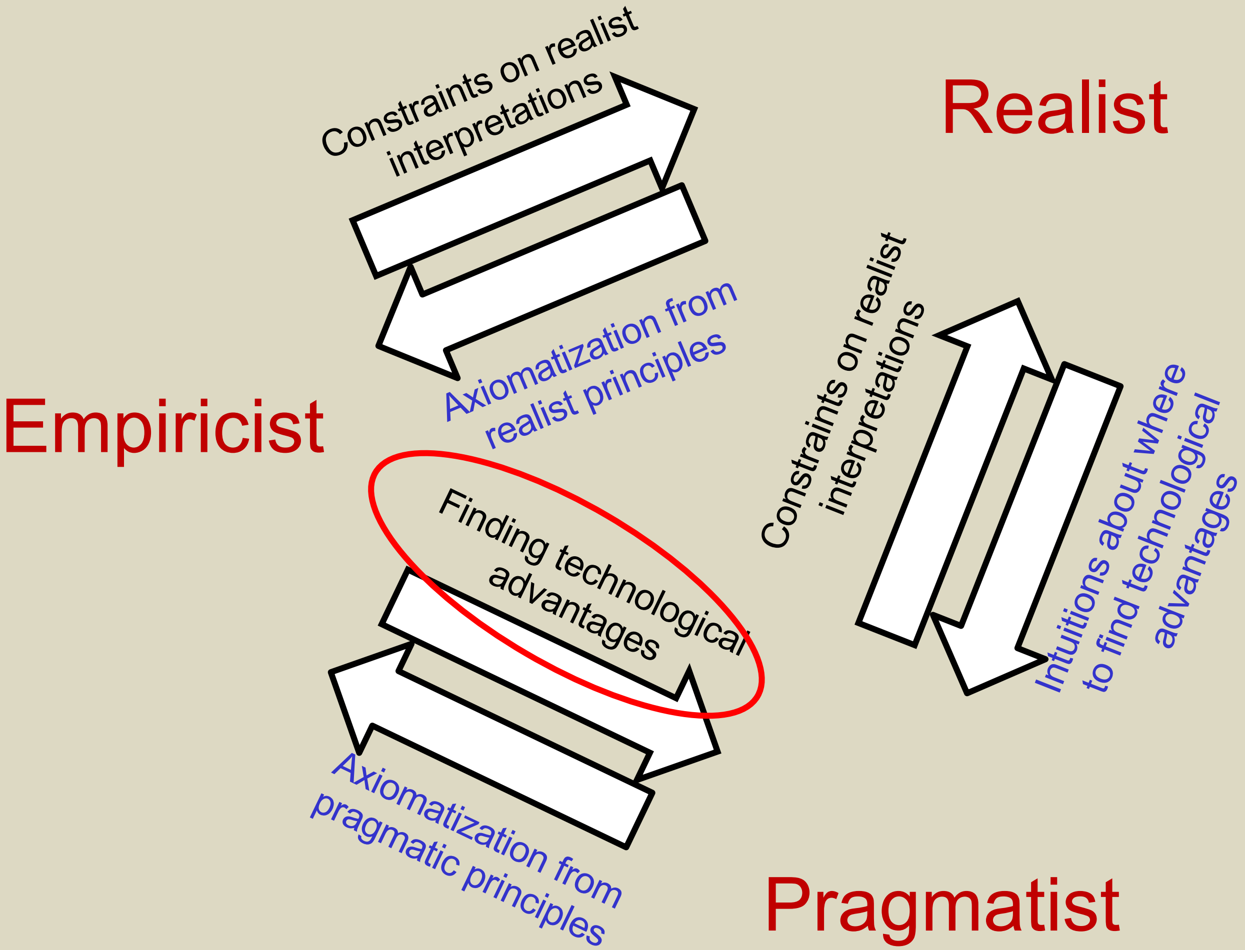
Information
theory

Resource
theories

Pragmatist

Just as physics evolves,
so too does the philosophy of science


Realist
Empiricist
Pragmatist
Constraints on realist interpretations
Axiomatization from realist principles
Constraints on realist interpretations
Intuitions about where to find technological advantages
Finding technological advantages
Axiomatization from pragmatic principles



Realist
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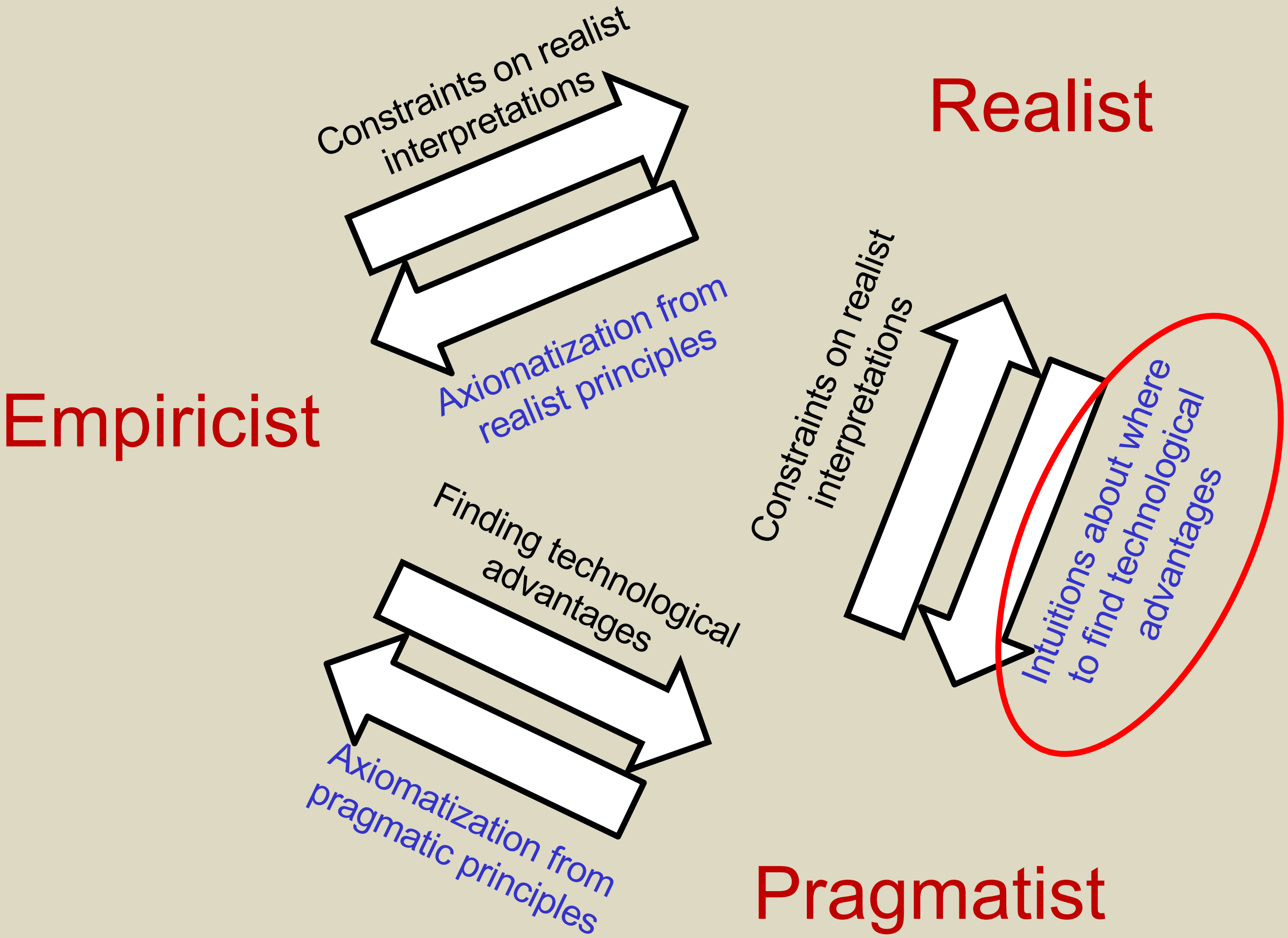
Axiomatization from pragmatic principles


Realist
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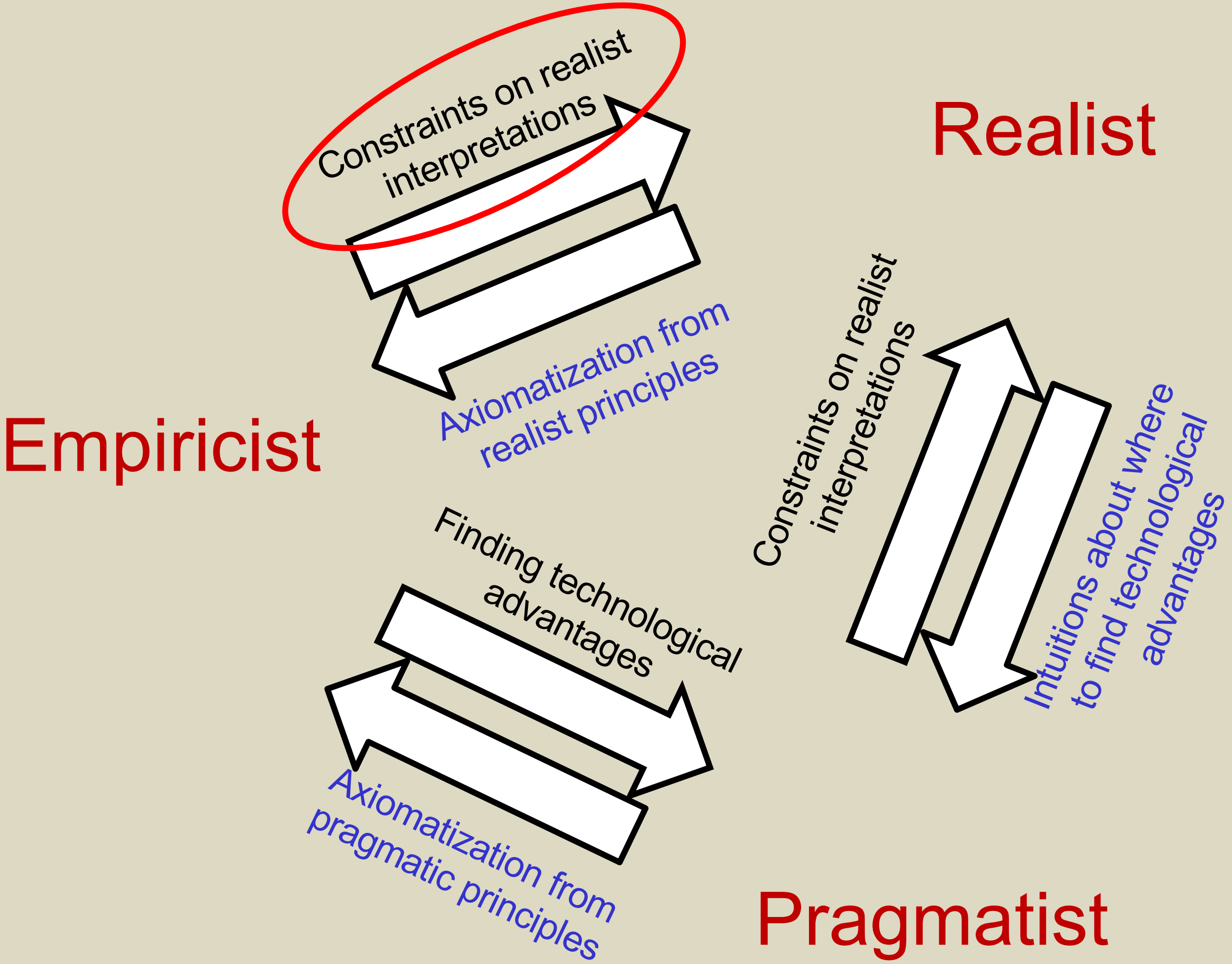
Constraints on realist interpretations
Realist
Axiomatization from realist principles
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Intuitions about where to find technological advantages
Finding technological advantages
Axiomatization from pragmatic principles
Pragmatist

Realist

Ontological models

GPTs w/ symmetries

Process theories

Interventionist Causal models

Empiricist

Generalized probabilistic theories

Theory of Bayesian inference

Device-independent paradigm

Information theory

Resource theories

Pragmatist

The methodology of foil theories

Foil: One that by contrast underscores or enhances the distinctive characteristics of another.
"I am resolved my husband shall not be a rival, but a foil to me" (Charlotte Brontë).

The methodology of foil theories:
it is only by studying the contrast class of a phenomenon that one can understand it

- Quantum theory over the real or quaternionic fields

Barnum, Graydon, and Wilce, arXiv:1606.09331 (2016)

- Theories with higher-order interference

Barnum, Muller, and Ududec, New Journal of Physics 16, 123029 (2014)
Dakic, Paterek, and Brukner, New Journal of Physics 16, 023028 (2014)
Lee and Selby, Foundations of Physics 47, 89 (2017)

- Generalized No-signalling Theory (Boxworld)

Barrett, Phys. Rev. A 75, 032304 (2007)
Short and Barrett, New Journal of Physics 12, 033034 (2010)

- Almost Quantum Theory

Sainz, Guryanova, Acin, and Navascues, arXiv:1707.02620 (2017)

- Epistemically restricted classical statistical theories

Spekkens, Phys. Rev. A 75, 032110 (2007)
Bartlett, Rudolph, Spekkens, Phys. Rev. A 86, 012103 (2012)

Workshop: Operational probabilistic theories as foils to quantum theory

July 2 to 13, 2007
University of Cambridge, Cambridge, UK
Funded by FQXi

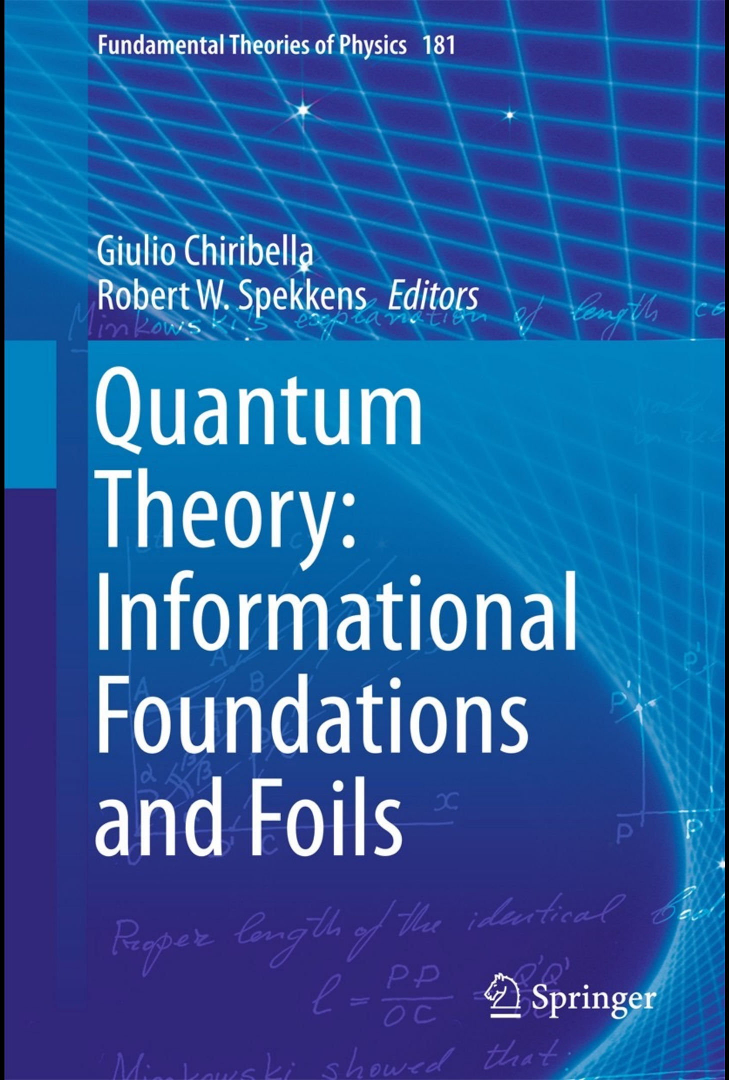
The interpretation of quantum theory is a subject of significant controversy; there is simply no agreement about what this theory is telling us about the world, represent. One strategy for progress on this front is to try to identify a set of *physical* principles that are sufficient to derive all aspects of the theory, to pick it consider the broadest possible class of such foil theories. Typically, however, one's preconceptions and tacit assumptions about the nature of reality tend to res observable consequences of experimental procedures, that is, operationally. Recent research into operational probabilistic theories has been improving our un *particular* foil theories rather than on identifying the similarities and differences of broad classes of such theories. This workshop will bring together research different programs of research, and broaden our perspectives on the issues.

Organizers
Jonathan Barrett (Perimeter Institute, Canada)
Tony Short (University of Bristol, UK)
Robert Spekkens (University of Cambridge, UK)

Invited participants
Marcus Appleby (Queen Mary London, UK)
Howard Barnum (Los Alamos National Laboratories, USA)
Oscar Dahlsten (University of Waterloo, Canada)
Fay Dowker (Imperial College, UK)
Chris Fuchs (Bell labs, Lucent technology, USA)
Philip Goyal (University of Cambridge, UK)
Lucien Hardy (Perimeter Institute, Canada)
Adrian Kent (University of Cambridge, UK)
Mathew Leifer (University of Waterloo, Canada)
Piero Mana (KTH, Sweden)
Joseph Renes (University Erlangen-Nuremberg, Germany)
Ruediger Schack (Royal Holloway College, UK)
Ben Toner (CWI, Netherlands)
Alex Wilce (Susquehanna University, USA)
William Wootters (Williams college, USA)

Dates: Invited participants arrive on Sunday, July 1st. The workshop begins on the morning of July 2nd, and ends at noon on Friday, July 13th.


Conceptual Foundations and Foils for Quantum Information Processing
May 9 - 13, 2011
Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada
The interplay between information-processing protocols and basic physical principles has attracted increasing interest in the past few years and has been the subject of many new and exciting results. Such investigations offer a new perspective on the foundations of quantum theory, a deeper understanding of the origin of quantum advantages for information-processing, and a framework for exploring the nature of information-processing within alternatives to quantum theory (foil theories).
Invited Speakers
Scott Aaronson, MIT
Antonio Acín, ICFO Barcelona
Howard Barnum, University of New Mexico
Jon Barrett, Royal Holloway*
Gilles Brassard, Université de Montréal
Nicolas Brunner, University of Bristol
Dan Browne, University College London*
Caslav Brukner, University of Vienna
Bob Coecke, University of Oxford
Roger Colbeck, Perimeter Institute
Mauro D'Ariano, University of Pavia
Chris Fuchs, Perimeter Institute
Lucien Hardy, Perimeter Institute
Marc Kaplan, Université de Montréal
Gen Kimura, Shibaura Institute of Technology*
Tsuyoshi Ito, Institute for Quantum Computing
Lluis Masanes, ICFO
Markus Mueller, Perimeter Institute
Jonathan Oppenheim, University of Cambridge
Paolo Perinotti, University of Pavia
Sandu Popescu, University of Bristol
Renato Renner, ETH Zurich
Valerio Scarani, National University of Singapore
Ben Schumacher, Kenyon College
Anthony Short, University of Cambridge
Stephanie Wehner, National University of Singapore
Alex Wilce, Susquehanna University
Andreas Winter, University of Bristol
*to be confirmed
Scientific Organizers
Giulio Chiribella, Perimeter Institute (main organizer)
Anne Broadbent, Institute for Quantum Computing
Robert Spekkens, Perimeter Institute
Deadline for registration is May 3, 2011
www.perimeterinstitute.ca/Conceptual_Foundations_and_Foils_for_QIP
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS
IQC Institute for Quantum Computing
Canada


Fundamental Theories of Physics 181
Giulio Chiribella
Robert W. Spekkens Editors
Quantum Theory: Informational Foundations and Foils
Springer

Operational foil theories

The framework of Generalized Probabilistic Theories (GPTs)

L. Hardy, arXiv:0101012 (2001)
J. Barrett, PRA **75**, 032304 (2007)
L. Hardy, arXiv:0912.4740 (2009)
G. Chiribella, G. D'Ariano, and P. Perinotti, PRA **81**, 062348 (2010)
etc.

Building on: Mackey, Ludwig, Kraus, etc.

