Nonclassicality

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We have all heard that quantum theory is weird and can’t be explained “classically”.

rigorously, what does that mean?

Wave-particle duality?

Entanglement?

No-cloning?

Remote steering?

Nonlocality?

Teleportation?

Quantum interference?

Coherent superposition?

etc...

Contextuality

Uncertainty relations?

Classically explainable!

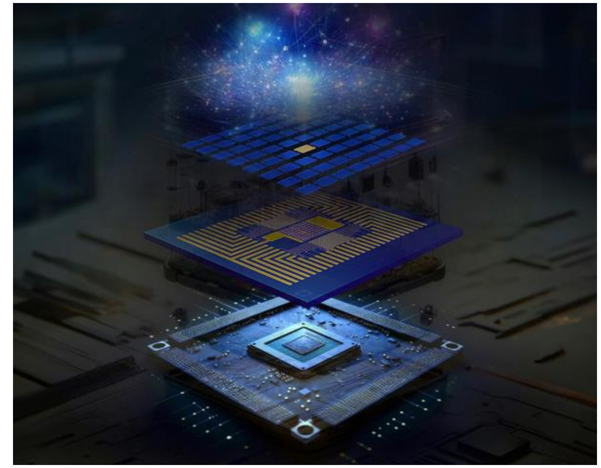
-noncommutativity
-complementarity
-interference
-no-cloning
-teleportation
-entanglement
-dense coding
-remote steering
-quantum eraser
-mmts must disturb
-ambiguity of mixtures
-no perfect state discr.
…

e.g., by Spekkens toy theory

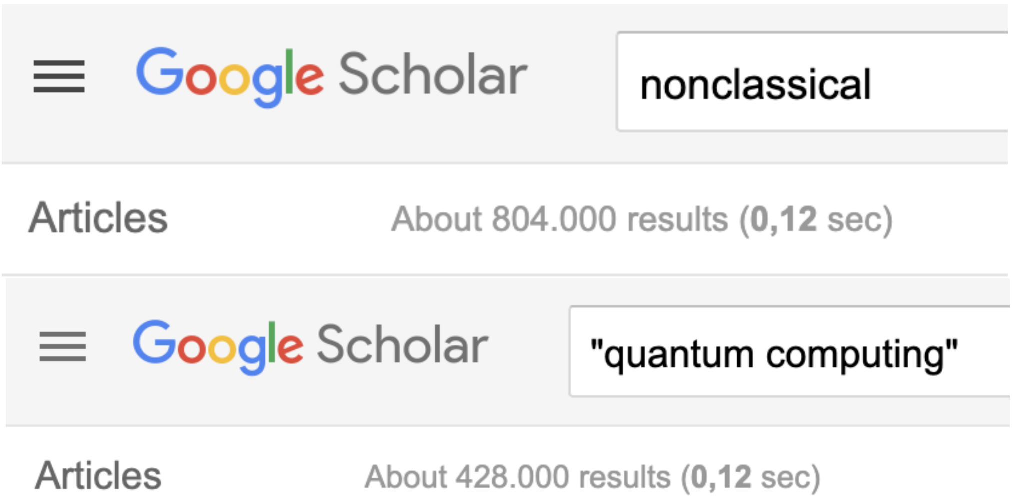
We need a principled way of dividing phenomena into
those which can be “explained classically”, and
those which are rigorous proofs of nonclassicality.

Why study (non)classicality?

Resources for quantum information processing



Intrinsic interest



Influencing how we interpret and extend quantum theory

quantum gravity?
quantum causal modeling?
quantum machine learning?
quantum thermodynamics?

A framework for theories

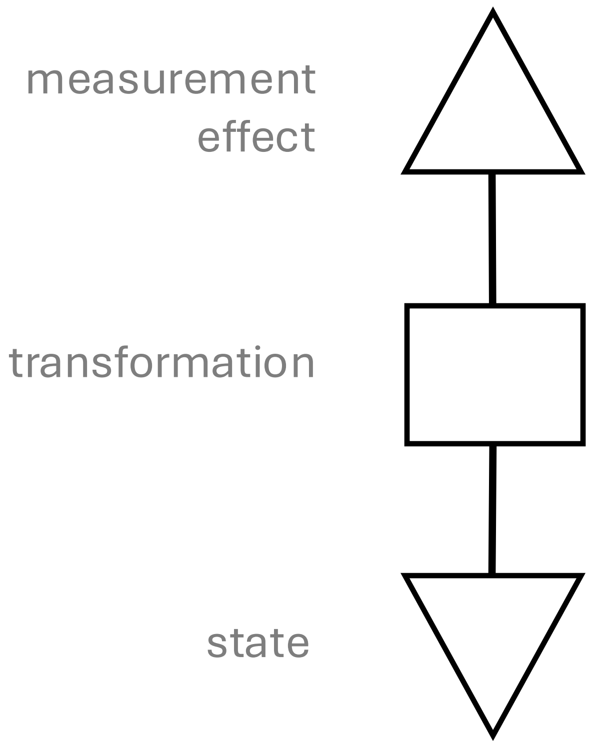
Generalized Probabilistic Theories

collection of systems

collection of processes

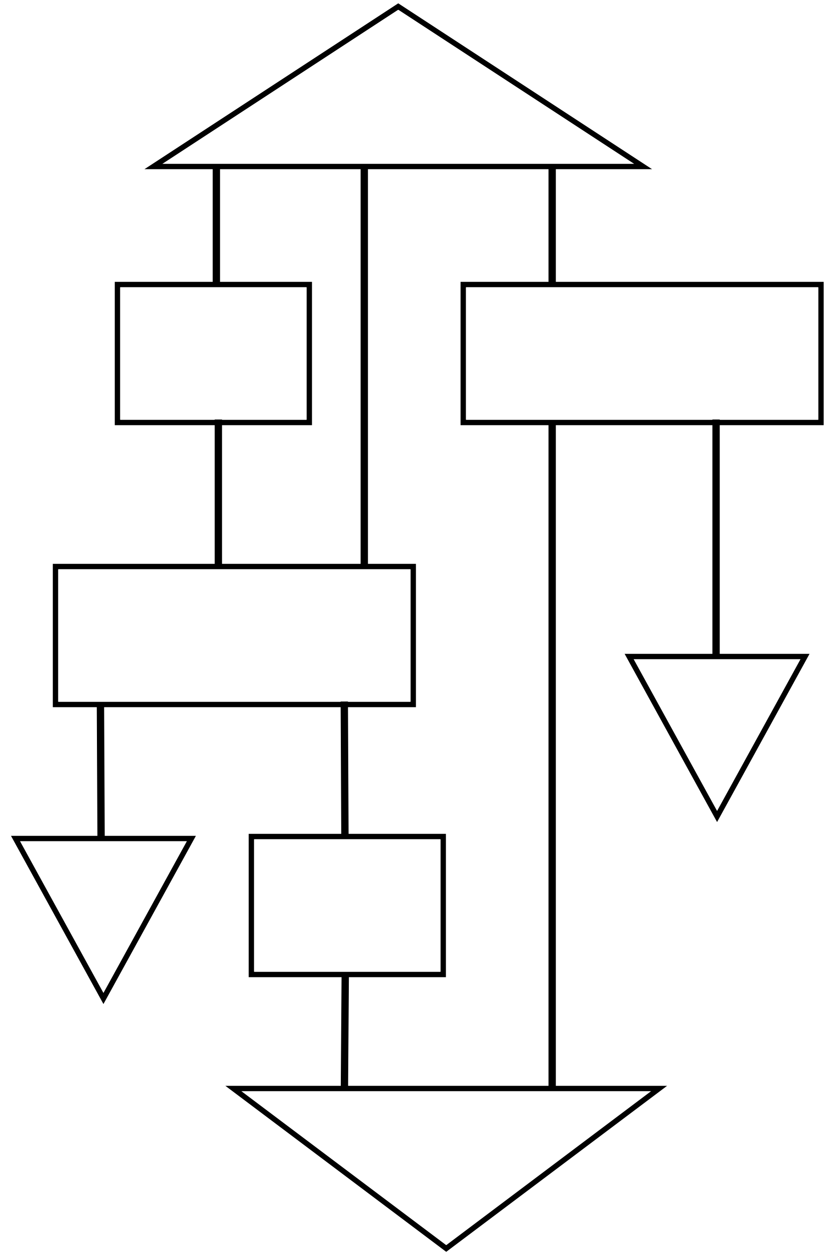
composition rule

⇒ observable probabilities



= prob(effect|transformed state)

Can generate arbitrary circuits/experiments by composition:

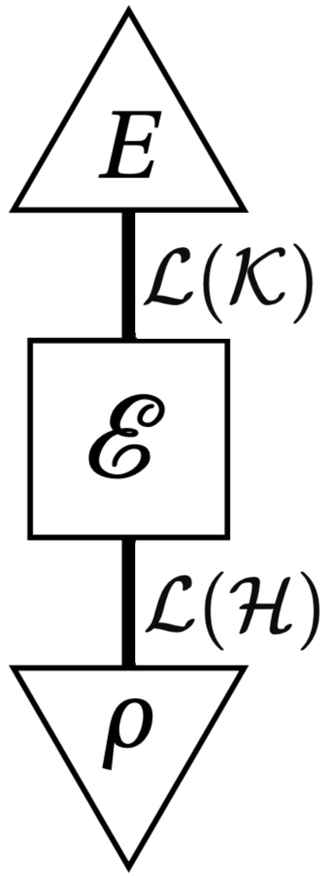


Quantum theory as a GPT

POVM element

CPTP maps

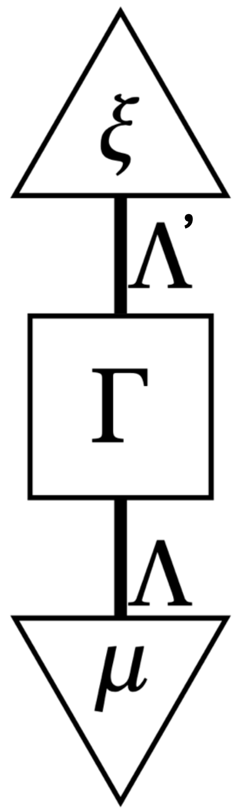
Density operator



Linear operators on Hilbert space

$$= \quad \mathrm{Tr}[E\mathcal{E}(\rho)]$$

The classical theory as a GPT



$$= \sum_{\lambda,\lambda'} \xi(\lambda')\Gamma(\lambda'|\lambda)\mu(\lambda)$$

Different theories are defined by their:

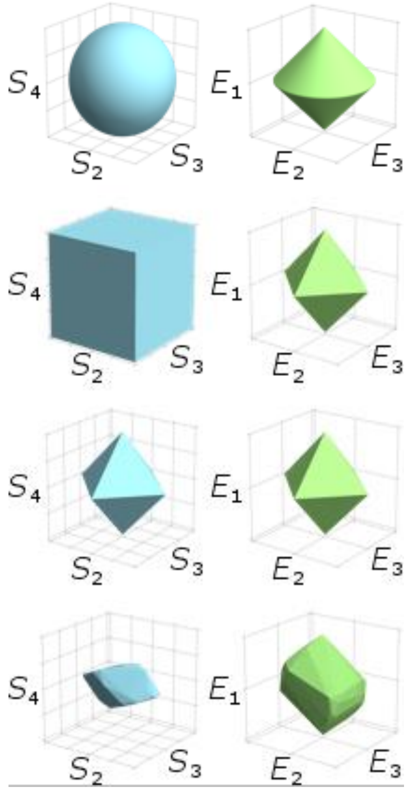
1. Convex geometry

qubit

Boxworld (3d)

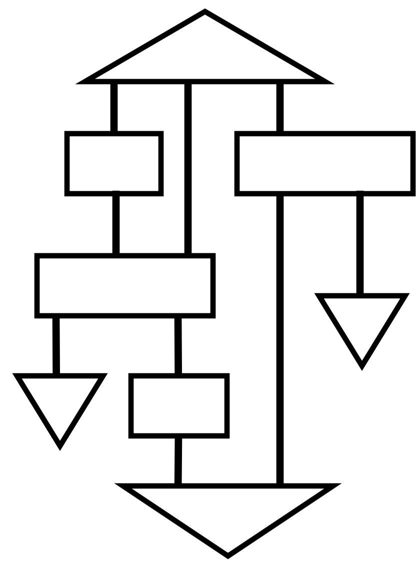
Spekkens toy theory

random GPT



2. Compositional structure

-multipartite states
-multipartite effects
-$T_1(T_2)=T_3$
-etc



GPT	GPT fragment
all possible systems, processes, and circuits	subset of systems, processes, and circuits
to describe a possible way the world could have been	to describe an experiment

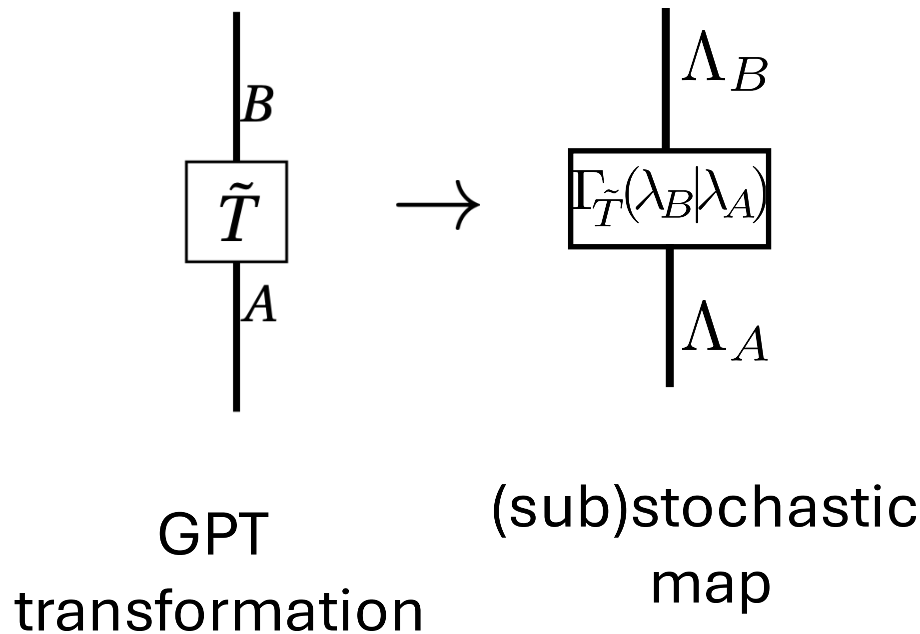
Which theories/fragments can be explained by the classical GPT?