Realist

Ontological models

GPTs w/ symmetries

Process theories

Interventionist Causal models

Generalized probabilistic theories

Empiricist

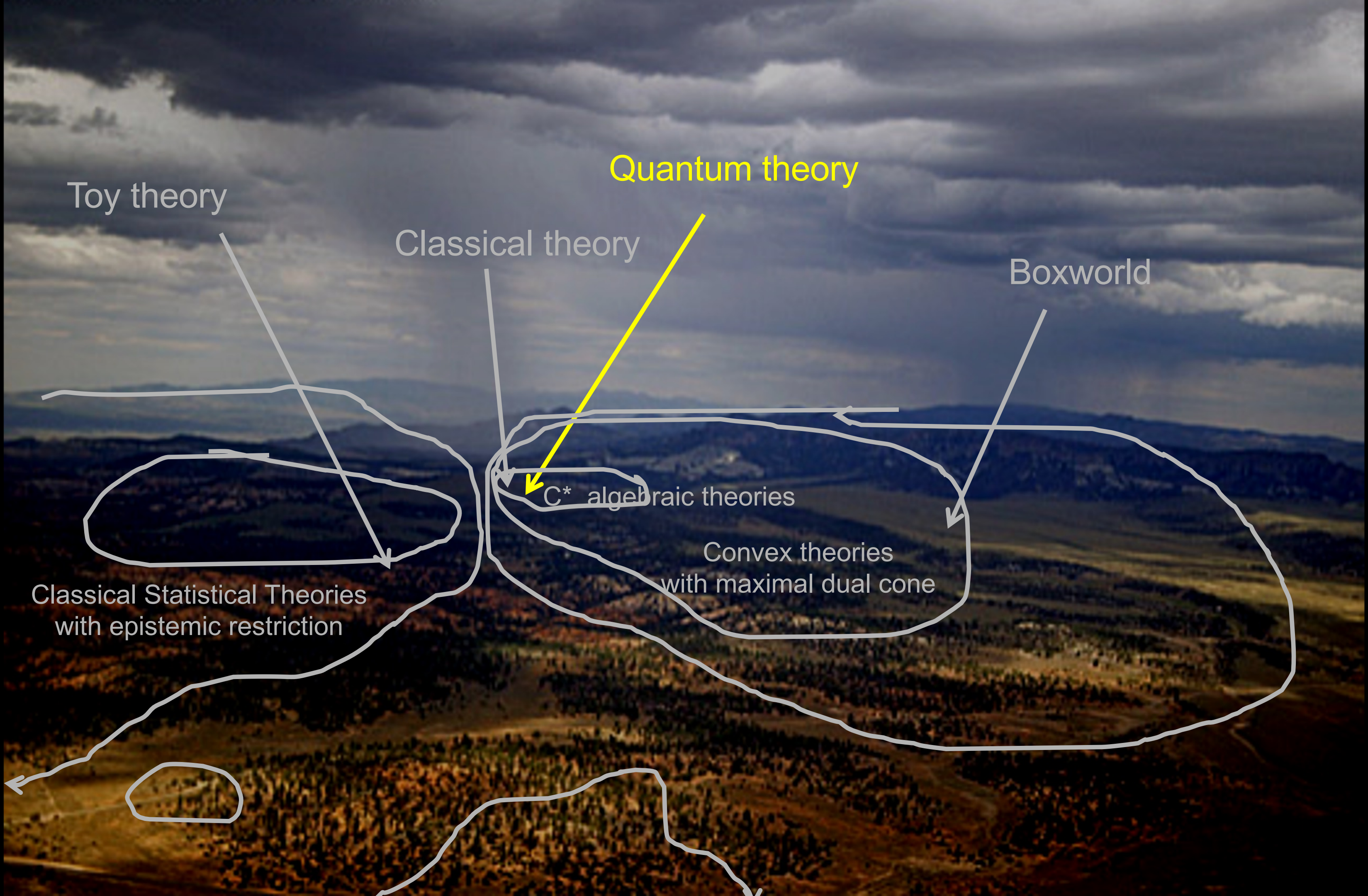
Theory of Bayesian inference

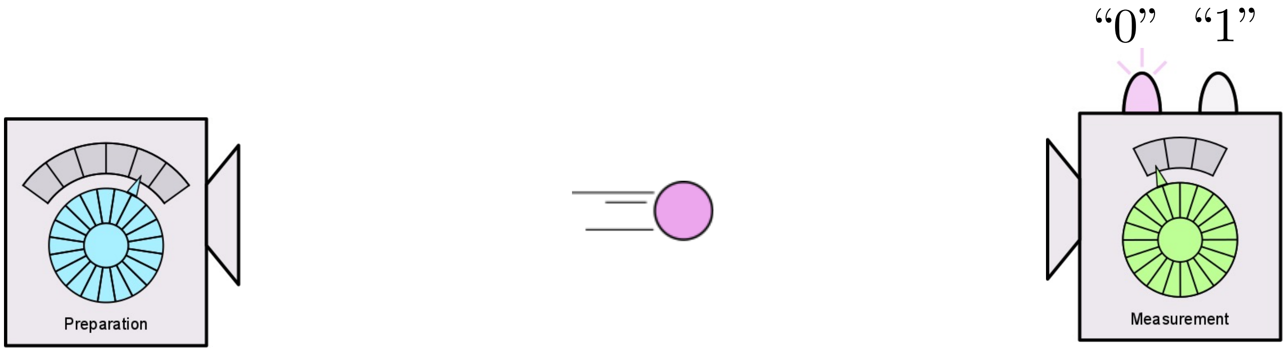
Device-independent paradigm

Information theory

Resource theories

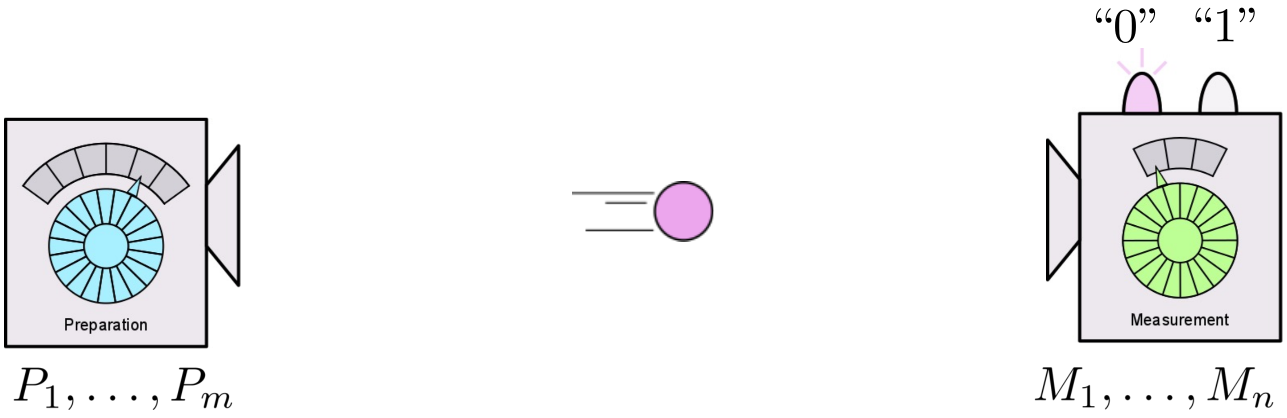
Pragmatist


Quantum theory
Toy theory
Classical theory
Boxworld
C* algebraic theories
Convex theories
with maximal dual cone
Classical Statistical Theories
with epistemic restriction



$P_1, \ldots, P_m$ $\qquad$ $M_1, \ldots, M_n$

$$\begin{pmatrix} 1 & p(0|P_1, M_2) & p(0|P_1, M_3) & p(0|P_1, M_4) & p(0|P_1, M_5) & \cdots \\ 1 & p(0|P_2, M_2) & p(0|P_2, M_3) & p(0|P_2, M_4) & p(0|P_2, M_5) & \cdots \\ 1 & p(0|P_3, M_2) & p(0|P_3, M_3) & p(0|P_3, M_4) & p(0|P_3, M_5) & \cdots \\ 1 & p(0|P_4, M_2) & p(0|P_4, M_3) & p(0|P_4, M_4) & p(0|P_4, M_5) & \cdots \\ 1 & p(0|P_5, M_2) & p(0|P_5, M_3) & p(0|P_5, M_4) & p(0|P_5, M_5) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$



$$
\begin{pmatrix}
1 & s_1^{(1)} & \cdots & s_{k-1}^{(1)} \\
1 & s_1^{(2)} & \cdots & s_{k-1}^{(2)} \\
1 & s_1^{(3)} & \cdots & s_{k-1}^{(3)} \\
1 & s_1^{(4)} & \cdots & s_{k-1}^{(4)} \\
1 & s_1^{(5)} & \cdots & s_{k-1}^{(5)} \\
\vdots & \vdots & \vdots & \vdots
\end{pmatrix}
\begin{pmatrix}
1 & e_0^{(2,0)} & e_0^{(3,0)} & e_0^{(4,0)} & e_0^{(5,0)} & \cdots \\
0 & e_1^{(2,0)} & e_1^{(3,0)} & e_1^{(4,0)} & e_1^{(5,0)} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
0 & e_{k-1}^{(2,0)} & e_{k-1}^{(3,0)} & e_{k-1}^{(4,0)} & e_{k-1}^{(5,0)} & \cdots
\end{pmatrix}
$$

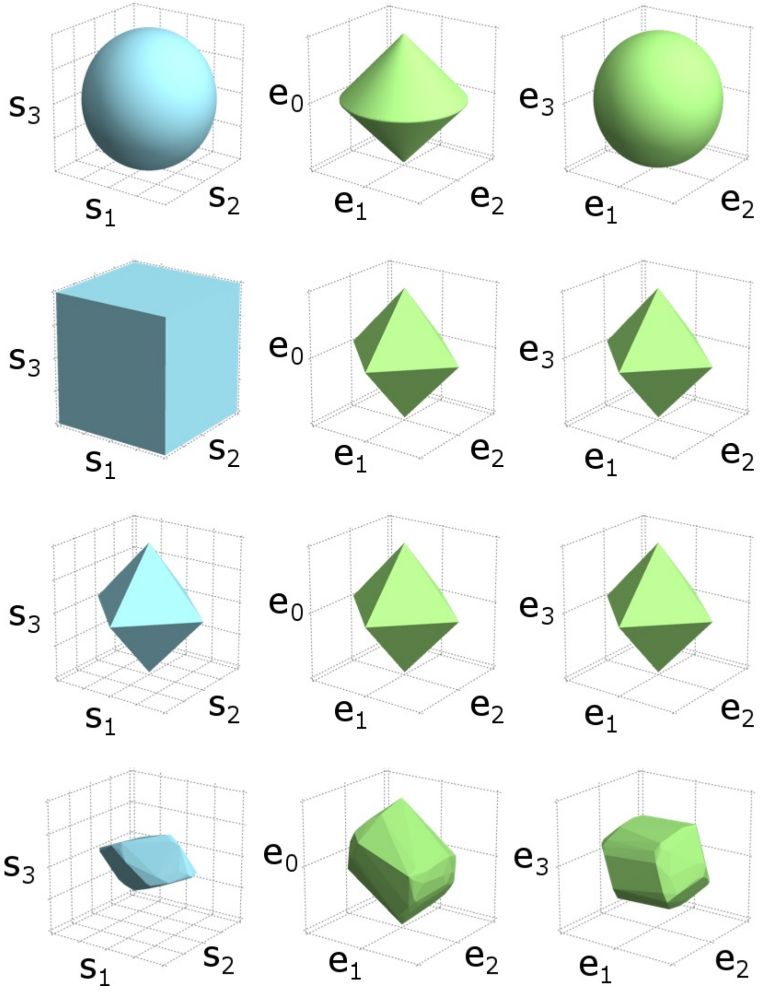
$$
\begin{pmatrix}
1 & s_1^{(1)} & \cdots & s_{k-1}^{(1)} \\
1 & s_1^{(2)} & \cdots & s_{k-1}^{(2)} \\
1 & s_1^{(3)} & \cdots & s_{k-1}^{(3)} \\
1 & s_1^{(4)} & \cdots & s_{k-1}^{(4)} \\
1 & s_1^{(5)} & \cdots & s_{k-1}^{(5)} \\
\vdots & \vdots & \vdots & \vdots
\end{pmatrix}
\begin{pmatrix}
1 & e_0^{(2,0)} & e_0^{(3,0)} & e_0^{(4,0)} & e_0^{(5,0)} & \cdots \\
0 & e_1^{(2,0)} & e_1^{(3,0)} & e_1^{(4,0)} & e_1^{(5,0)} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
0 & e_{k-1}^{(2,0)} & e_{k-1}^{(3,0)} & e_{k-1}^{(4,0)} & e_{k-1}^{(5,0)} & \cdots
\end{pmatrix}
$$

$$
p\left(0 | P_i, M_j\right) = \left(1, s_1^{(i)}, \ldots, s_{k-1}^{(i)}\right) \cdot \left(e_0^{(j,0)}, \ldots, e_{k-1}^{(j,0)}\right) = \mathbf{s}^{(i)} \cdot \mathbf{e}^{(j,0)}
$$

GPT state

GPT effect

Examples of GPTs for systems with k=4



Qubit

Generalized No-signalling Theory (Boxworld)

Convex hull of my toy theory

Randomly generated example

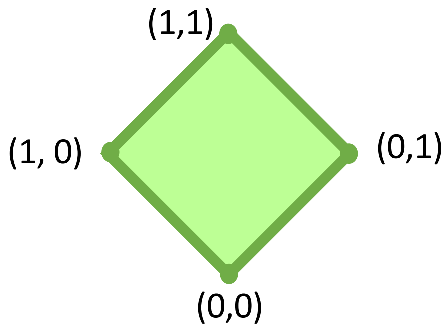
2-level classical system (bit)

k=2

$$p(0) = p(b=0)p(0|b=0) + p(b=1)p(0|b=1)$$

$$(p(b=0), p(b=1))$$

$$(p(0|b=0), p(0|b=1))$$



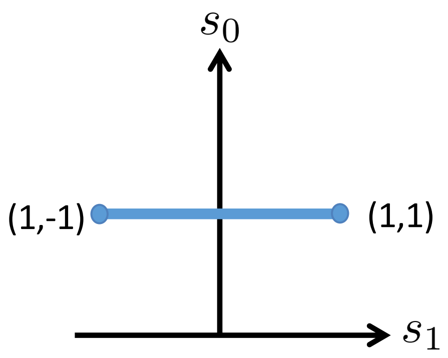
2-level classical system (bit)

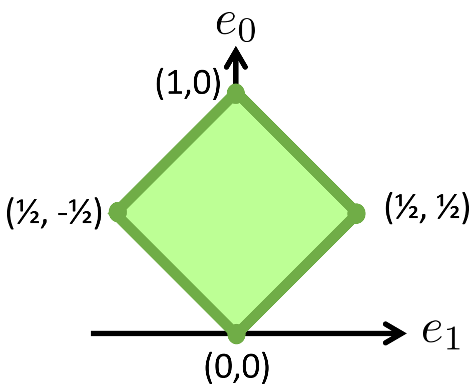
k=2

$$p(0) = p(b=0)p(0|b=0) + p(b=1)p(0|b=1)$$
$$= s_0 e_0 + s_1 e_1$$

$$\mathbf{s} = (s_0, s_1)$$

$$\mathbf{e} = (e_0, e_1)$$





$$s_0 = p(b=0) + p(b=1) = 1$$
$$s_1 = p(b=0) - p(b=1)$$

$$e_0 = \frac{1}{2}(p(0|b=1) + p(0|b=0))$$
$$e_1 = \frac{1}{2}(p(0|b=1) - p(0|b=0))$$

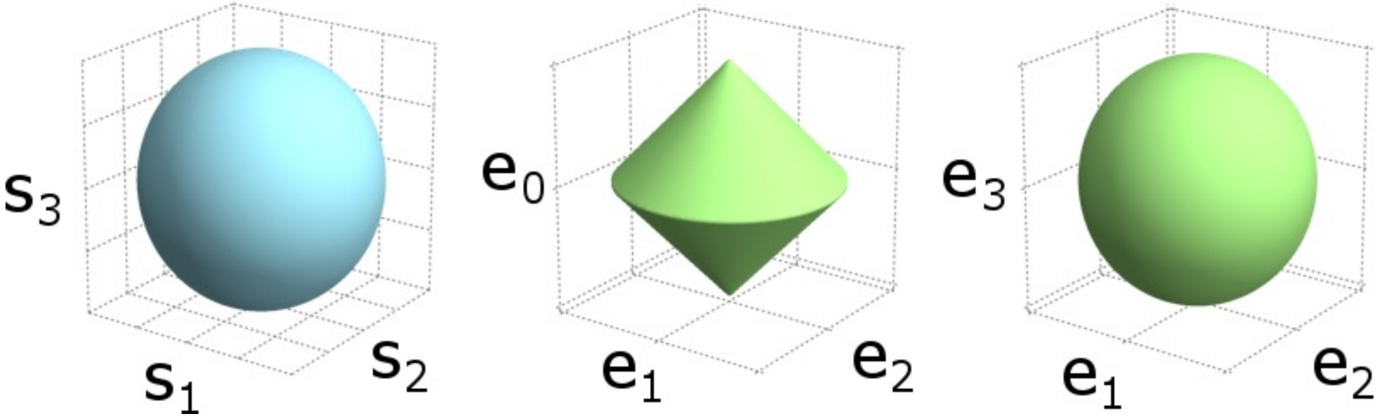
Qubit

$$\rho = \frac{1}{2}\left(\mathbb{I} + s_X\sigma_X + s_Y\sigma_Y + s_Z\sigma_Z\right)$$

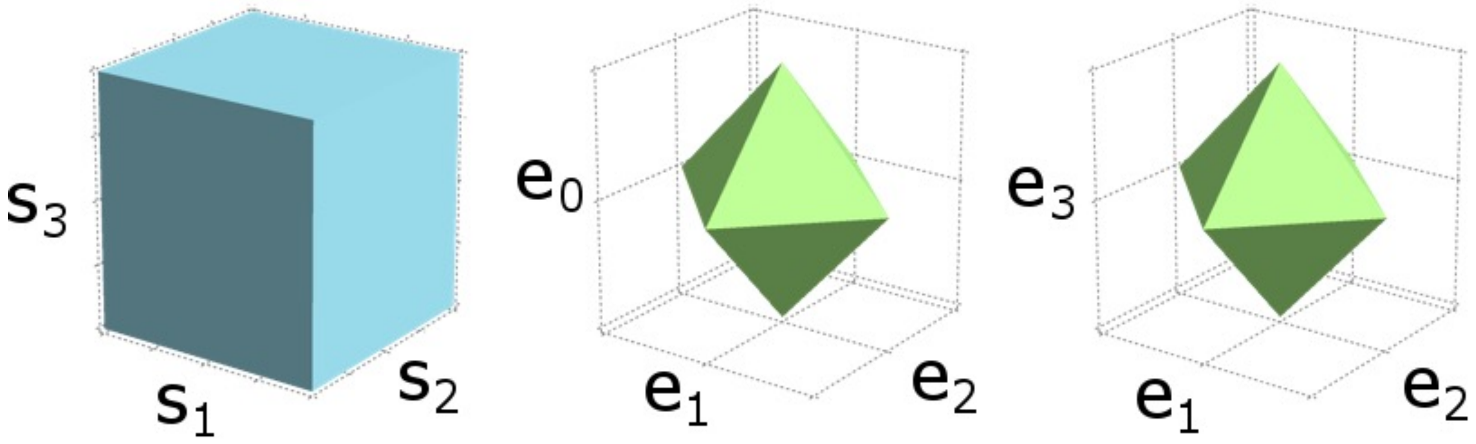
$$E = \frac{1}{2}\left(e_0\mathbb{I} + e_X\sigma_X + e_Y\sigma_Y + e_Z\sigma_Z\right)$$

$$\mathrm{Tr}(\rho E) = \frac{1}{2}(e_o + s_\mathrm{X}e_\mathrm{X} + s_\mathrm{Y}e_\mathrm{Y} + s_\mathrm{Z}e_\mathrm{Z})$$

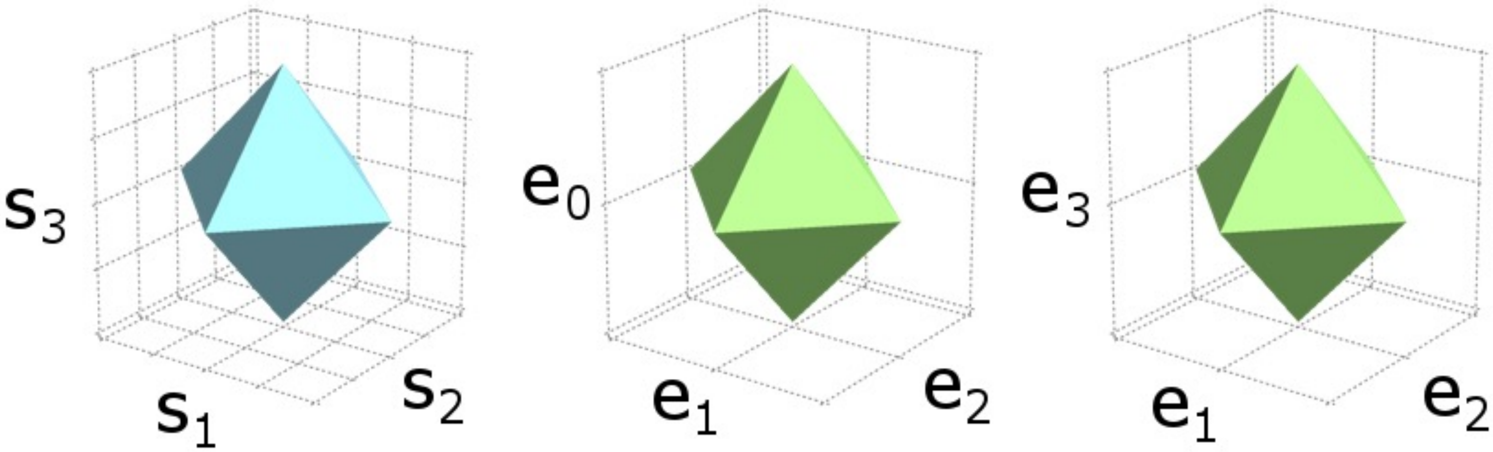
$$= \underbrace{(1, s_\mathrm{X}, s_\mathrm{Y}, s_\mathrm{Z})}_{\mathbf{s}} \cdot \underbrace{\frac{1}{2}(e_0, e_\mathrm{X}, e_\mathrm{Y}, e_\mathrm{Z})}_{\mathbf{e}}$$