Answer: theories/fragments that "fit inside" the classical GPT (in a structure-preserving way)

Γ : GPT → Classical GPT

which:

1) preserves the predictions
2) is linear
3) is diagram-preserving



The probabilities assigned to any complete circuit must be the same after applying $\boldsymbol{\Gamma}$ as before

Linearity

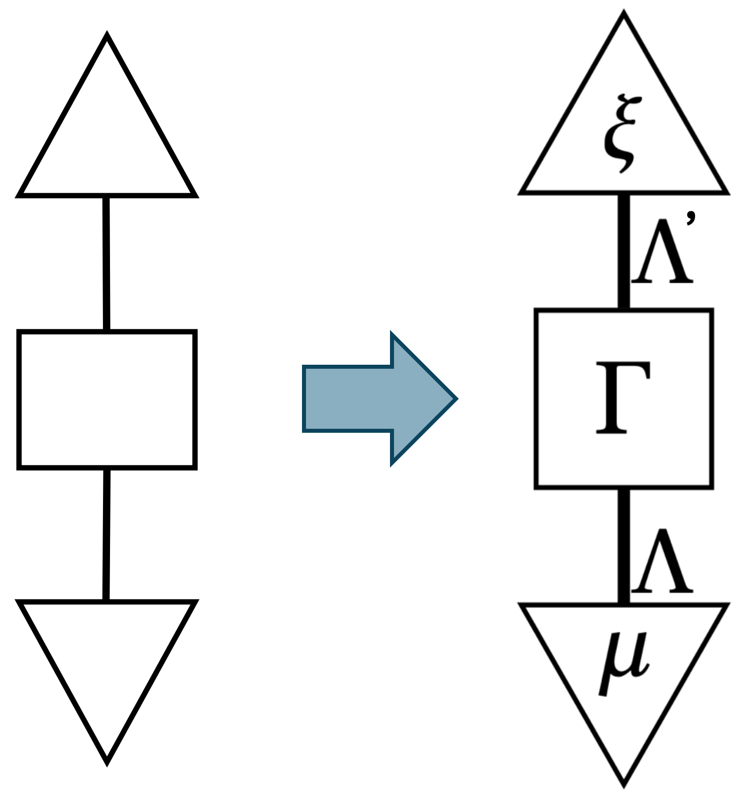
preservation of
convex geometry

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

$$\frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda) = \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$

Diagram-preservation

preservation of
compositional structure



Linearity

preservation of
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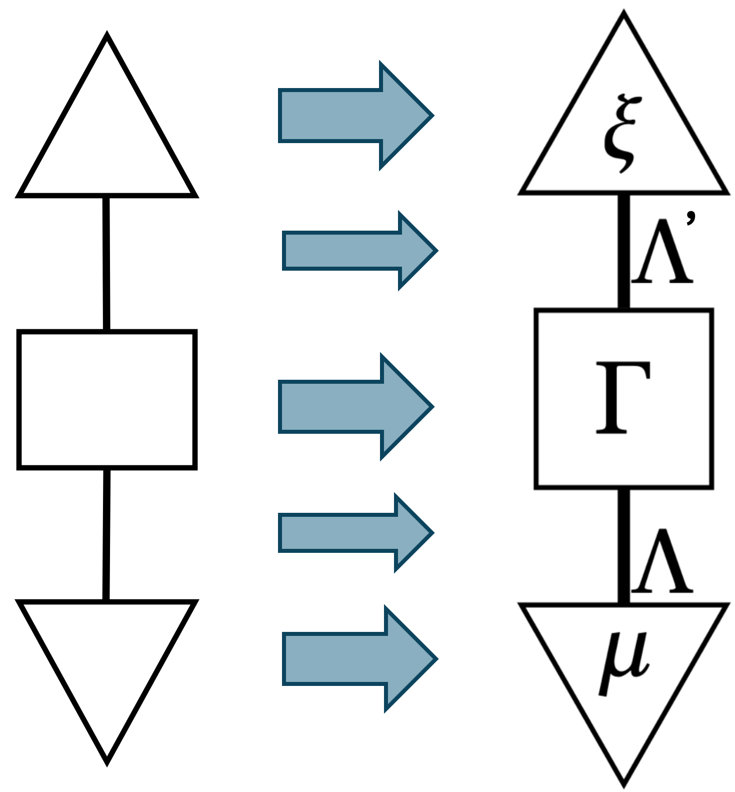
$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

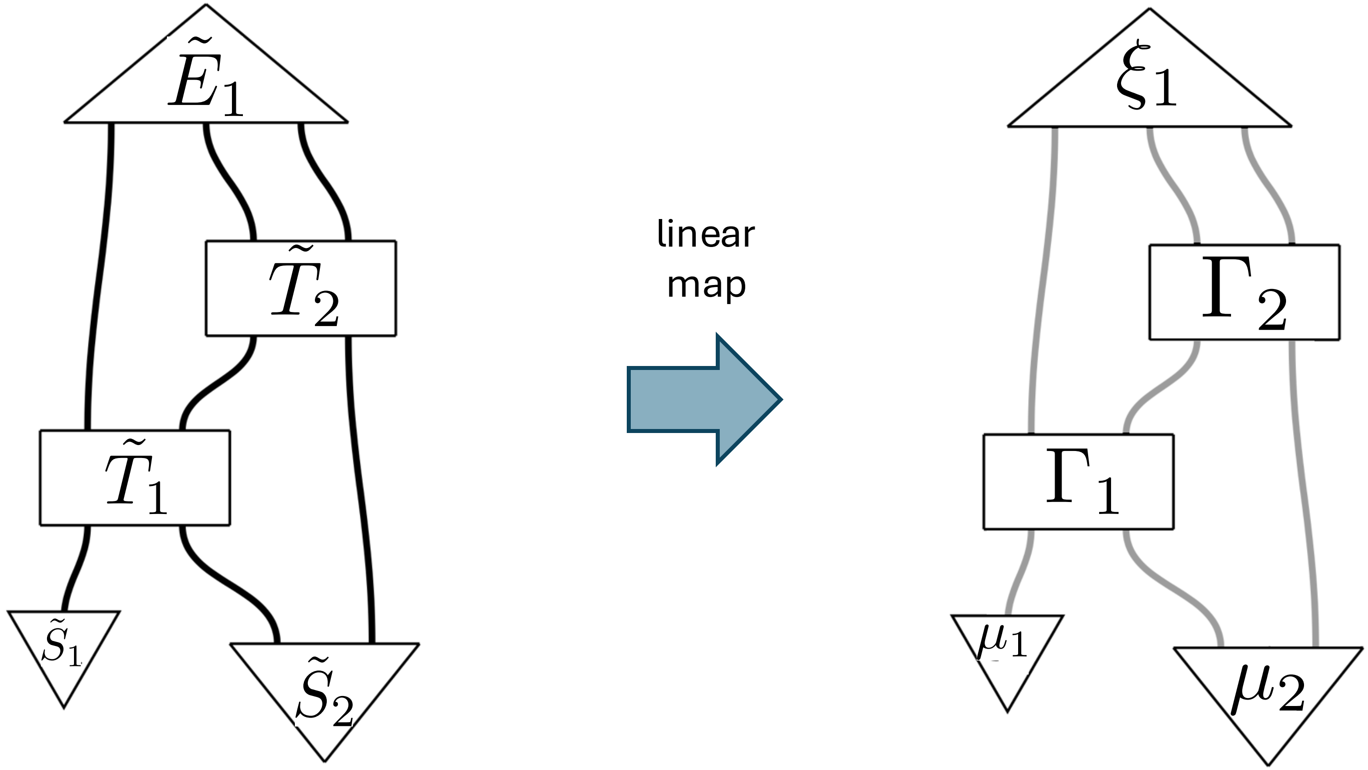
$$\Downarrow$$

$$\frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda) = \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$

Diagram-preservation

preservation of
compositional structure





GPT systems

GPT processes

random variables

(sub)stochastic processes

This is *the* notion of “classical-explainability” for a GPT.

...or for a GPT fragment! So it applies to <u>*experiments*</u> as well

Equivalent to the notion of
“Generalized Contextuality”

For more details, go to Youtube:
Noncontextuality, by David Schmid | Solstice of Foundations 2022

Leibniz's principle in action

indistinguishability (even in principle!)

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

Linearity

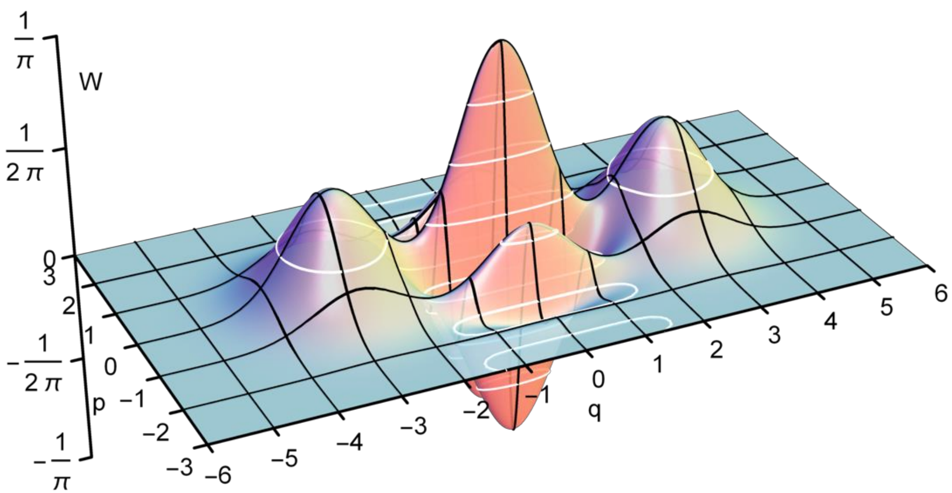
$$\frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda) = \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$

sameness in the (classical) explanation

Example: Wigner function (when it is positive)

linear map from quantum processes to real-valued vectors

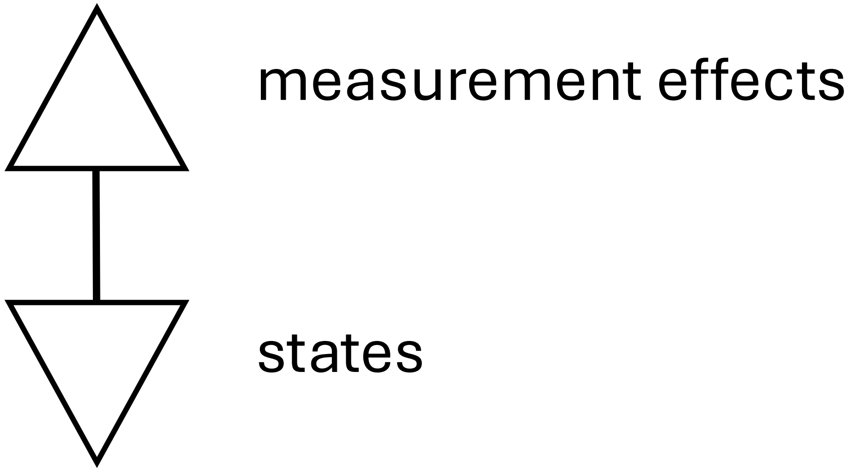
$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \langle x-y|\hat{\rho}|x+y\rangle e^{2ipy/\hbar}\, dy$$



Noncontextuality generalizes this to <u>arbitrary</u> linear functions over <u>arbitrary</u> classical variables