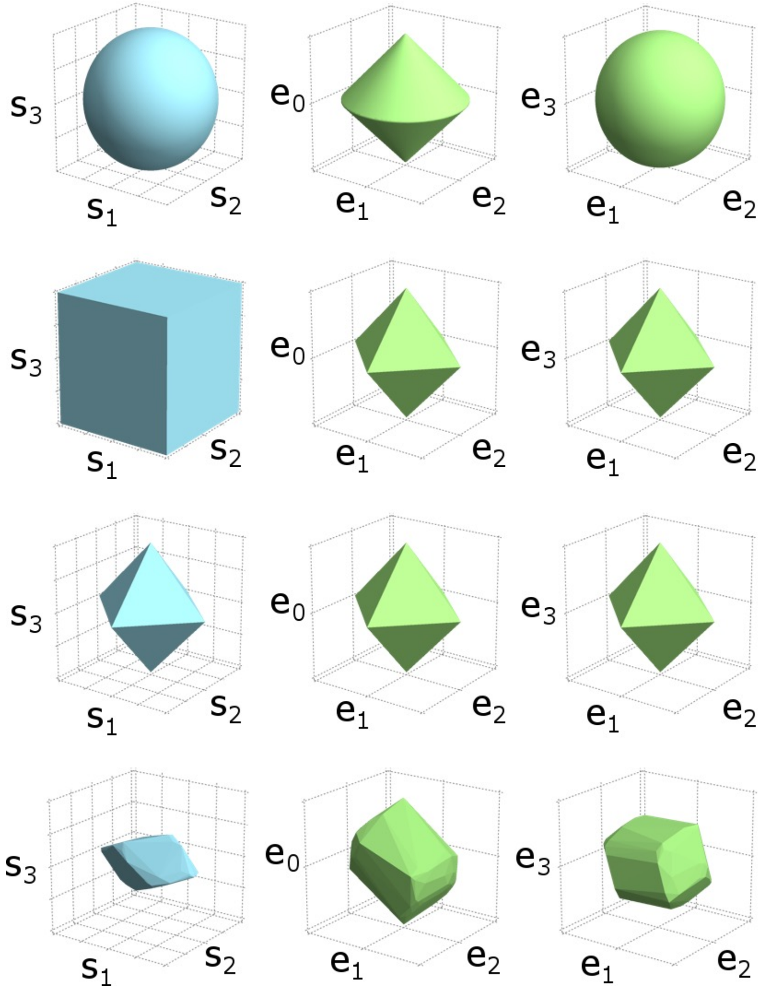
Generalised no-signalling theory (Boxworld)



Convex hull of my toy theory



Examples of GPTs for systems with k=4



Qubit

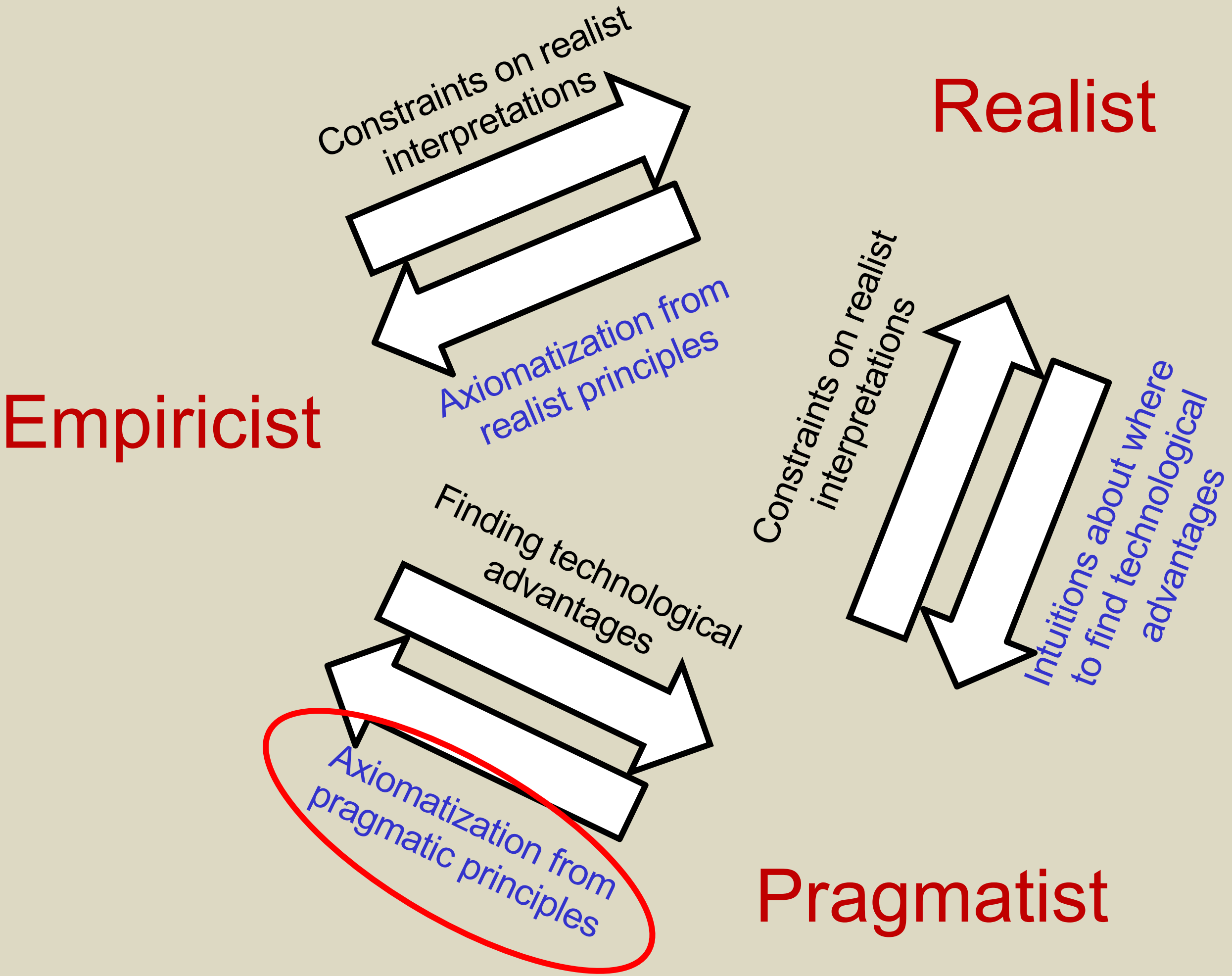
Generalized No-signalling Theory (Boxworld)

Convex hull of my toy theory

Randomly generated example

Composition

A little bit about the quantum reconstruction program


Realist
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Constraints on realist interpretations
Axiomatization from realist principles
Constraints on realist interpretations
Intuitions about where to find technological advantages
Finding technological advantages
Axiomatization from pragmatic principles

The principle of tomographic locality

Tomographic locality appears in many axiomatizations of quantum theory

L. Hardy, Quantum theory from five reasonable axioms (2001), arXiv:quant-ph/0101012
J. Barrett, Information processing in generalized probabilistic theories, Phys Rev A 75, 032304 (2007)
A. Wilce, Four and a half axioms for finite dimensional quantum mechanics (2009), arXiv:0912.5530
B. Dakic and C. Brukner, Quantum theory and beyond: Is entanglement special? (2009), arXiv:0911.0695 J.
G. Chiribella, G. M. D'Ariano, and P. Perinotti, Probabilistic theories with purification, Phys. Rev. A 81, 062348 (2010)
M. Zaopo, Information theoretic axioms for quantum theory (2012), arXiv:1205.2306
L. Hardy, Reconstructing quantum theory (2013), arXiv:1303.1538
L. Masanes, M. P. Muller, R. Augusiak, and D. Perez-Garcıa, Existence of an information unit as a postulate of quantum theory, Proceedings of the National Academy of Sciences 110, 16373–16377 (2013).
M. P. Muller and L. Masanes, Information-theoretic postulates for quantum theory, in Quantum Theory: Informational Foundations and Foils (Springer Netherlands, 2015) p. 139–170.
P. A. Hohn and C. S. P. Wever, Quantum theory from questions, Phys. Rev. A 95, 012102 (2017).
H. Selby, C. M. Scandolo, and B. Coecke, Reconstructing quantum theory from diagrammatic postulates, Quantum 5, 445 (2021).
M. Muller, Probabilistic theories and reconstructions of quantum theory, SciPost Physics Lecture Notes , 028 (2021)

Tomographic completeness

$$\begin{pmatrix} 1 & s_1^{(1)} & \cdots & s_{k-1}^{(1)} \\ 1 & s_1^{(2)} & \cdots & s_{k-1}^{(2)} \\ 1 & s_1^{(3)} & \cdots & s_{k-1}^{(3)} \\ 1 & s_1^{(4)} & \cdots & s_{k-1}^{(4)} \\ 1 & s_1^{(5)} & \cdots & s_{k-1}^{(5)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} 1 & e_0^{(2,0)} & e_0^{(3,0)} & e_0^{(4,0)} & e_0^{(5,0)} & \cdots \\ 0 & e_1^{(2,0)} & e_1^{(3,0)} & e_1^{(4,0)} & e_1^{(5,0)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\ 0 & e_{k-1}^{(2,0)} & e_{k-1}^{(3,0)} & e_{k-1}^{(4,0)} & e_{k-1}^{(5,0)} & \cdots \end{pmatrix}$$

Any set of k GPT states (effects) that spans the space of GPT states (effects) can be used to do complete tomography on the effects (states)

State tomography for a single qubit

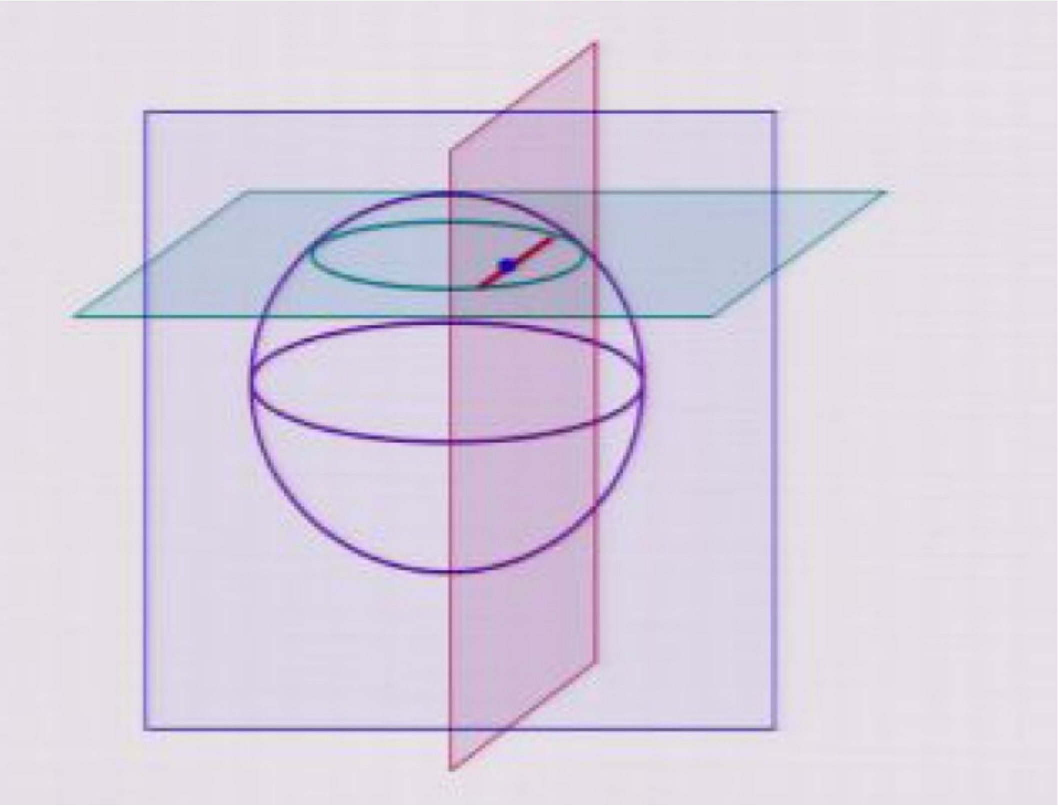
A basis for the 4d space of 1-qubit Hermitian operators

I

X

Y

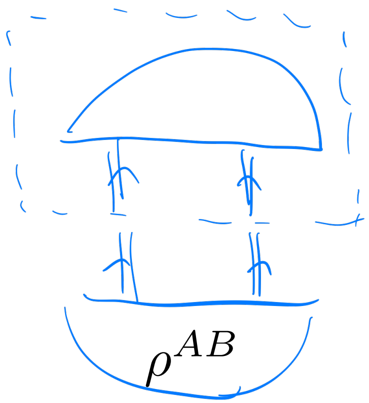
Z



State tomography for a pair of qubits

A basis for the 16d space of 2-qubit Hermitian operators

$$\begin{array}{cccc} I\otimes I & X\otimes I & Y\otimes I & Z\otimes I \\ I\otimes X & X\otimes X & Y\otimes X & Z\otimes X \\ I\otimes Y & X\otimes Y & Y\otimes Y & Z\otimes Y \\ I\otimes Z & X\otimes Z & Y\otimes Z & Z\otimes Z \end{array}$$



Tomographic locality:
A theory satisfies tomographic locality if, for the purpose of achieving a tomographic characterization of a bipartite state (i.e., inferring the state from the measurement statistics it induces), it is sufficient to use only local measurements. `

Real Quantum Theory
fails to satisfy the
principle of
tomographic locality

Defining Real Quantum Theory

Choose a basis relative to which to express all Hermitian operators as matrices
Denote complex conjugate of a matrix O as O^* or $\mathcal{C}(O)$

Real Quantum Theory: The density operators and effects are all and only those that are invariant under complex conjugation
$\mathcal{C}(O) = O$

$$O = \mathsf{Re}(O) + i\mathsf{Im}(O)$$

$$\mathsf{Re}(O) = \frac{1}{2}(O + O^*)$$

$$\mathsf{Im}(O) = \frac{1}{2i}(O - \varnothing^*)$$

$\mathcal{C}(O) = O$ if and only if $O = \mathsf{Re}(O)$

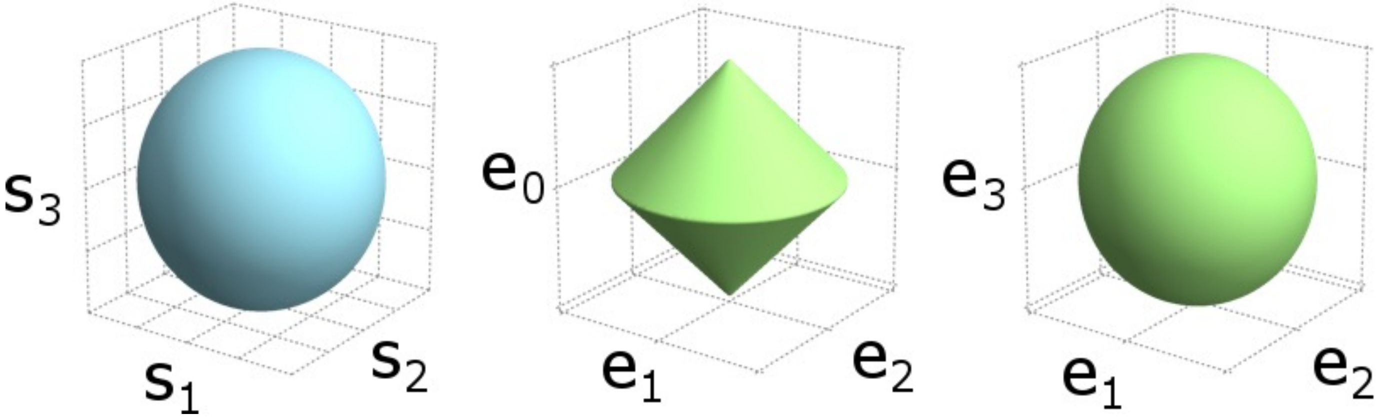
Recall qubit

$$\rho = \frac{1}{2}\left(\mathbb{I} + s_X\sigma_X + s_Y\sigma_Y + s_Z\sigma_Z\right)$$

$$E = \frac{1}{2}\left(e_0\mathbb{I} + e_X\sigma_X + e_Y\sigma_Y + e_Z\sigma_Z\right)$$

$$\mathrm{Tr}(\rho E) = \frac{1}{2}(e_o + s_\mathrm{X}e_\mathrm{X} + s_\mathrm{Y}e_\mathrm{Y} + s_\mathrm{Z}e_\mathrm{Z})$$

$$= \underbrace{(1, s_\mathrm{X}, s_\mathrm{Y}, s_\mathrm{Z})}_{\mathbf{s}} \cdot \underbrace{\frac{1}{2}(e_0, e_\mathrm{X}, e_\mathrm{Y}, e_\mathrm{Z})}_{\mathbf{e}}$$



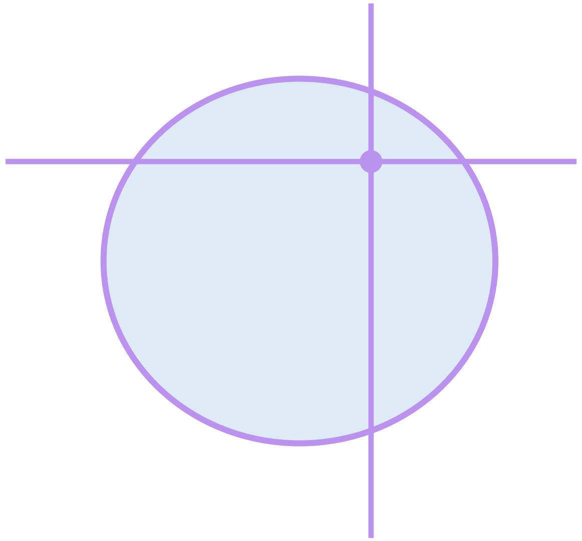
State tomography for a single rebit

A basis for the 3d space of 1-rebit Hermitian operators

I

X

Z



A basis for the 10d space of 2-rebit Hermitian operators

$$
\begin{array}{ccc}
I \otimes I & X \otimes I & Z \otimes I \\
I \otimes X & X \otimes X & Z \otimes X \\
I \otimes Z & X \otimes Z & Z \otimes Z
\end{array}
\quad \text{and} \quad Y \otimes Y
$$

Basis of 9d space of Hermitian operators that arise from products of 1-rebit Hermitian operators

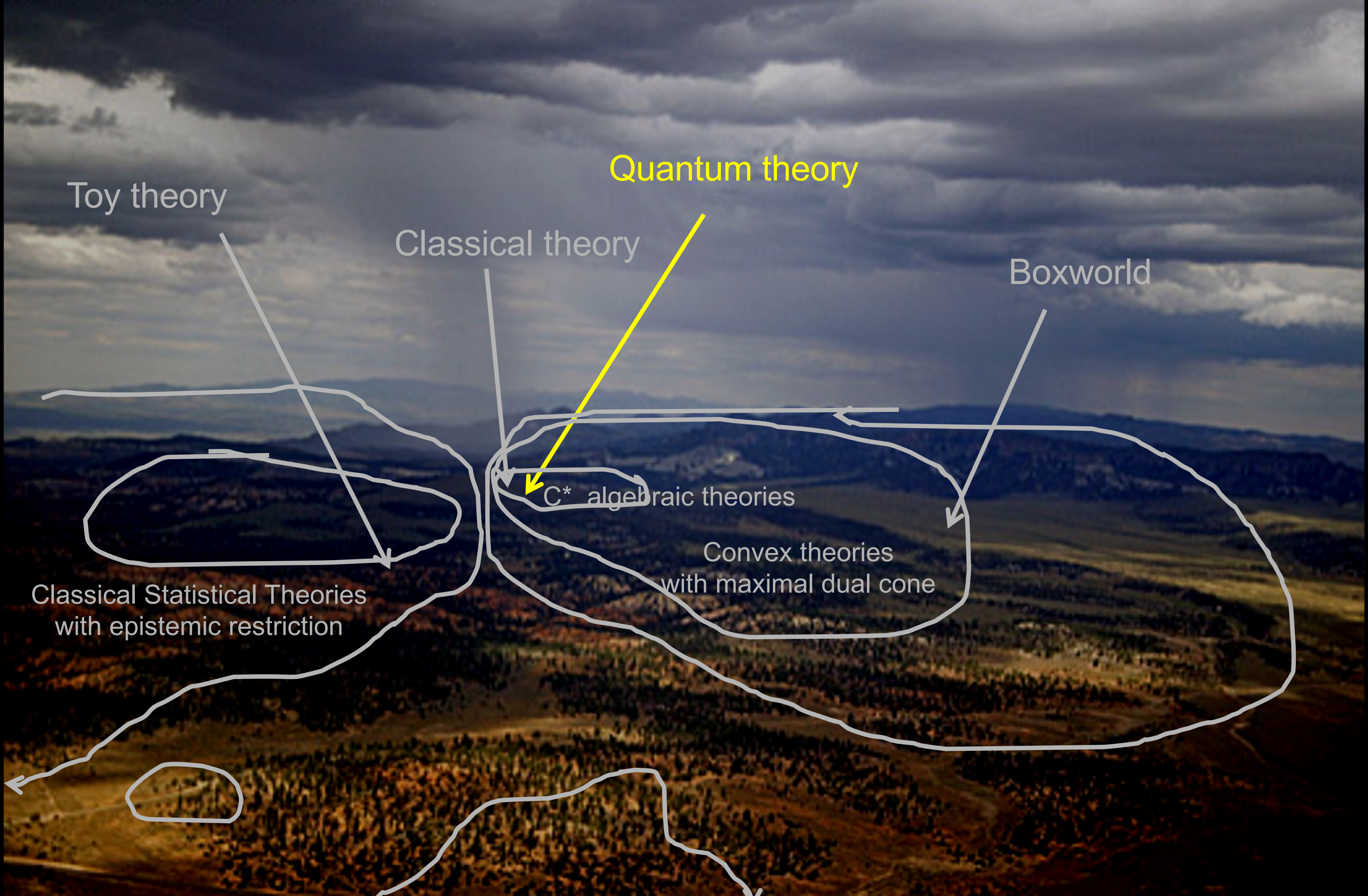
A pair of distinct 2-rebit states that are indistinguishable using local measurements in Real QT

$$\rho_+^{AB} = \frac{1}{2}|\psi^+\rangle\langle\psi^+| + \frac{1}{2}|\phi^-\rangle\langle\phi^-|$$
$$= \frac{1}{4}(I \otimes I + Y \otimes Y)$$

$$\rho_-^{AB} = \frac{1}{2}|\psi^-\rangle\langle\psi^-| + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$$
$$= \frac{1}{4}(I \otimes I - Y \otimes Y)$$

So, for the purpose of achieving a tomographic characterization of a bipartite state, it is NOT sufficient to use only local measurements

Tomographic Locality fails!