Understanding this geometrically

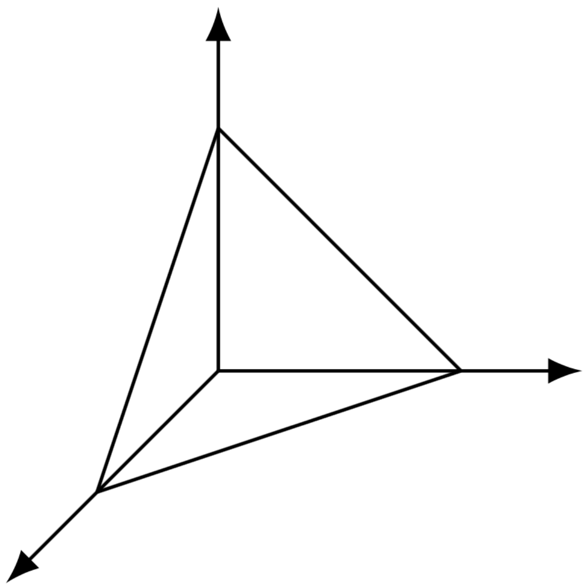
Prepare-measure circuit



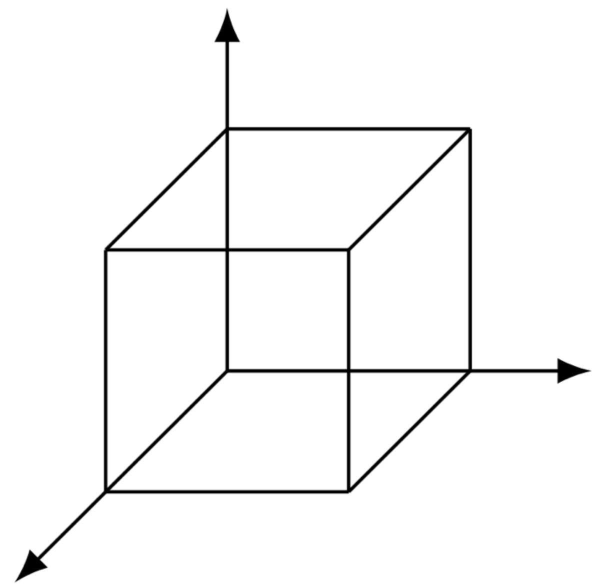
The classical GPT

d=3

state space: simplex



effect space: dual of simplex



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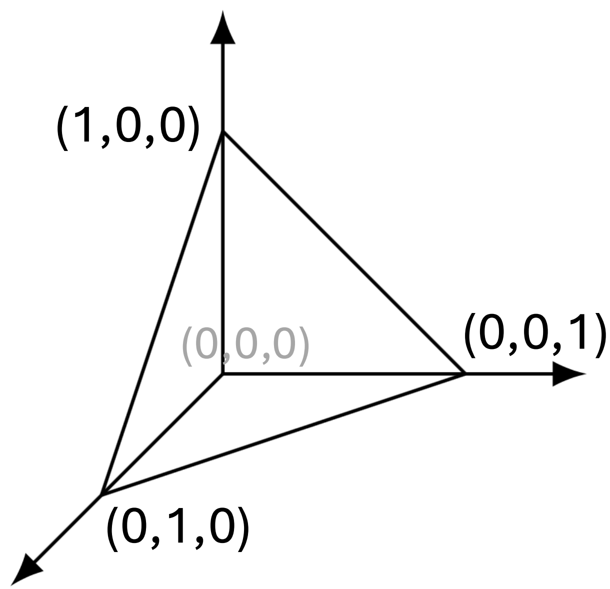
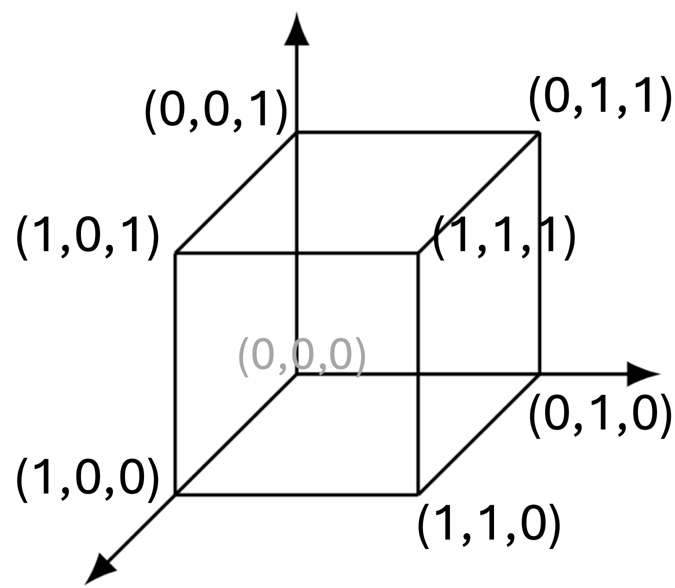


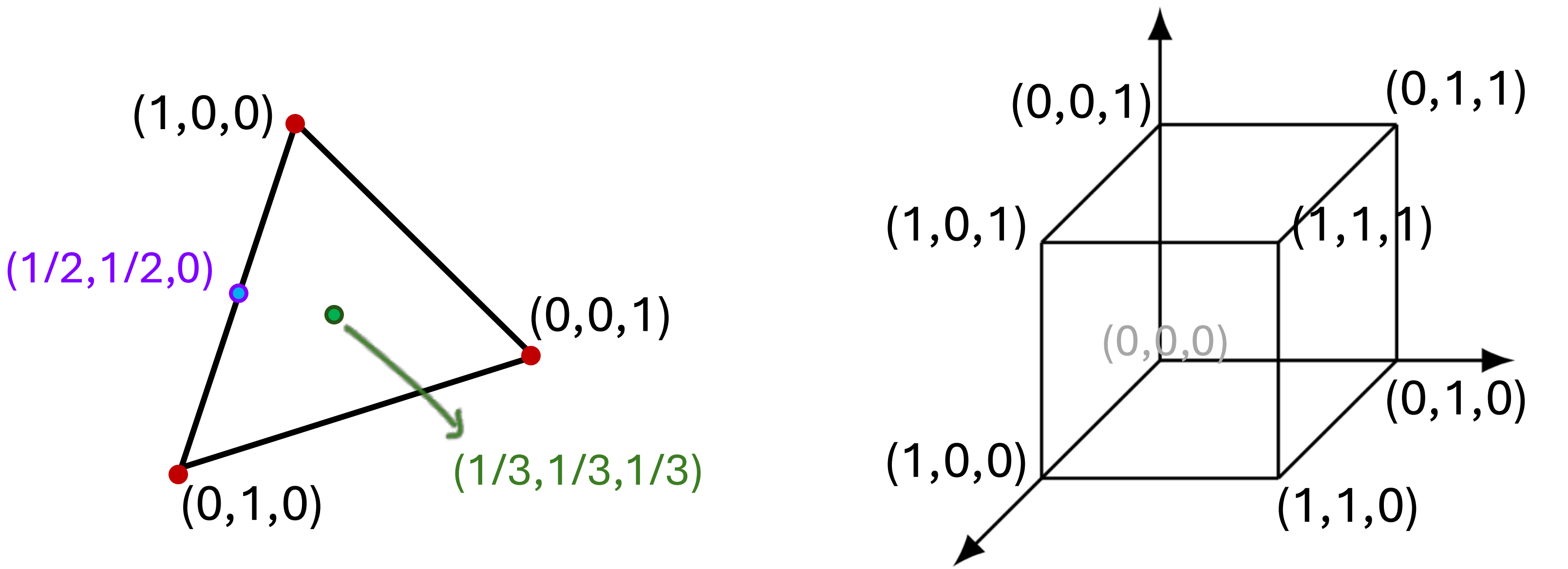
effect space: dual of simplex



normalized states

effects

d=3

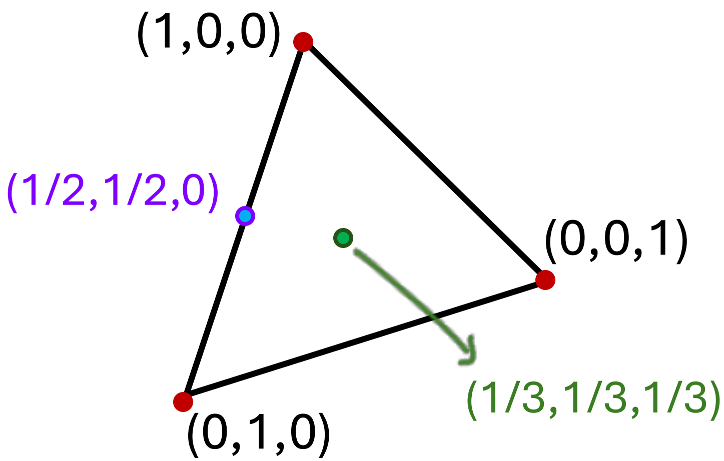


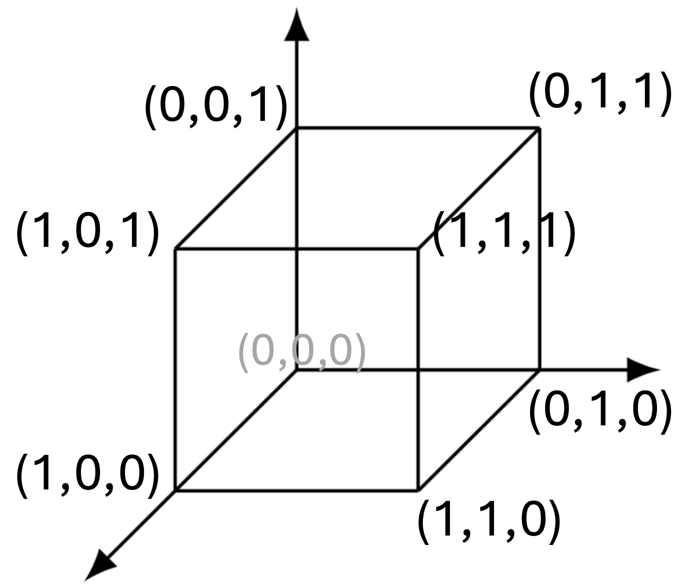
Classical statistical theory: probability distributions over a set of classical states

normalized states

effects

d=3



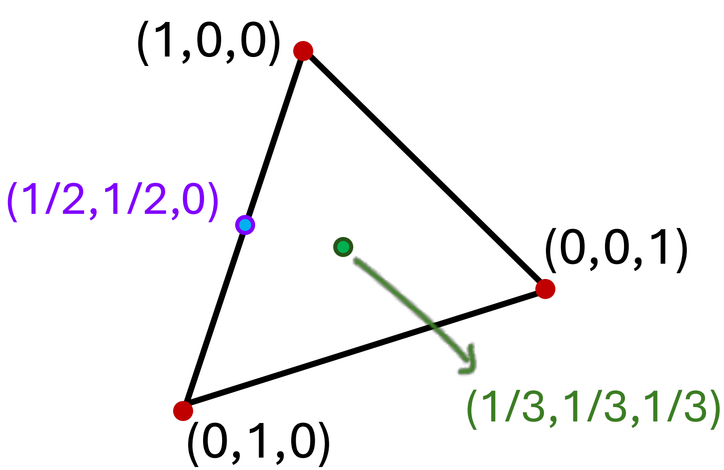


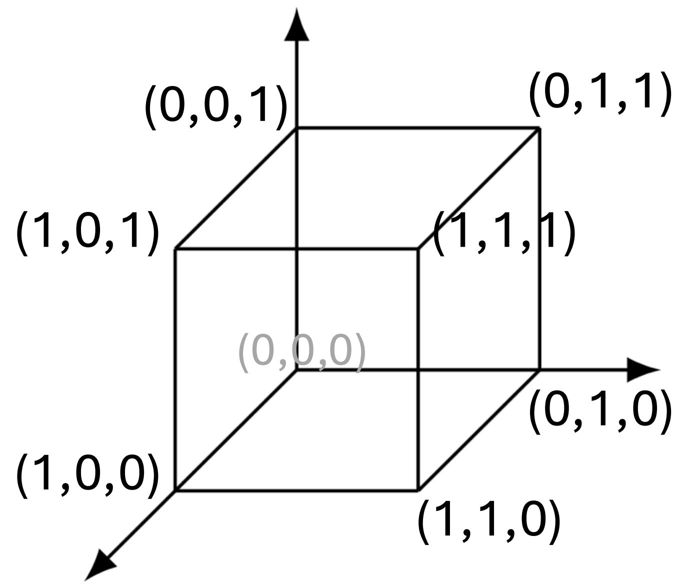
Every mixed state decomposes into pure states in a unique way
⇒ One can always imagine that there is a true state of the system, and any mixed state can be *uniquely* interpreted as uncertainty about the true state.

normalized states

effects

d=3





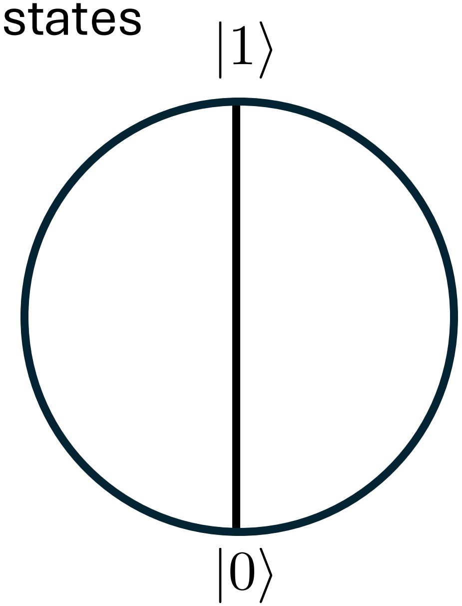
All logically possible measurements are *physically possible* and *compatible*.
⇒ One can determine the exact state of the system in a single measurement.

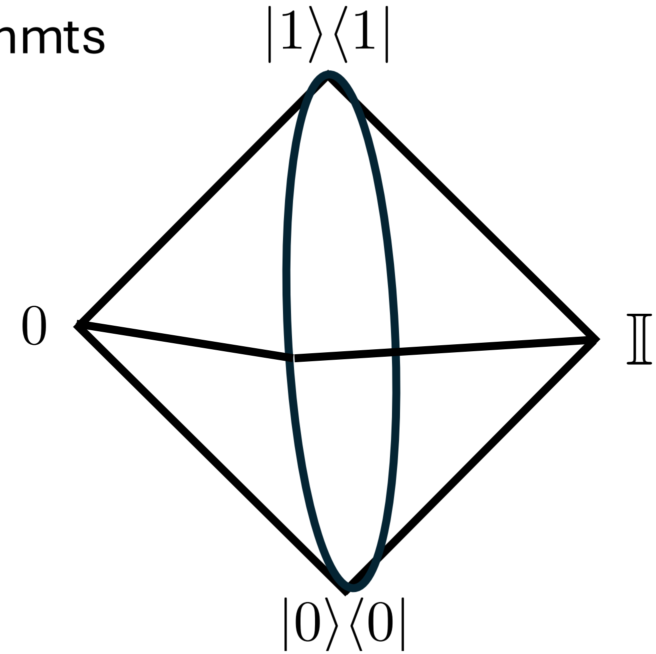
note that every simplicial system fits inside quantum theory