Quantum theory
Toy theory
Classical theory
Boxworld
C* algebraic theories
Convex theories
with maximal dual cone
Classical Statistical Theories
with epistemic restriction

Realist foil theories

Realist

Ontological models

GPTs w/ symmetries

Process theories

Interventionist Causal models

Generalized probabilistic theories

Empiricist

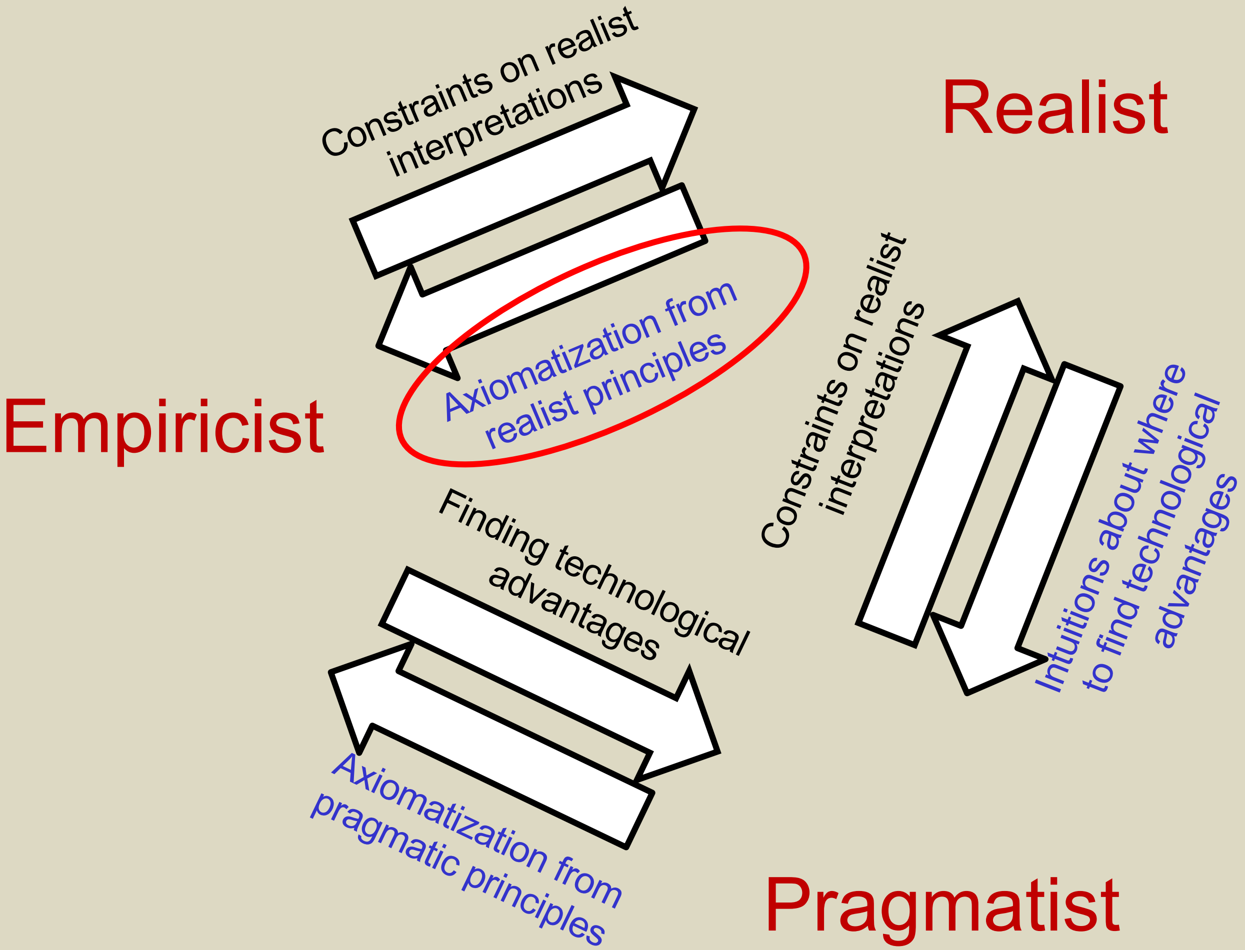
Theory of Bayesian inference

Device-independent paradigm

Information theory

Resource theories

Pragmatist



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Constraints on realist interpretations
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Constraints on realist interpretations
Intuitions about where to find technological advantages
Finding technological advantages
Axiomatization from pragmatic principles

A toy theory

RWS, PRA 75, 032110 (2007)
RWS, arXiv:1409.5041

Recall: particle mechanics

Configuration space: $\mathbb{R}^n \ni (x_1, x_2, \ldots, x_n)$

Phase space: $\equiv \mathbb{R}^{2n} \ni (x_1, p_1, x_2, p_2, \ldots, x_n, p_n) \equiv m$

Functionals on phase space: $F : \mathbb{R}^{2n} \to \mathbb{R}$

$$X_k(m) = x_k$$
$$P_k(m) = p_k$$

Poisson bracket of functionals:

$$[F, G](m) \equiv \sum_{i=1}^{n} \left(\frac{\partial F}{\partial X_i}\frac{\partial G}{\partial P_i} - \frac{\partial F}{\partial P_i}\frac{\partial G}{\partial X_i}\right)(m)$$

“bit mechanics” $\mathbb{Z}_2 = \{0,1\}$

Configuration space: $(\mathbb{Z}_2)^n \ni (x_1, x_2, \ldots, x_n)$

Phase space: $\equiv (\mathbb{Z}_2)^{2n} \ni (x_1, p_1, x_2, p_2, \ldots, x_n, p_n) \equiv m$

Functionals on phase space: $F : (\mathbb{Z}_2)^{2n} \to \mathbb{Z}_2$

$$X_k(m) = x_k$$
$$P_k(m) = p_k$$

Poisson bracket of functionals:

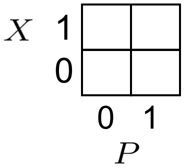
$$[F,G](m) \equiv \sum_{i=1}^{n} (F[m+e_{x_i}] - F[m])(G[m+e_{p_i}] - G[m]) - (F[m+e_{p_i}] - F[m])(G[m+e_{q_i}] - G[m])$$

The epistemic restriction:

An observer can only have knowledge of the values of a set of canonical variables that commute relative to the Poisson bracket and is maximally ignorant otherwise.

Statistical states

A single bit

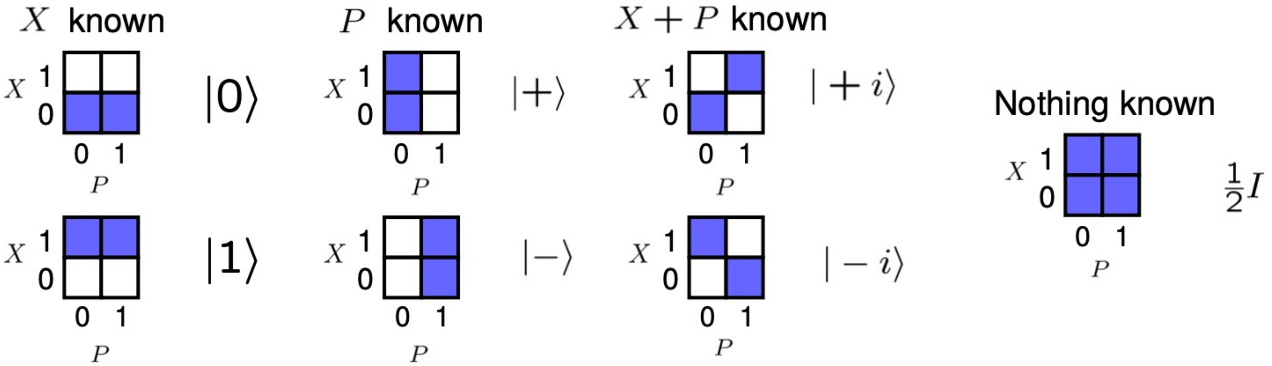


Canonical variables

$$aX + bP \qquad a, b \in \mathbb{Z}_2$$ Addition is mod2

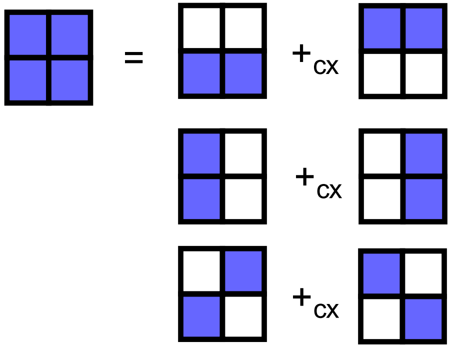
$$X, \quad P, \quad X + P$$

Statistical distributions



$$|\pm\rangle = \sqrt{2}^{-1}(|0\rangle \pm |1\rangle) \qquad |\pm i\rangle = \sqrt{2}^{-1}(|0\rangle \pm i|1\rangle)$$

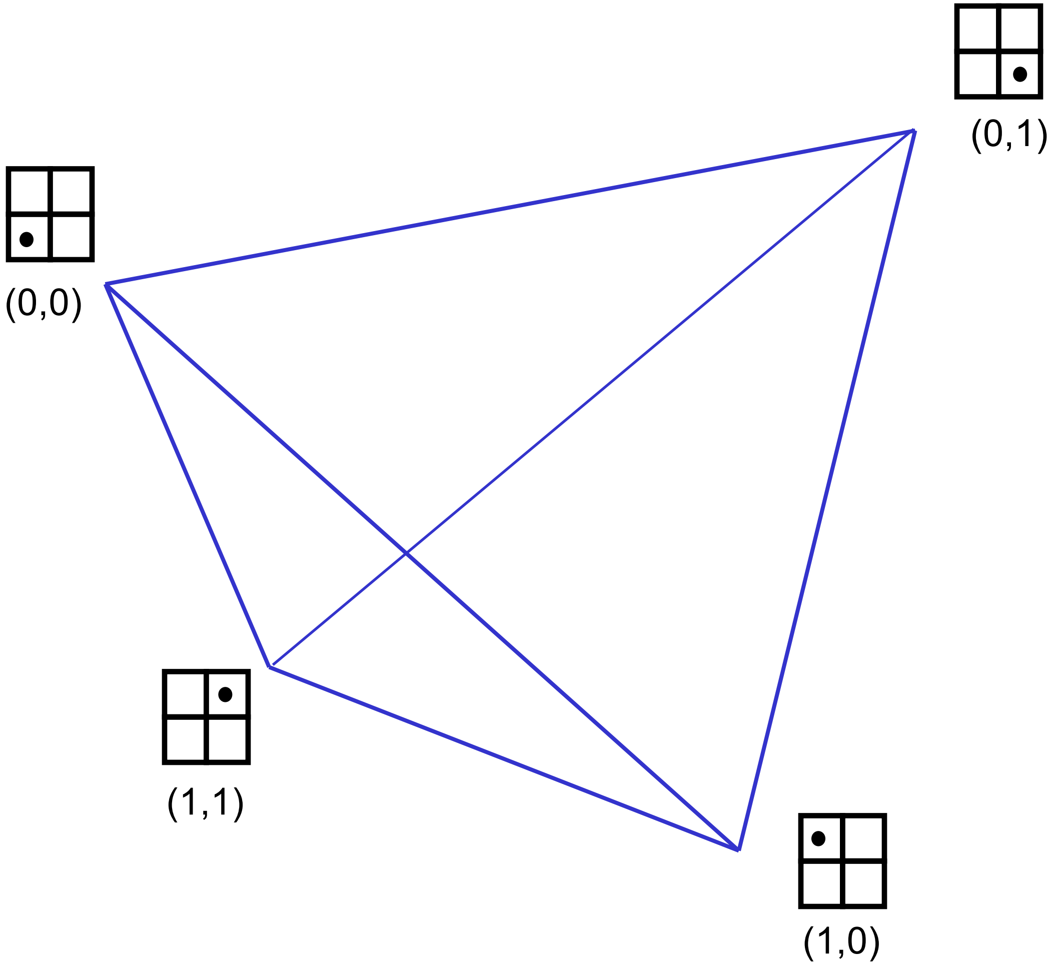
Convex combination



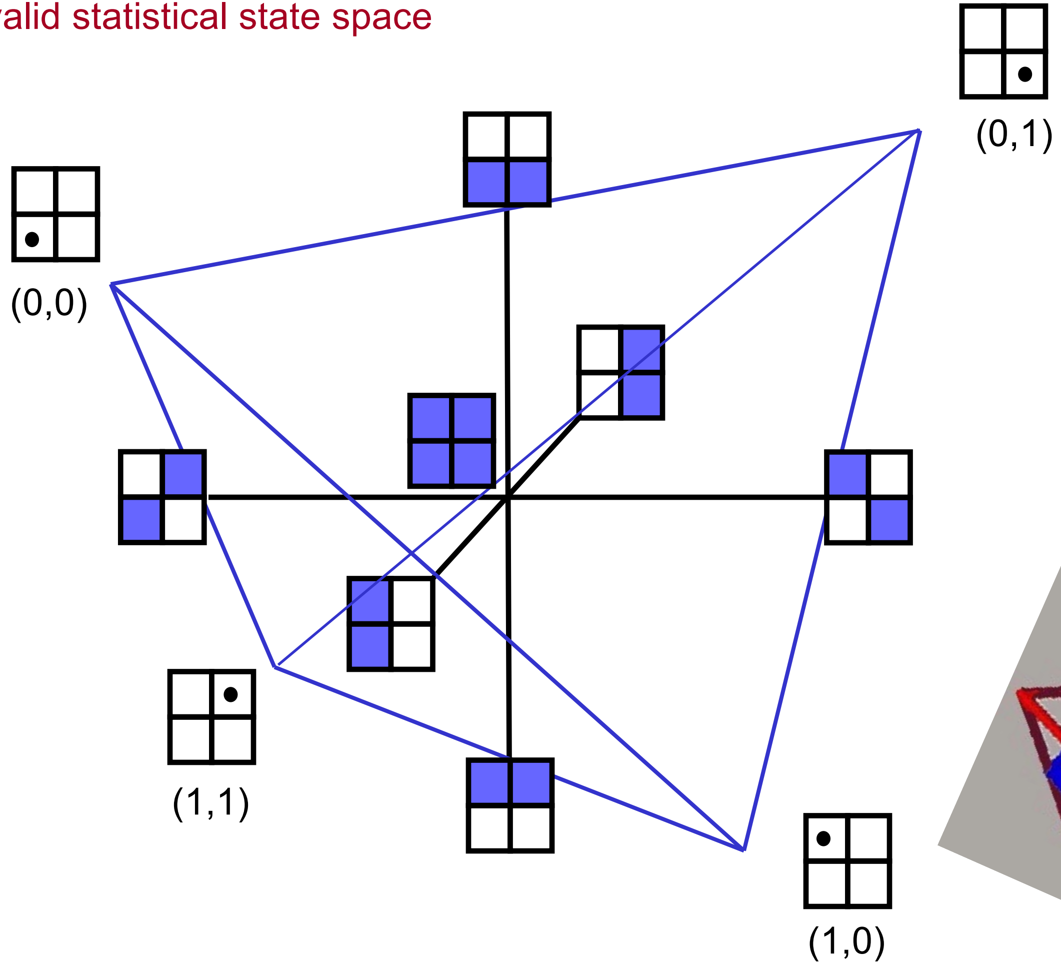
$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$
$$= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$
$$= \frac{1}{2}|+i\rangle\langle +i| + \frac{1}{2}|-i\rangle\langle -i|$$

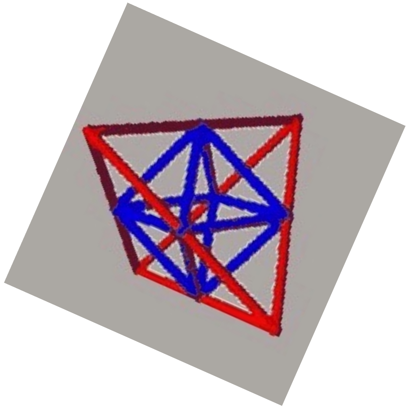
States of non-maximal knowledge are mixed
States of maximal knowledge are pure

There is a multiplicity of decompositions
of mixed states into pure states

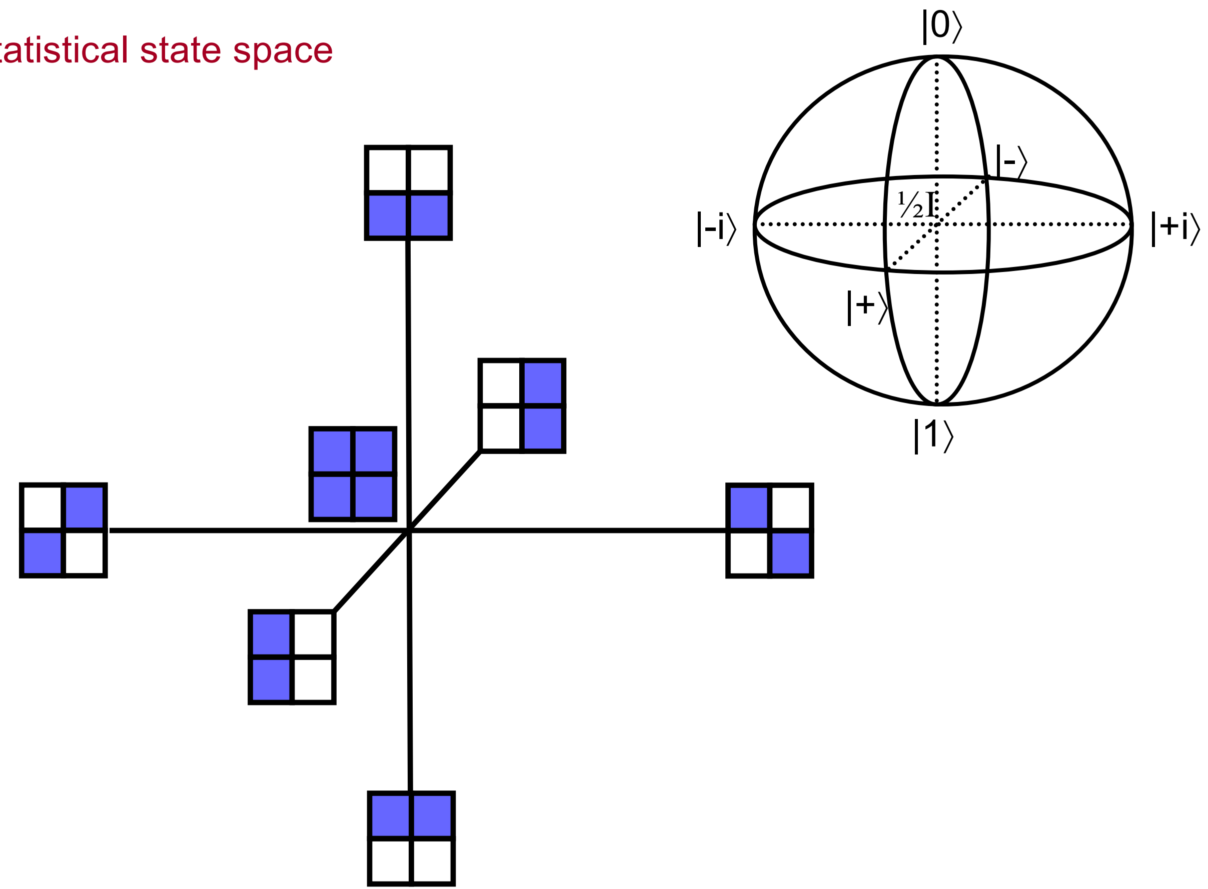

(0,1)
(0,0)
(1,1)
(1,0)

The valid statistical state space





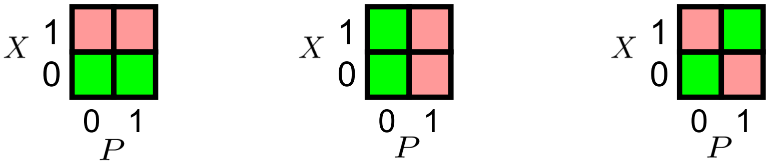
The valid statistical state space



Measurements

One can only measure a Poisson-commuting set of canonical variables

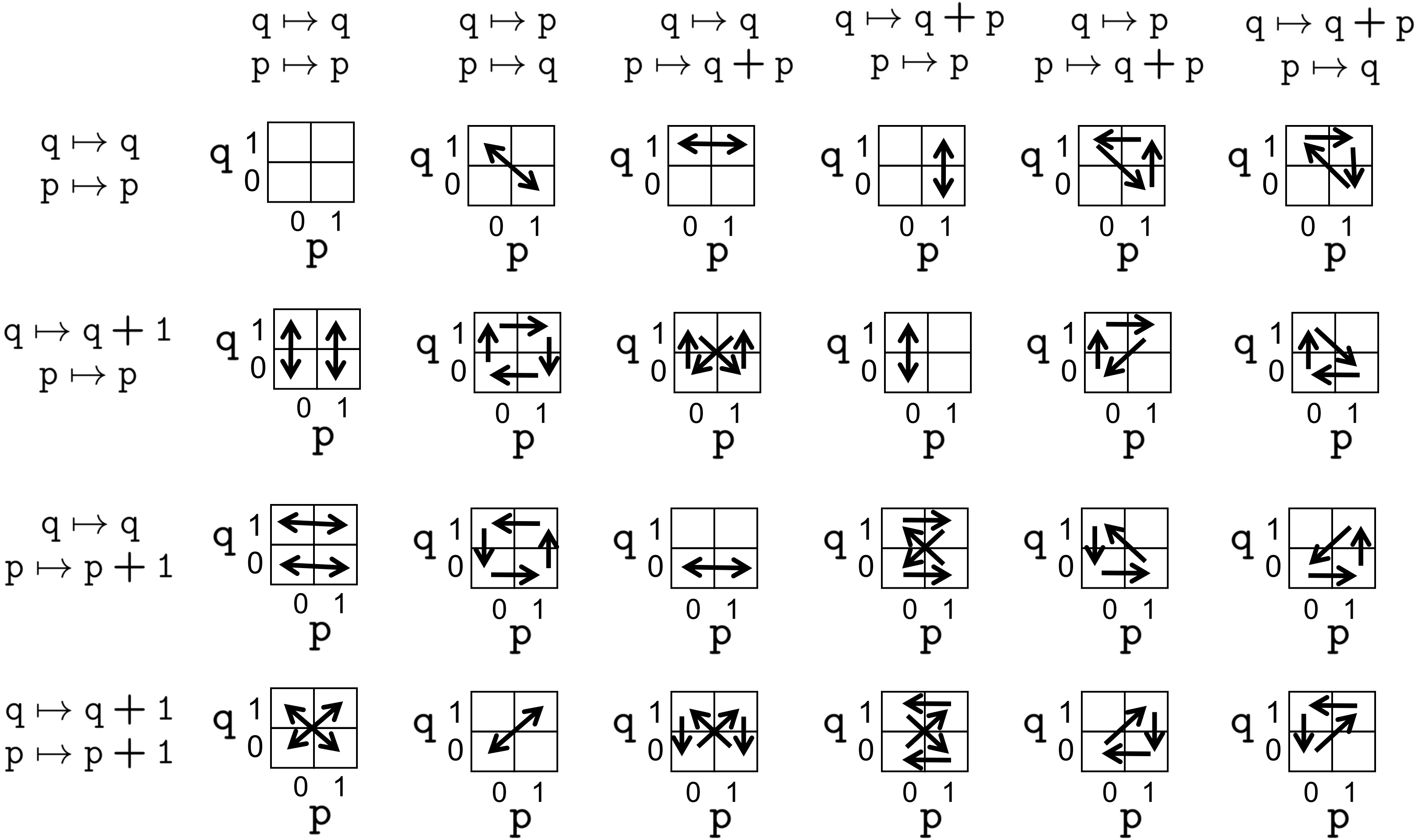
X $\quad$ P $\quad$ $X + P$



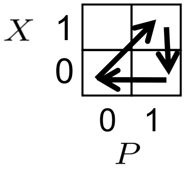
$\{|0\rangle, |1\rangle\}$ $\quad$ $\{|+\rangle, |-\rangle\}$ $\quad$ $\{|+i\rangle, |-i\rangle\}$

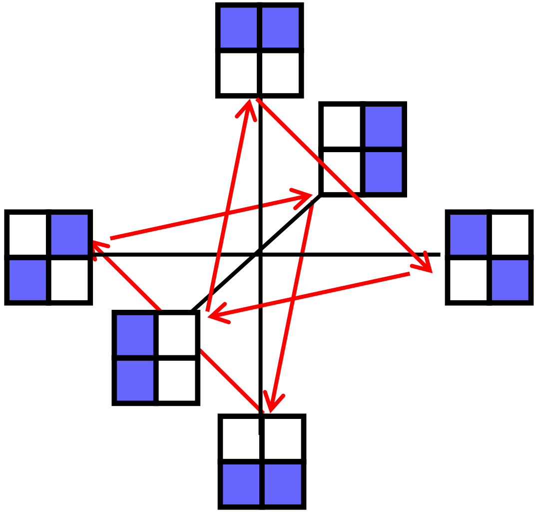
Information about the complementary variable is lost

Reversible transformations

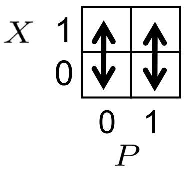

q ↦ q
p ↦ p
q ↦ p
p ↦ q
q ↦ q
p ↦ q + p
q ↦ q + p
p ↦ p
q ↦ p
p ↦ q + p
q ↦ q + p
p ↦ q
q ↦ q
p ↦ p
q ↦ q + 1
p ↦ p
q ↦ q
p ↦ p + 1
q ↦ q + 1
p ↦ p + 1
q
p
0
1

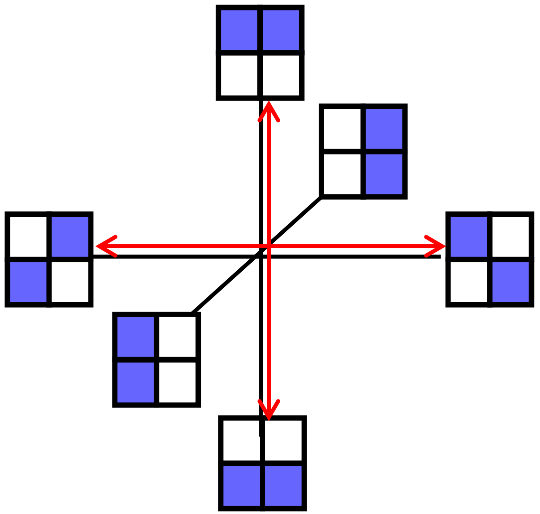
A 3-cycle



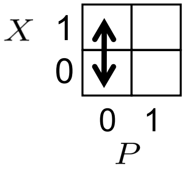


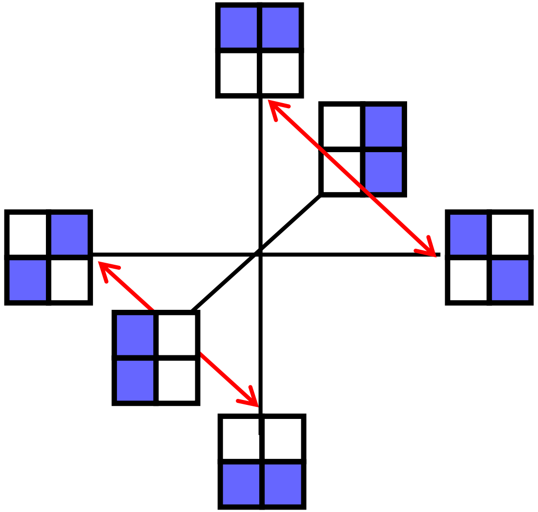
A pair of 2-cycles



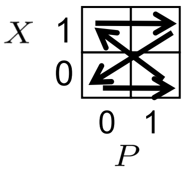


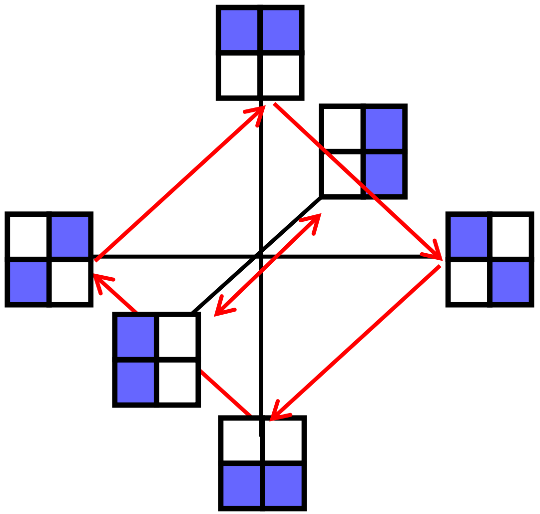
A 2-cycle





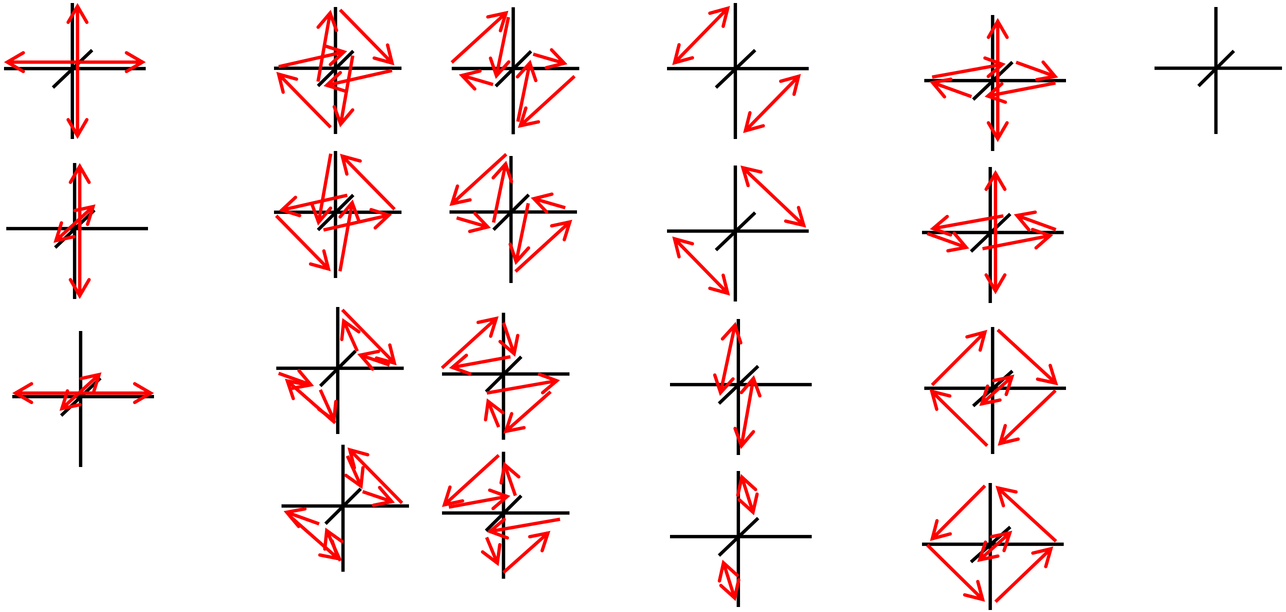
A 4-cycle

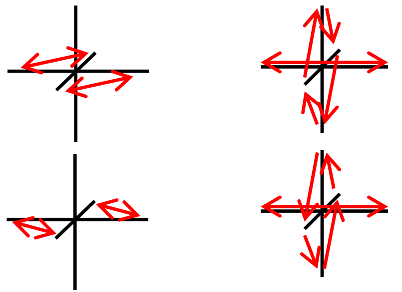


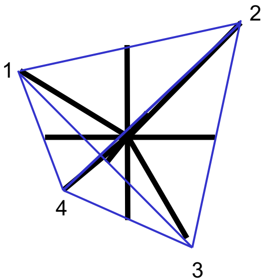


Reversible transformations:

Pairs of 2-cycles | 3-cycles | 2-cycles | 4-cycles | identity



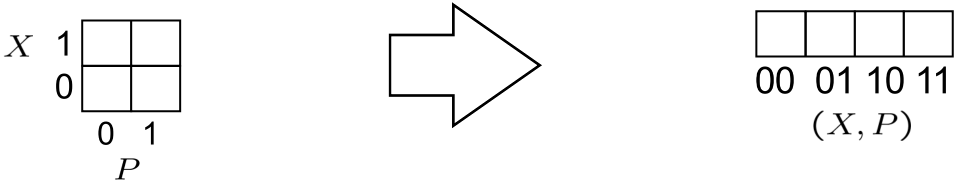


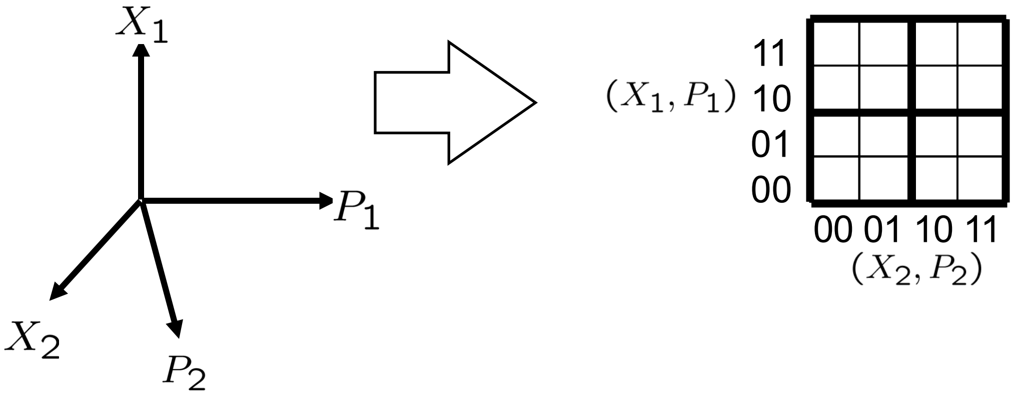


Symmetries of the tetrahedron under rotations and reflections

A pair of bits

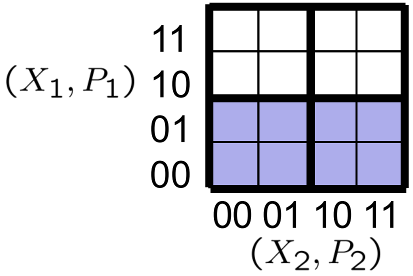
Canonical variables $a_1 X_1 + b_1 P_1 + a_2 X_2 + b_2 P_2$



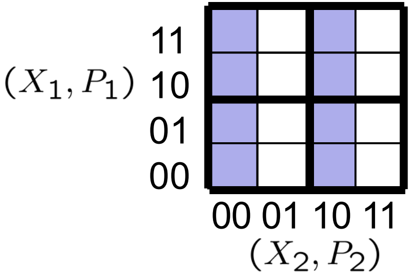


1 variable known

X_1 known

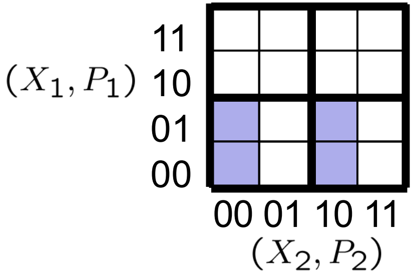


P_2 known



2 variables known

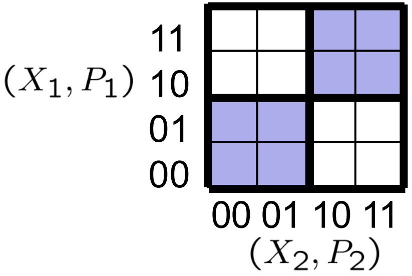
X_1 and P_2 known



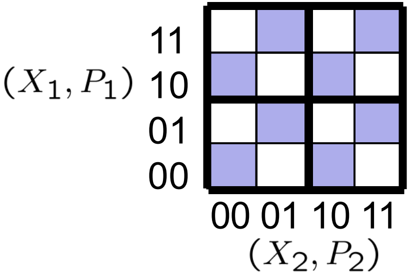
$$|0\rangle \otimes |+\rangle$$

1 variable known

$X_1 + X_2$ known

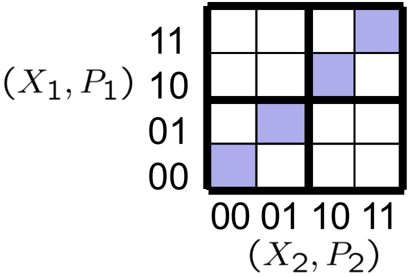


$P_1 + P_2$ known



2 variables known

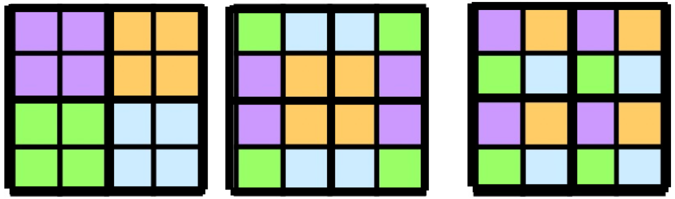
$X_1 + X_2$ and $P_1 + P_2$ known



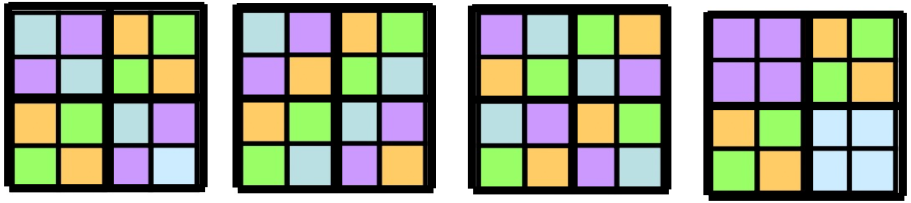
$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$$

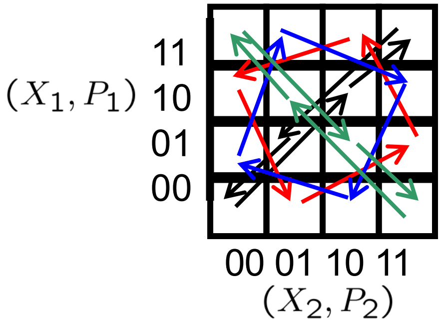
Measurements

Product basis Measurements



Entangled basis Measurements




11
(X_1, P_1) 10
01
00
00 01 10 11
(X_2, P_2)

Operational
phenomenology
reproduced

No cloning

Quantum: A cloning process for a set $\{|\psi_i\rangle\}$ satisfies

$$|\psi_i\rangle\,|\chi\rangle \rightarrow |\psi_i\rangle\,|\psi_i\rangle$$

Example: $\{|1\rangle\,, |+\rangle\}$

$$\begin{aligned} |1\rangle\,|0\rangle &\rightarrow |1\rangle\,|1\rangle \\ |+\rangle\,|0\rangle &\rightarrow |+\rangle\,|+\rangle \end{aligned}$$

Overlaps are:

$$|\,\langle 1|+\rangle\,\langle 0|0\rangle\,|^2 \neq |\,\langle 1|+\rangle\,\langle 1|+\rangle\,|^2$$

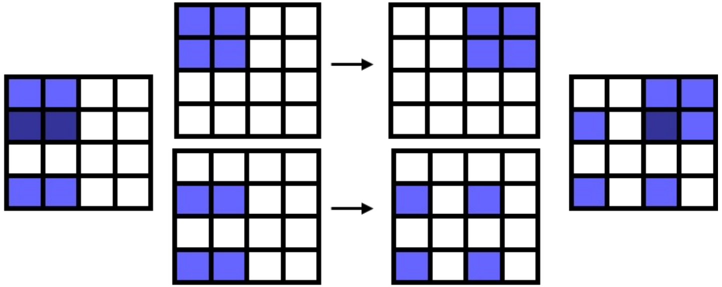
$$1/2 \neq 1/4$$

Toy theory: A cloning process for a set $\{(a_i \vee b_i)\}$ satisfies

$$(a_i \vee b_i)\cdot(c \vee d) \rightarrow (a_i \vee b_i)\cdot(a_i \vee b_i)$$

Example: $\{(3 \vee 4), (1 \vee 3)\}$

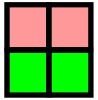
$$\begin{aligned} (3 \vee 4)\cdot(1 \vee 2) &\rightarrow (3 \vee 4)\cdot(3 \vee 4) \\ (1 \vee 3)\cdot(1 \vee 2) &\rightarrow (1 \vee 3)\cdot(1 \vee 3) \end{aligned}$$