states

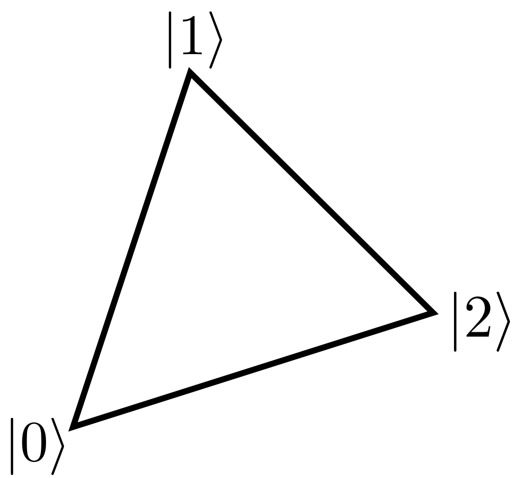
mmts

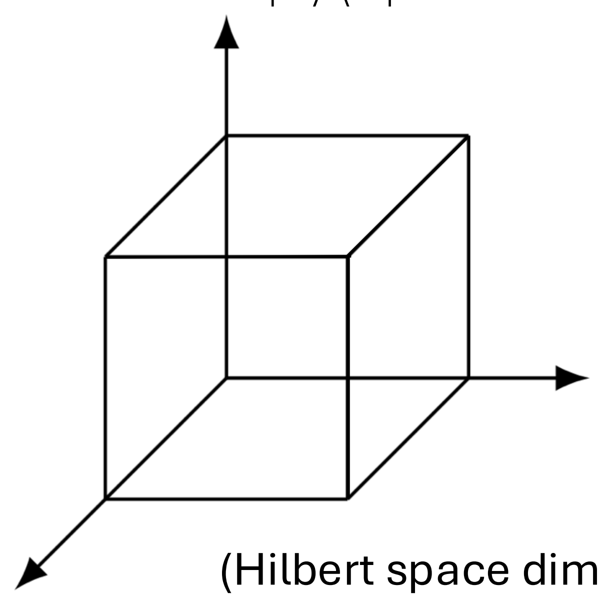
d = 2





d = 3

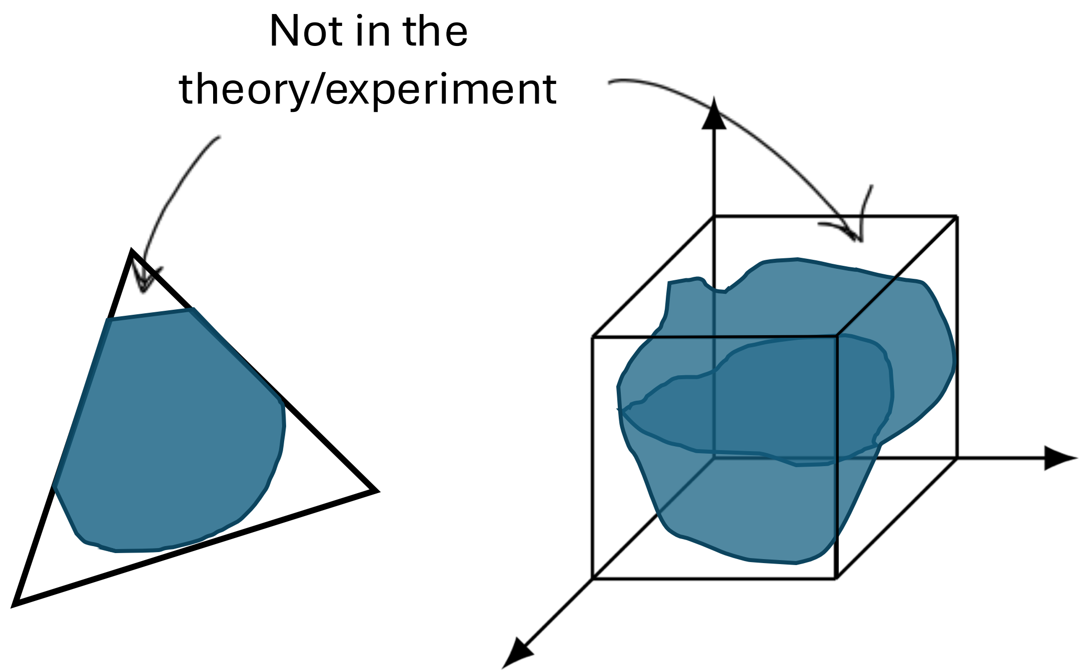




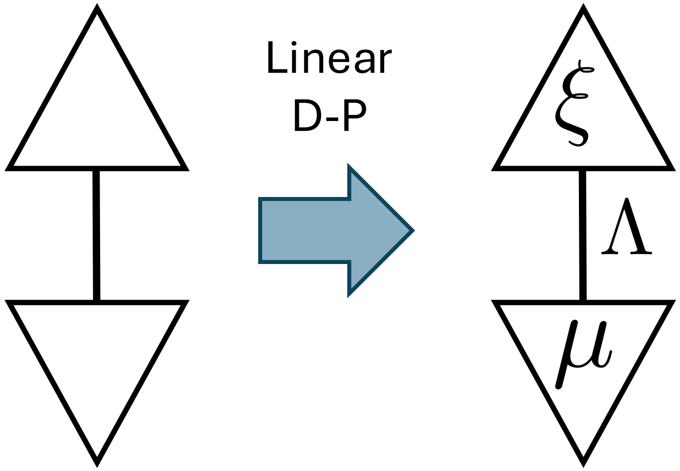
(Hilbert space dimension 3+ required)

simplicial = strictly classical

Intuitively: any theory/fragment that “fits inside” the simplicial GPT is <u>classically explainable</u>