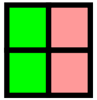
Overlaps are: $1/2 \neq 1/4$

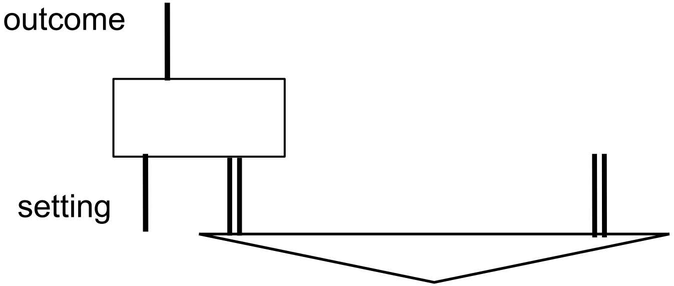
EPR steering

$\{|0\rangle, |1\rangle\}$

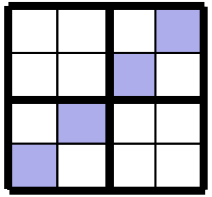


$\{|+\rangle, |-\rangle\}$



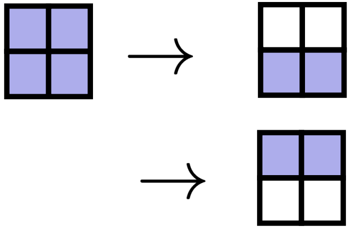


$\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$



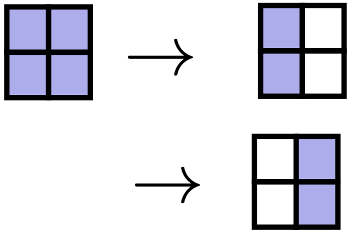
$$\frac{1}{2}I \rightarrow |0\rangle\langle 0|$$

$$\rightarrow |1\rangle\langle 1|$$



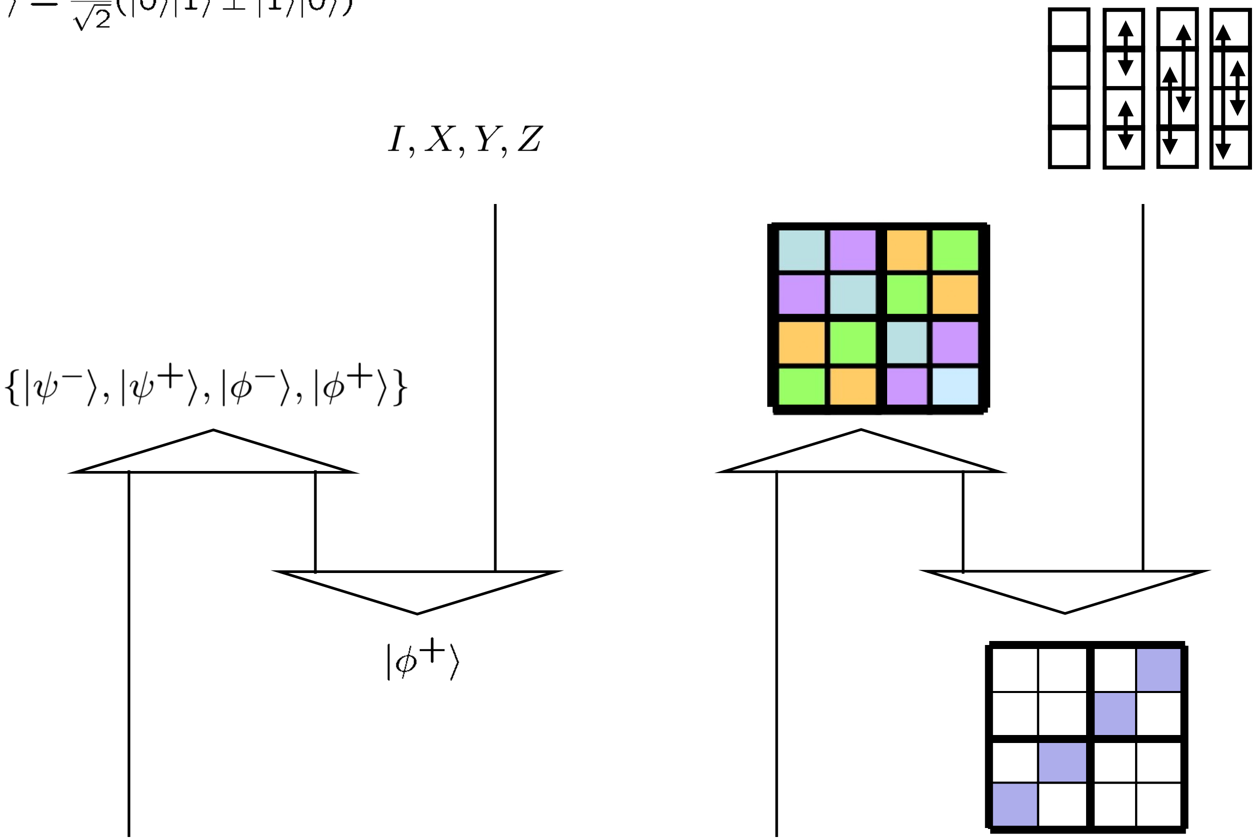
$$\frac{1}{2}I \rightarrow |+\rangle\langle +|$$

$$\rightarrow |-\rangle\langle -|$$



Teleportation

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$$
$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle)$$



Operational phenomenology reproduced:

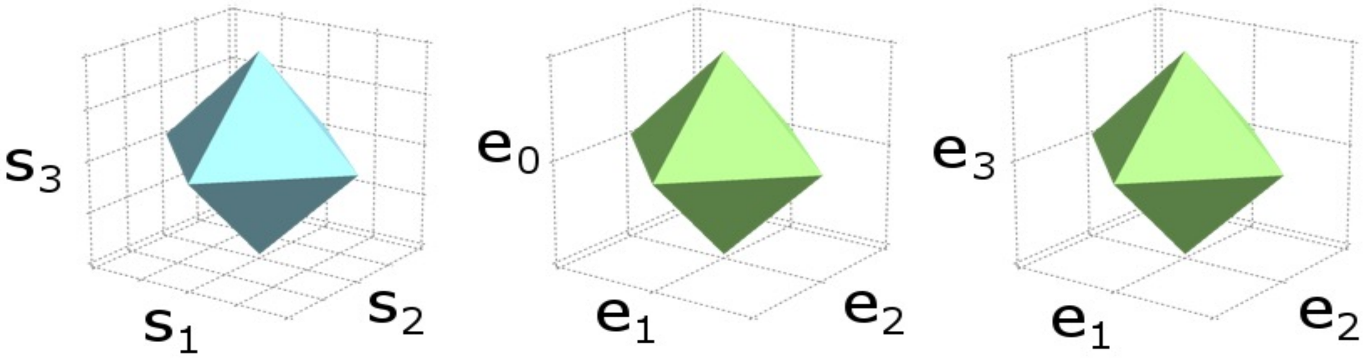
The TRAP phenomenology for:

Uncertainty relations
Wave-particle duality
No cloning
No broadcasting
Noncommutativity
EPR steering
Information gain-disturbance
Teleportation

Entanglement manipulation
Error correction
Metrology
State update rule
error-free state discrimination
Unambiguous state discrimination
Quantum eraser
Delayed Choice

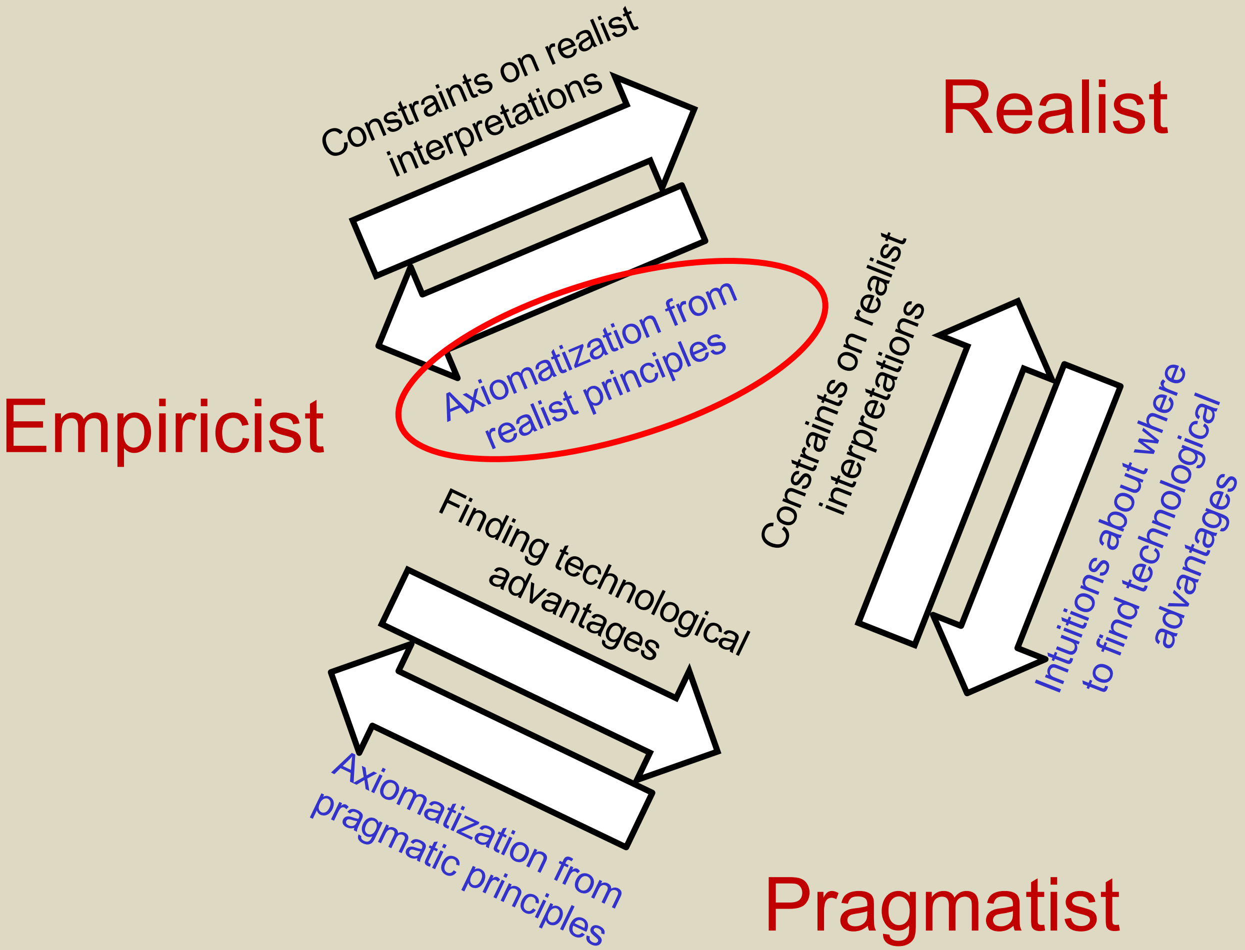
The toy theory as a GPT

GPT characterization of convex closure of toy theory



What lesson can we draw from the fact that the toy theory reproduces so many operational features of quantum theory?

It suggests strongly that different quantum states represent different ways of knowing, not different ways of being



Realist
Empiricist
Pragmatist
Constraints on realist interpretations
Axiomatization from realist principles
Constraints on realist interpretations
Intuitions about where to find technological advantages
Finding technological advantages
Axiomatization from pragmatic principles

Resource theories

Realist

Ontological models

GPTs w/ symmetries

Process theories

Interventionist Causal models

Empiricist

Generalized probabilistic theories

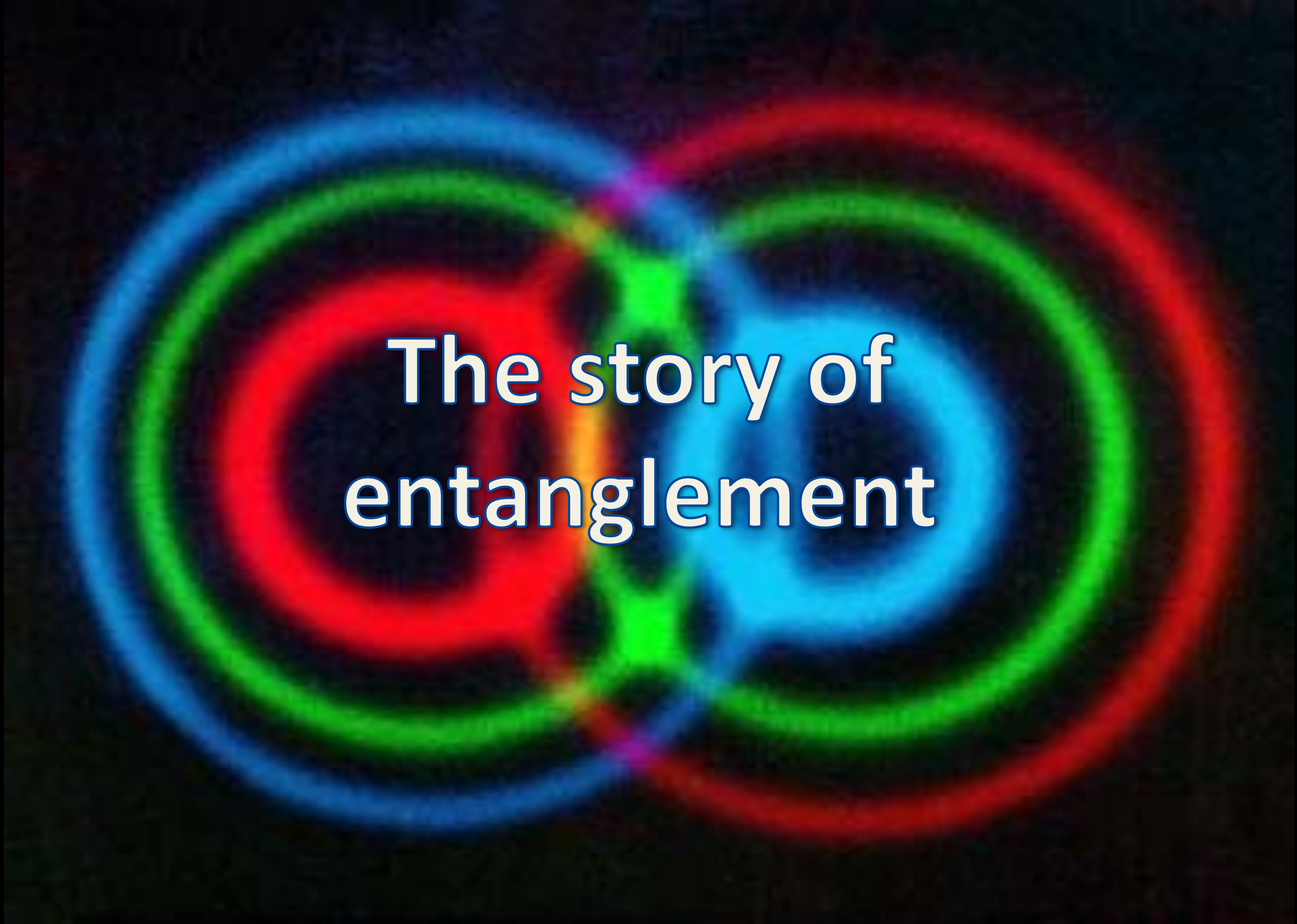
Theory of Bayesian inference

Device-independent paradigm

Information theory

Resource theories

Pragmatist


The story of
entanglement

Quantum resource theories

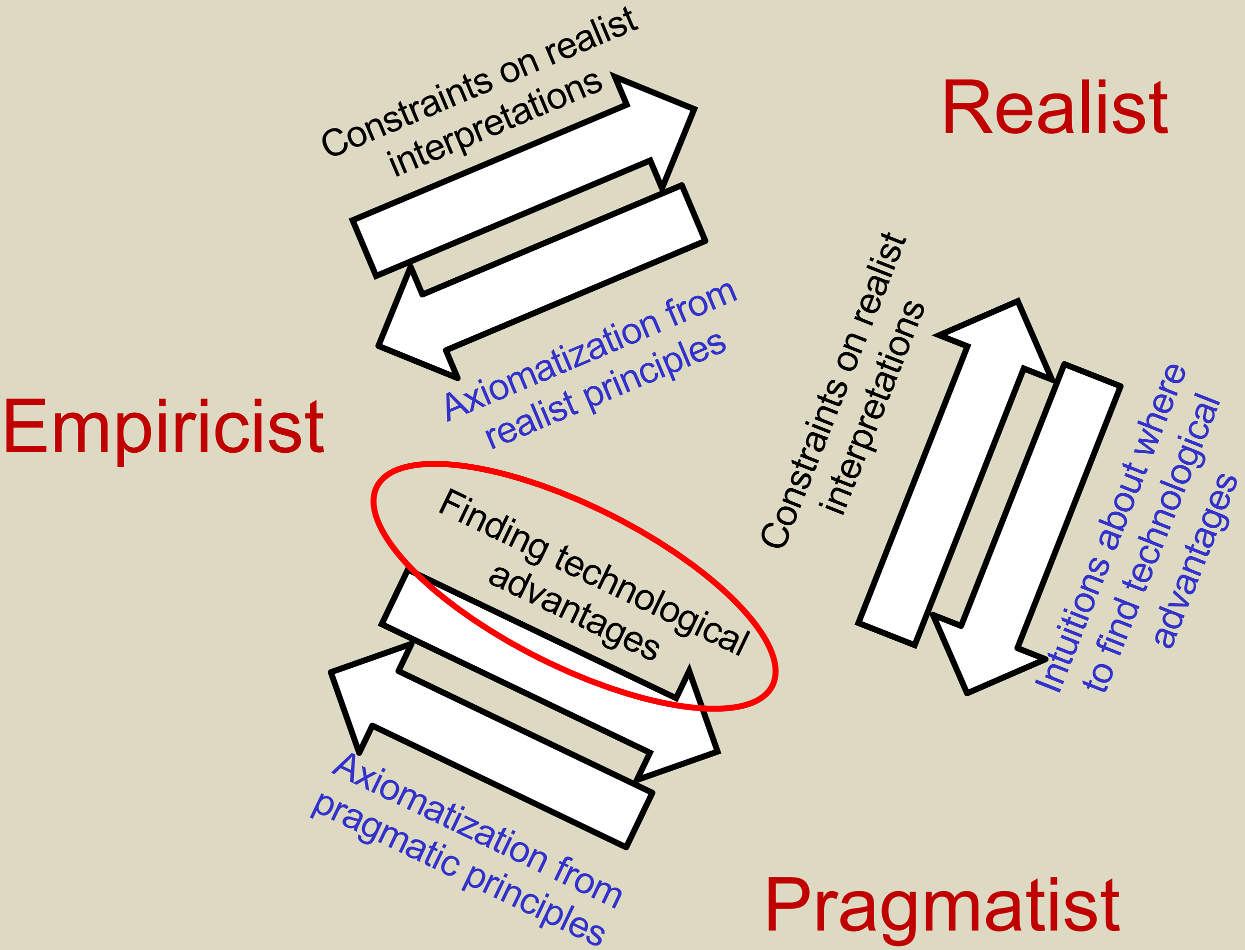
	Entanglement	Asymmetry	Athermality
Free operations	Local Operations and classical communication	Symmetric operations	Thermal operations
Resources	Entangled states Entangling operations (Quantum channels)	Asymmetric states Asymmetric operations (Ability to prepare reference frames)	Athermal states Athermal operations (Ability to do work)

Typical technical problems:

- What are the necessary and sufficient conditions under which one state can be converted to another **deterministically** by the free operations?
- **Stochastically**?
- **Catalytically**?
- Rates of conversion for **arbitrarily many copies**?
- How does one define **measures of the resource**?
- What are the necessary and sufficient conditions under which a **resource state can simulate a resource transformation**?
- For a given **operational task** that uses the resource, what measure quantifies performance?

Why do it?

Characterizing a resource theory helps with characterizing the possible limits of performance on operational tasks that it powers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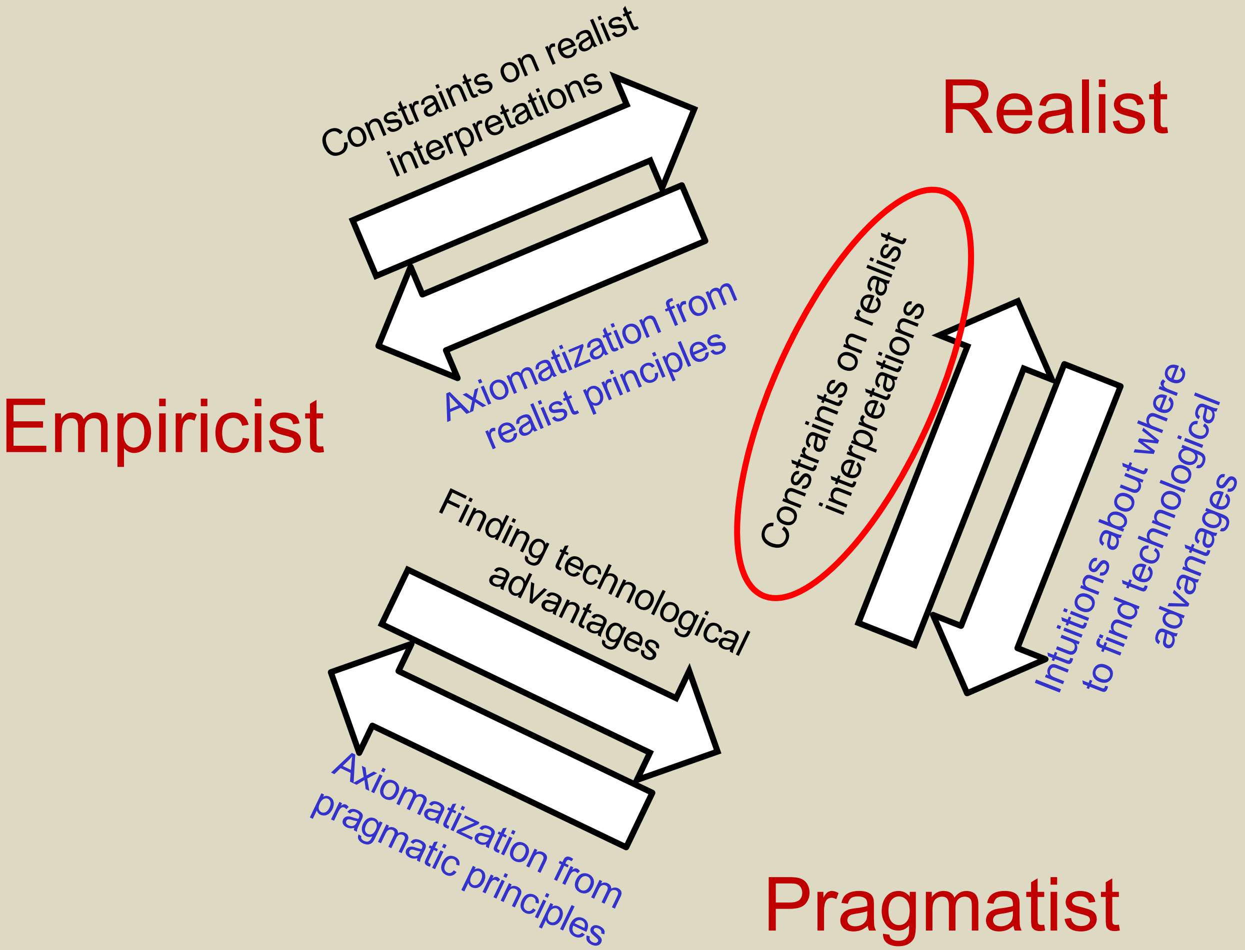
Why do it?

The discipline of solving technical problems tends to lead to clarification of conceptual issues



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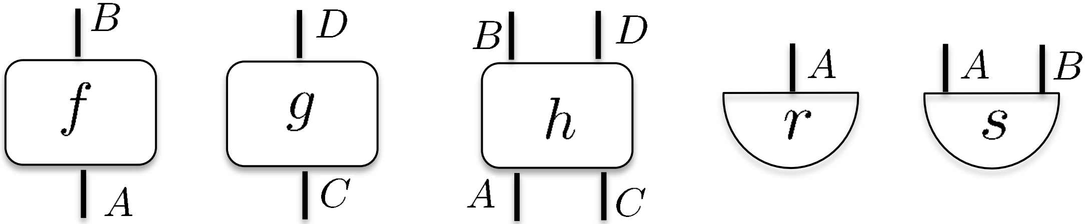
Process theory T:

System types A, B, C,…
(including the trivial system)

Processes f, g, h, r, s, …

Closed under parallel and sequential composition



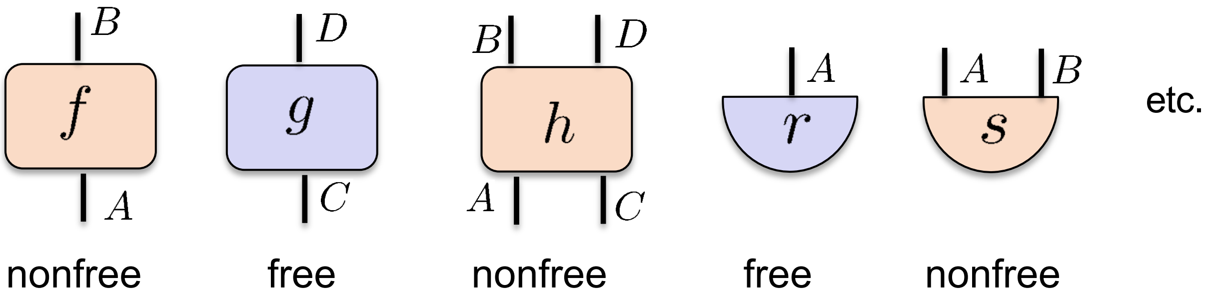
etc.

Coecke, Fritz, RWS, Information and Computation **250**, 59 (2016).

Process theory T: System types A, B, C,…
(including the trivial system)

Processes f, g, h, r, s, …

Closed under parallel and sequential composition

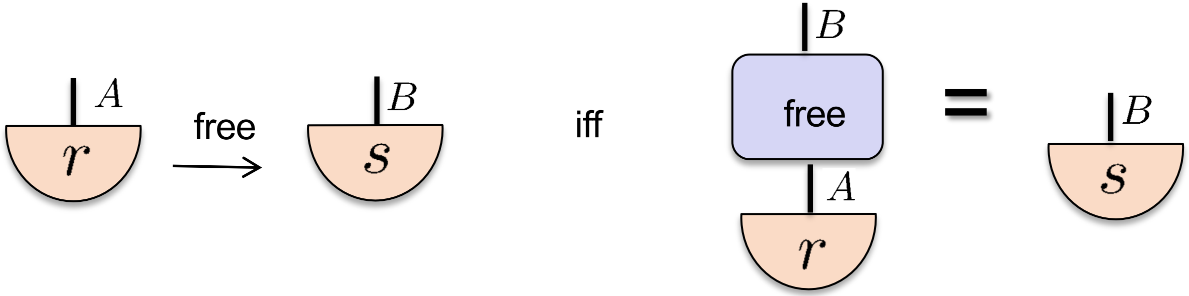


Subtheory consisting of “free” processes T_{free}:

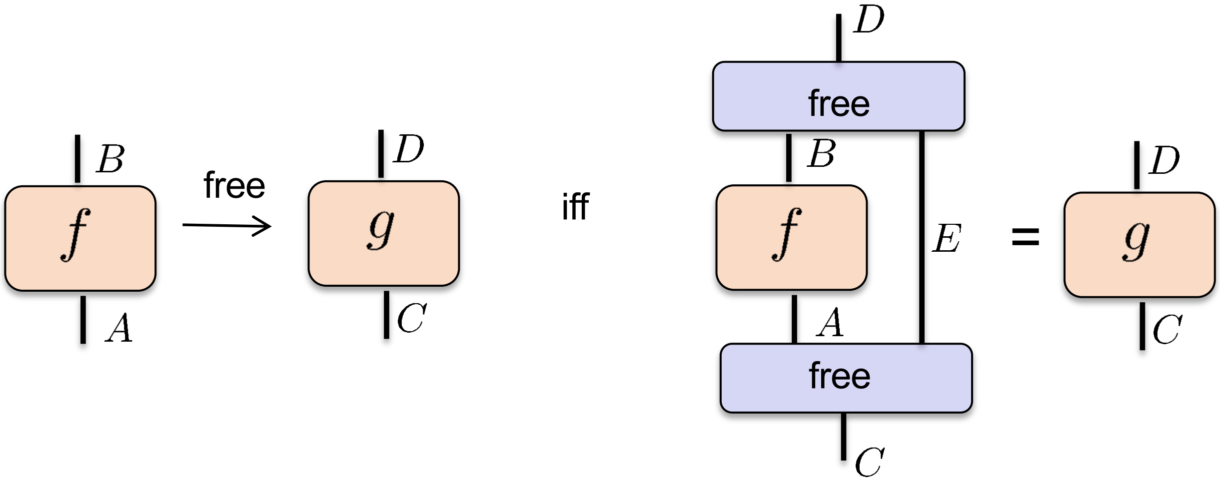
A resource theory is a
partitioned process theory
(T, T_{free})

Coecke, Fritz, RWS, Information and Computation **250**, 59 (2016).

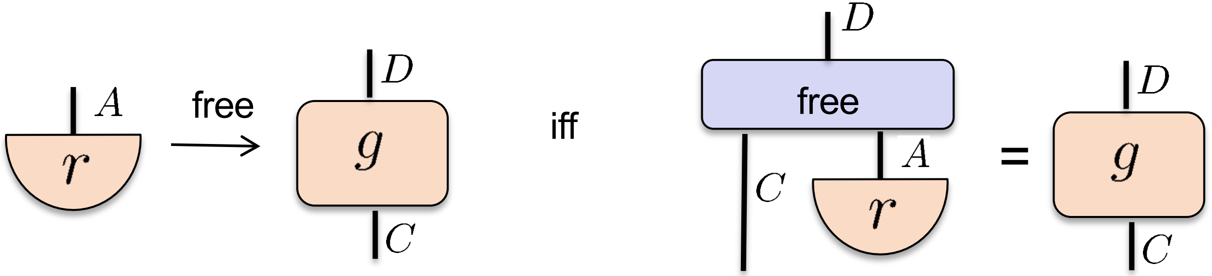
Conversion of state resources:



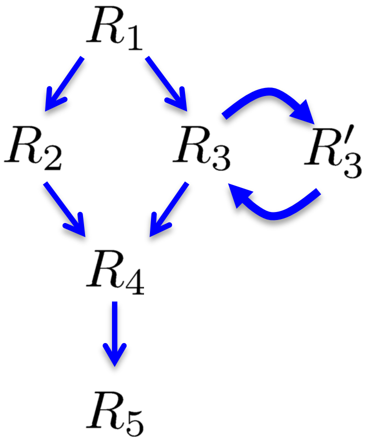
Conversion of channel resources:



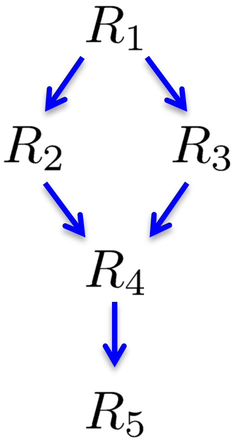
Conversion of state resource to channel resource:



Conversion relations induce a preorder of resources



Quotienting equivalences, one gets a partial order of resources



The nature of the partial order teaches us about the resource

Properties of a partial order

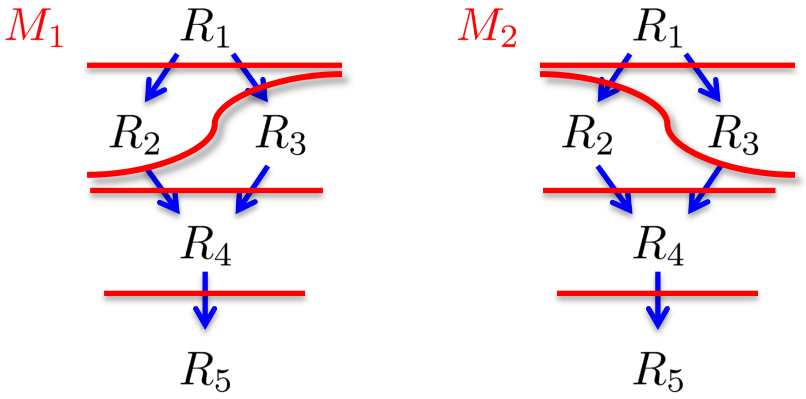
- Totally ordered (no incomparable elements) or not
- weak (incomparability relation is transitive) or not
- Height (cardinality of the largest chain)
- Width (cardinality of the largest antichain)
- Locally finite (finite number of inequivalent elements between any two ordered elements) or not

Measures of a resource

Def'n: A function *M* from resources to the reals is a resource monotone if

$$\forall R_1, R_2 : R_1 \xrightarrow{free} R_2 \quad \Longrightarrow \quad M(R_1) \geq M(R_2)$$

Equivalently, M must respect the partial order



If it is not a total order, there cannot be "one measure to rule them all"

A family of monotones $\{M_i\}_i$ is complete if it completely characterizes the pre-order,

$$\forall R_1, R_2 : R_1 \xrightarrow{free} R_2 \iff \forall i : M_i(R_1) \geq M_i(R_2)$$

The resource theory of asymmetry


Clock synchronization


Frame alignment


Global positioning

Pierre Curie
(1859 –1906)

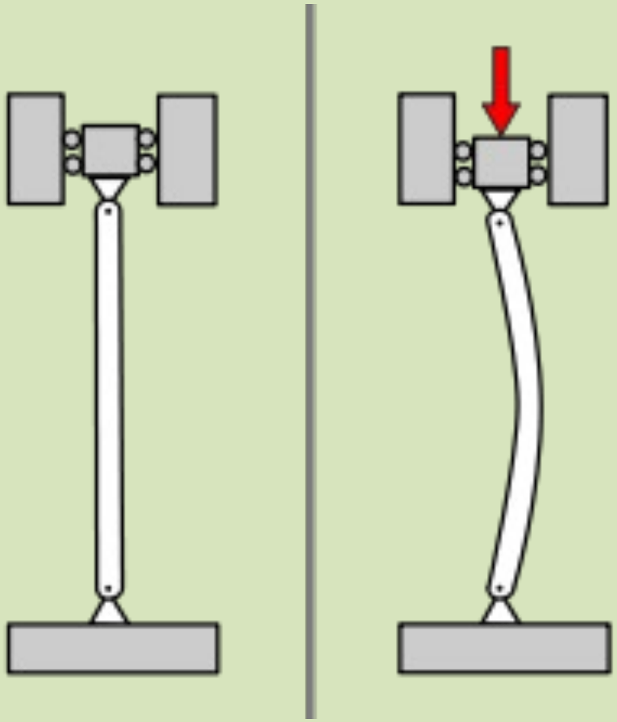
Curie’s principle

Any asymmetry in a physical effect must be found in its causes

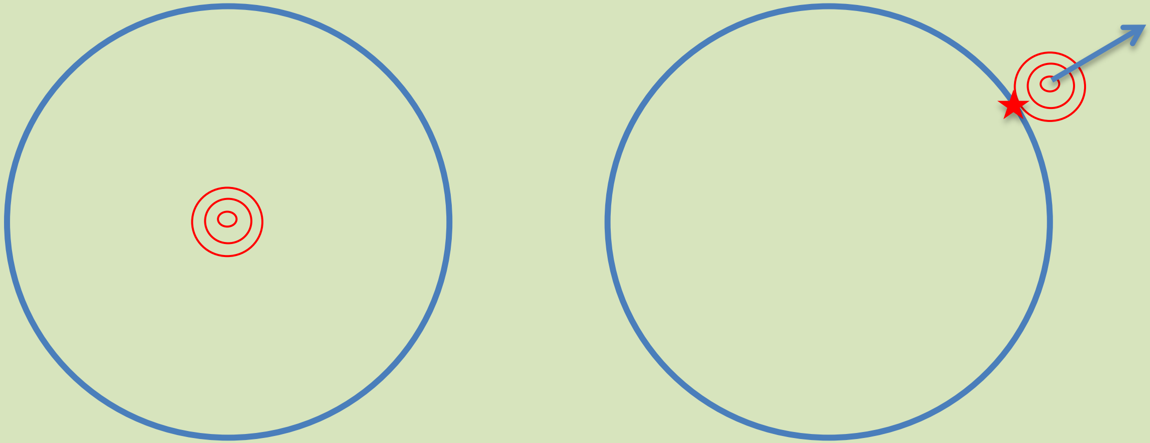
A quantitative version of Curie’s principle

The measure of asymmetry in a physical effect cannot be greater than the measure of asymmetry in its cause

Violation of Curie's principle?



Violation of Curie's principle?

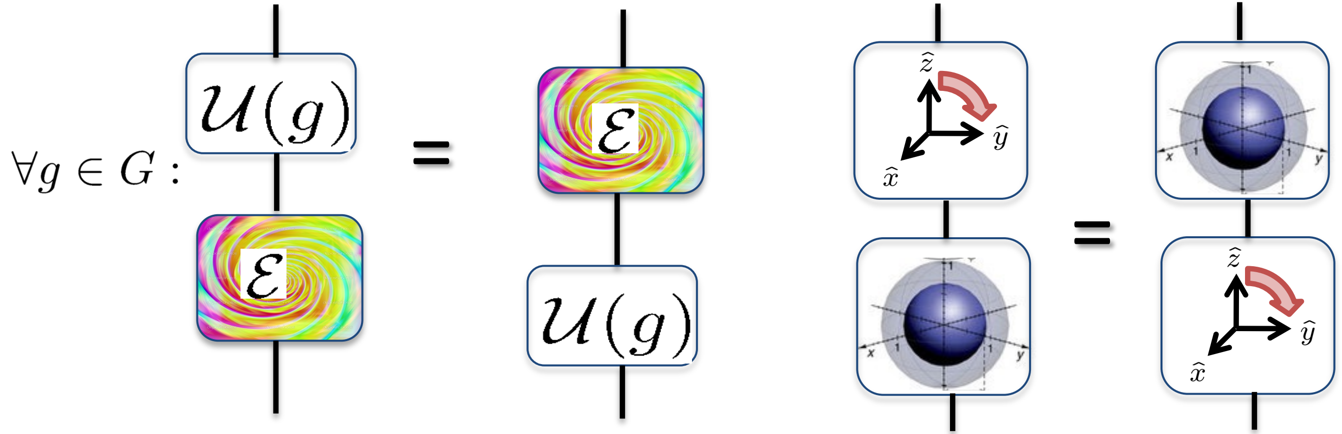


Symmetric operations

A symmetry is defined by a group G
and a unitary representation $g \in G \to U(g)$

A symmetric operation is any completely-positive trace-preserving map $\mathcal{E}$ that commutes with the action of the group

$$\forall g \in G : \quad \mathcal{E}[U(g)\rho U^\dagger(g)] = U(g)\mathcal{E}[\rho]U^\dagger(g)$$



Measures of asymmetry

Def'n: A function *A* from states to the reals is a measure of asymmetry if

$$\rho \xrightarrow{sym} \sigma \quad \Rightarrow \quad A(\rho) \geq A(\sigma)$$

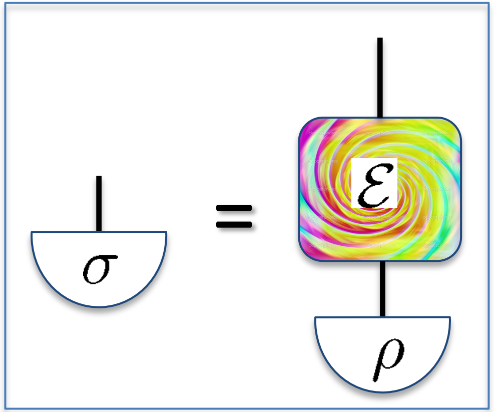
How can we find measures of asymmetry?

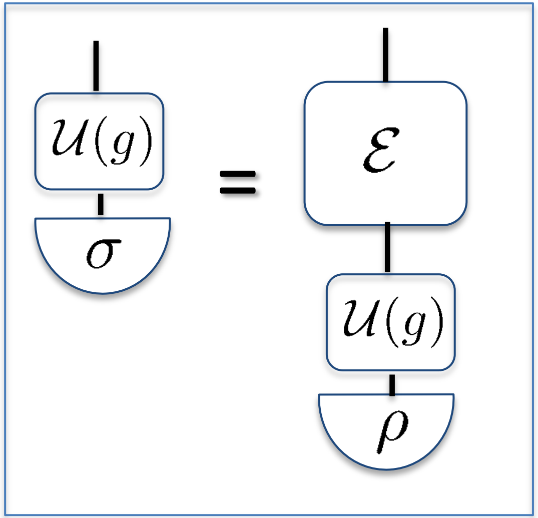
Bridge lemma:

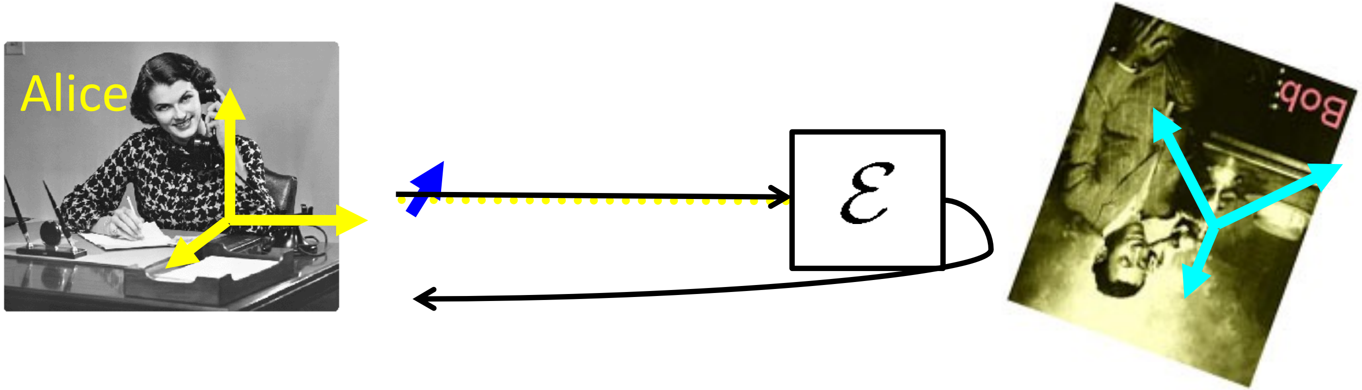
$$\rho \xrightarrow{sym} \sigma$$

$\Leftrightarrow$

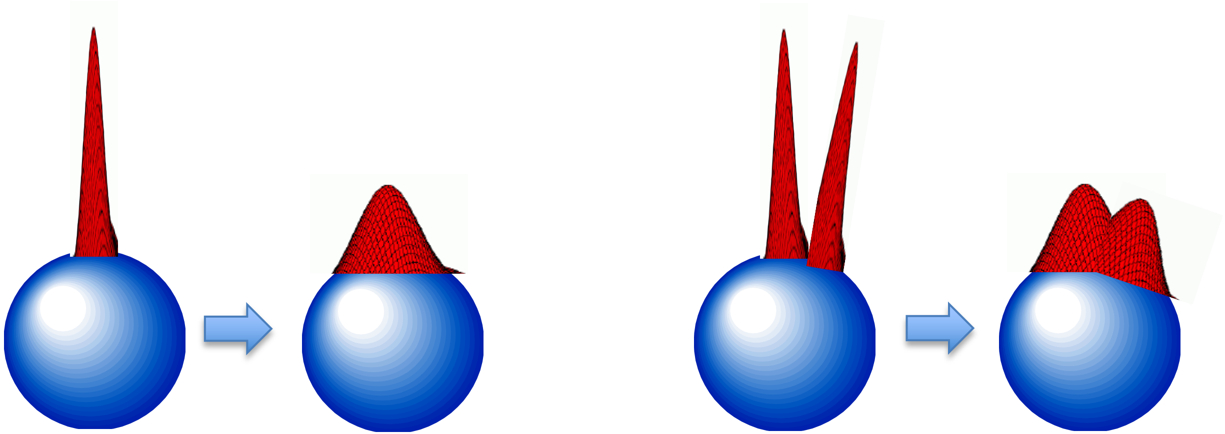
$$\forall g \in G: \quad U(g)\rho U^{\dagger}(g) \rightarrow U(g)\sigma U^{\dagger}(g)$$






Alice
$\mathcal{E}$
Bob

Classical analogue



Measure of asymmetry of the state ρ = Measure of information about G encoded in the orbit of ρ

$$\{U(g)\rho U^\dagger(g) : g \in G\}$$

Holevo asymmetry

$$A_p^{\mathrm{Hol}}(\rho) \equiv S(\mathcal{G}_p(\rho)) - S(\rho)$$

$$\mathcal{G}_p(\rho) \equiv \int dg\ p(g)\ U(g)\rho U^\dagger(g)$$

$$S(\rho) \equiv -\mathrm{tr}(\rho \log \rho)$$

l_1-norm-based asymmetry

$$A_L^{l_1}(\rho) \equiv \|[\rho, L]\|_1$$

$$\|A\|_1 \equiv \mathrm{tr}(\sqrt{A^\dagger A})$$

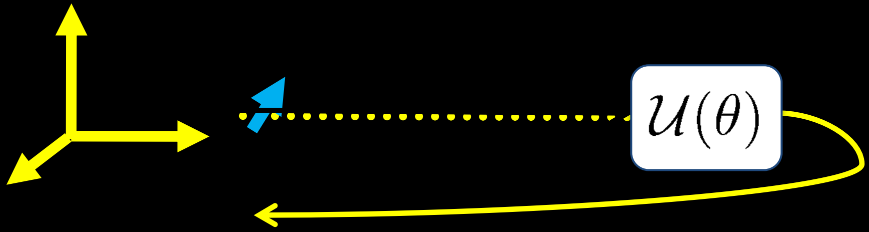
Wigner-Yanase-Dyson skew information

$$A_{L,s}^{\mathrm{skew}}(\rho) \equiv \mathrm{tr}(\rho L^2) - \mathrm{tr}(\rho^s L \rho^{1-s} L)$$

$$s \in (0,1) \cup (1,\infty)$$

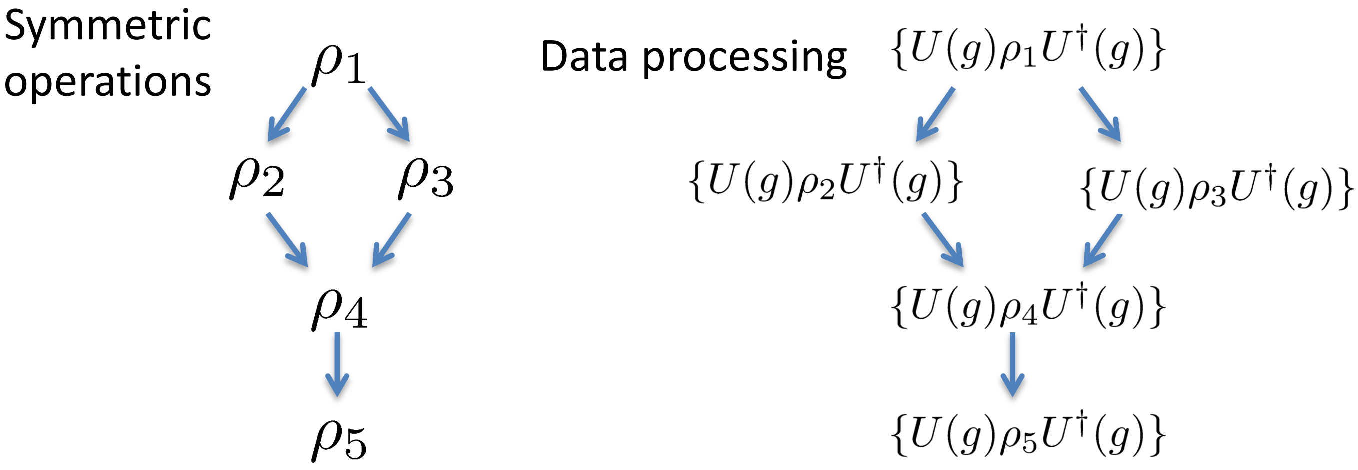
Marvian and RWS, Nature Commun. 5, 3821 (2014)

Quantum Metrology



Variance in unbiased estimator of a phase θ $\leq \dfrac{1}{4\left(\mathrm{tr}(\rho L^2)-\mathrm{tr}(\rho^{1/2}L\rho^{1/2}L)\right)}$

Marvian and RWS, Nature Commun. 5, 3821 (2014)



Symmetric operations
ρ_1
ρ_2
ρ_3
ρ_4
ρ_5
Data processing
$\{U(g)\rho_1 U^\dagger(g)\}$
$\{U(g)\rho_2 U^\dagger(g)\}$
$\{U(g)\rho_3 U^\dagger(g)\}$
$\{U(g)\rho_4 U^\dagger(g)\}$
$\{U(g)\rho_5 U^\dagger(g)\}$

Symmetric operations

$$\rho_1 \rightarrow \rho_2, \quad \rho_1 \rightarrow \rho_3$$

$$\rho_2 \rightarrow \rho_4, \quad \rho_3 \rightarrow \rho_4$$

$$\rho_4 \rightarrow \rho_5$$

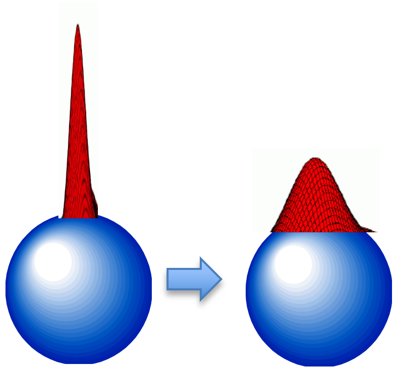
Data processing

$$\{U(g)\rho_1 U^\dagger(g)\} \rightarrow \{U(g)\rho_2 U^\dagger(g)\}, \quad \{U(g)\rho_1 U^\dagger(g)\} \rightarrow \{U(g)\rho_3 U^\dagger(g)\}$$

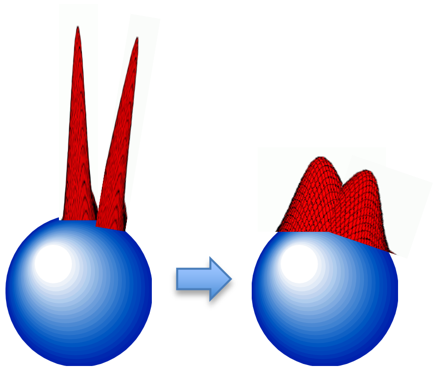
$$\{U(g)\rho_2 U^\dagger(g)\} \rightarrow \{U(g)\rho_4 U^\dagger(g)\}, \quad \{U(g)\rho_3 U^\dagger(g)\} \rightarrow \{U(g)\rho_4 U^\dagger(g)\}$$

$$\{U(g)\rho_4 U^\dagger(g)\} \rightarrow \{U(g)\rho_5 U^\dagger(g)\}$$

No increase of asymmetry under symmetric processing



No increase of information under data processing

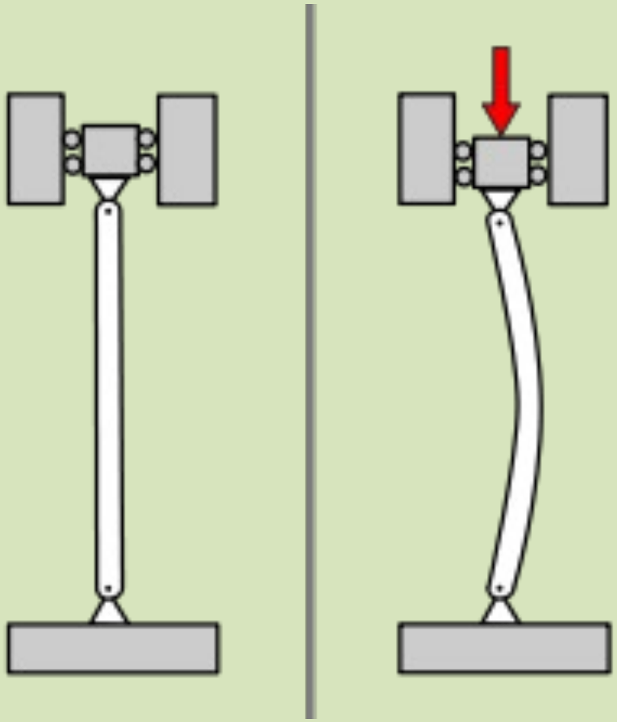


Principle that information cannot be increased by data processing

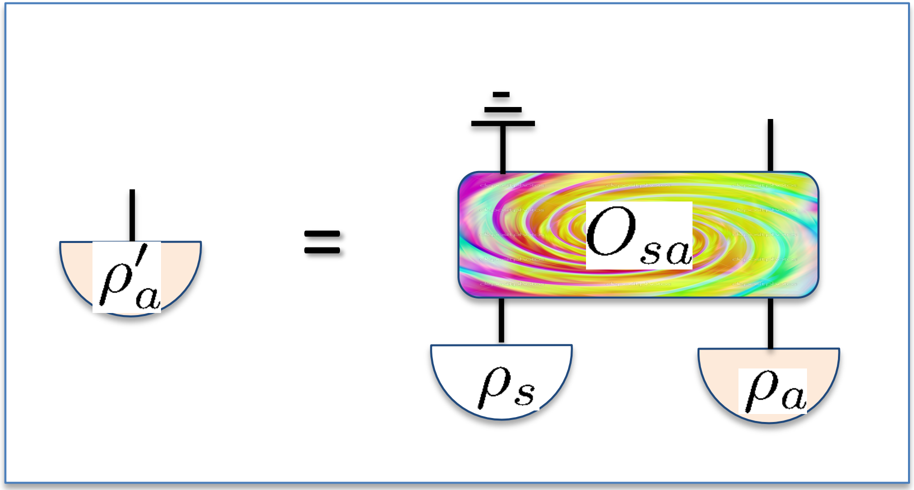
⇨

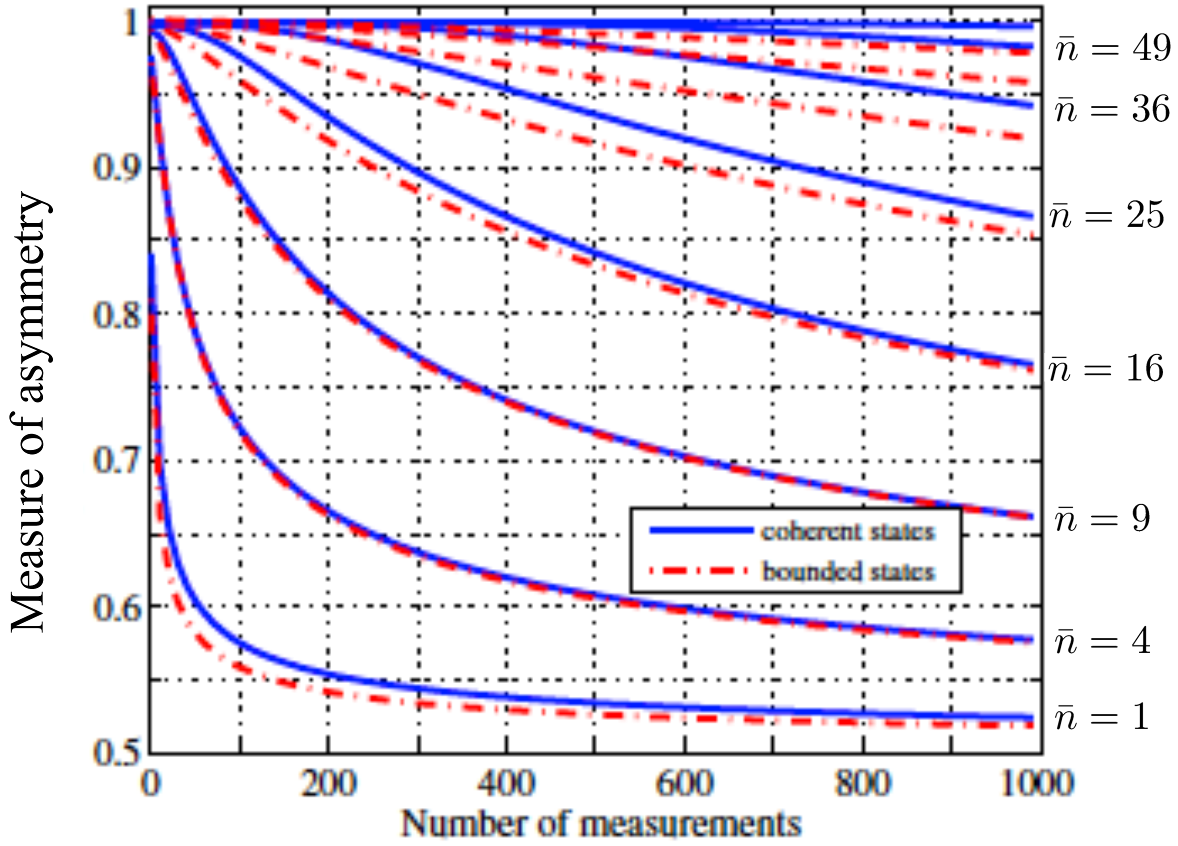
quantitative Curie's principle

Violation of Curie's principle?



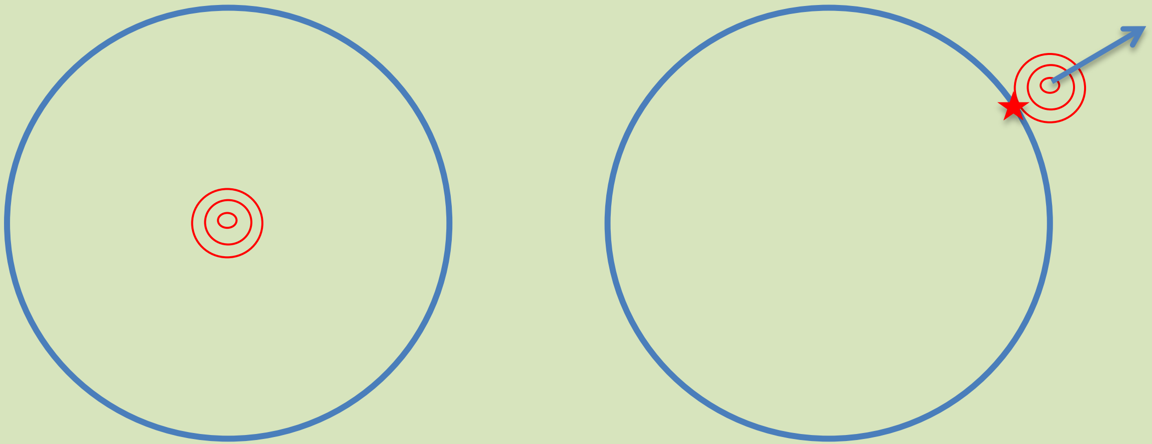
Degradation of asymmetric states





Bartlett, Rudolph, RWS, Turner, New J. Phys. 8, 58 (2006)

Violation of Curie's principle?



Some principles for theory construction

Slow Revolutions in Physics

The use of action principles
(Fermat, Maupertuis, Lagrange, Hamilton, Feynman,…)

The use of symmetry principles
(Lagrange, Curie, Noether, Wigner,…)

The use of thermodynamic principles
(Carnot, Clausius, Kelvin, Gibbs, Boltzmann,…)

The use of information-theoretic principles
(Szilard, Jaynes, Wheeler, Bennett, …)

“Information is physical”

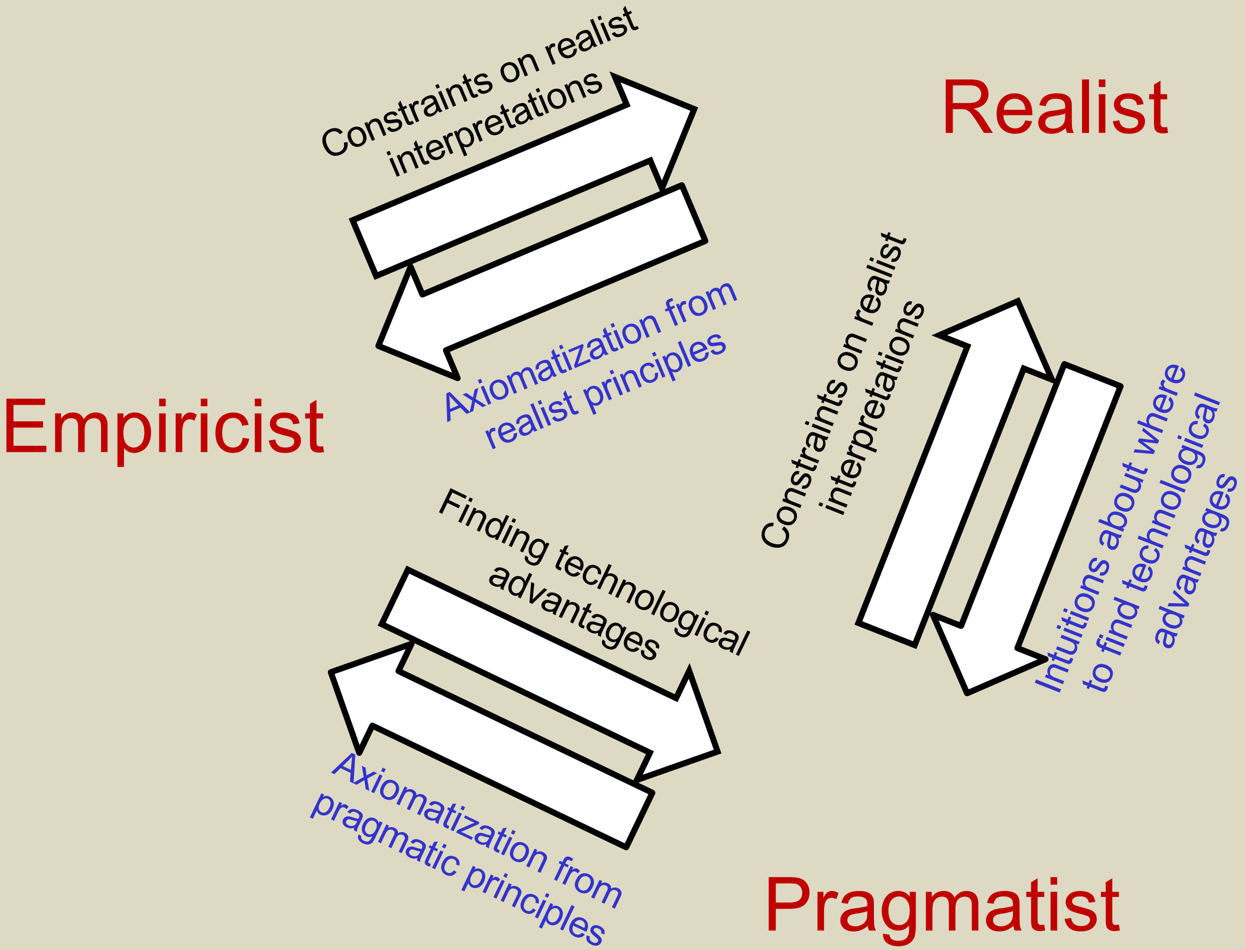
The possibilities for computation, communication and cryptography are determined by our best physical theories

“Physics is informational”

Adopting an information-theoretic perspective on physical theories can deepen our understanding of them and lead to new and transformative developments

The role of slow revolutions in theory discovery

Action principles + thermodynamic principles (de Broglie, Planck, Einstein)	→ Early quantum theory
Symmetry principles (Einstein)	→ Relativity theory
Action principles + symmetry principles (Dirac, Feynman)	→ QED
Information-theoretic principles	→ ???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Thanks!