Prepare-measure circuit



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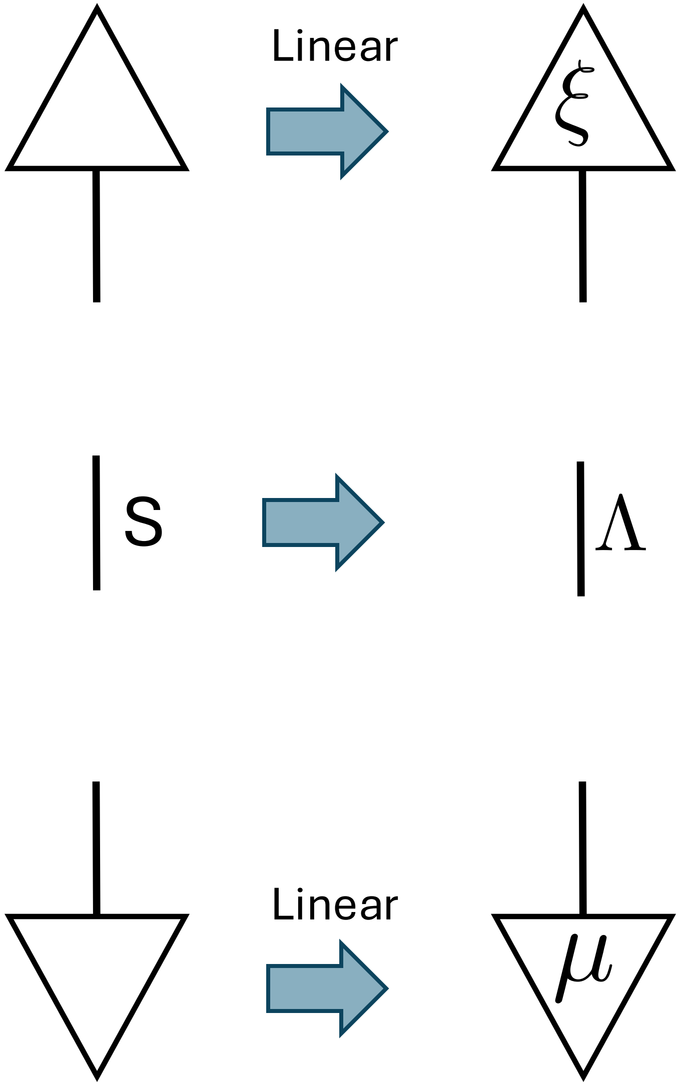
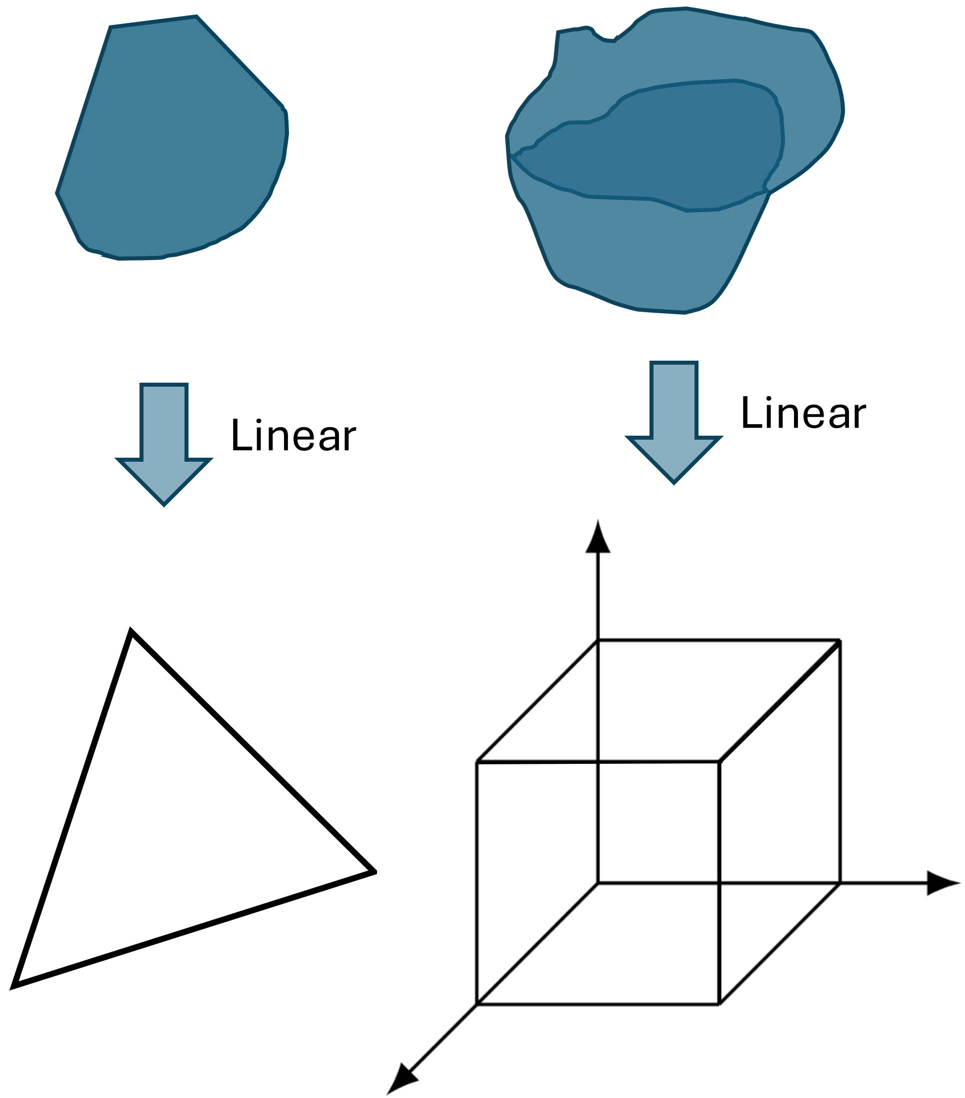


Diagram-preservation

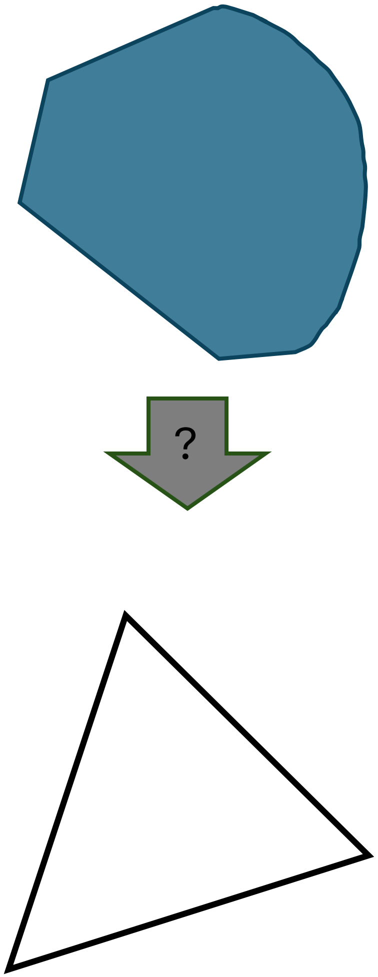
A prepare-measure GPT (fragment) is classically explainable iff there exists

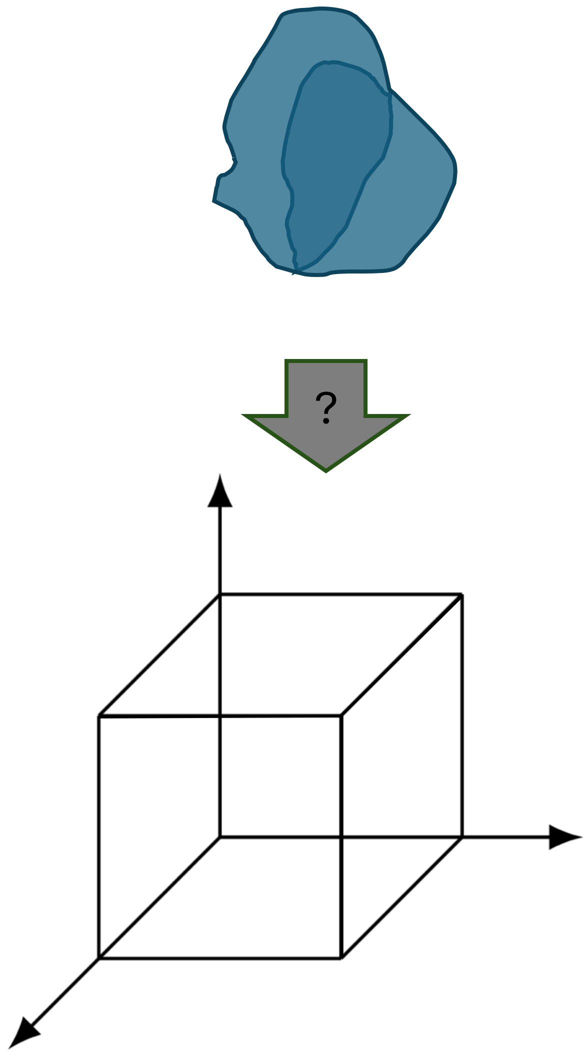
1) a linear map taking states into a simplex, and
2) a linear map taking effects into the dual to that simplex, such that
3) probabilities are preserved

“simplex embedding”



Which GPTs/fragments are simplex-embeddable?





-may need to jointly rescale states and effects

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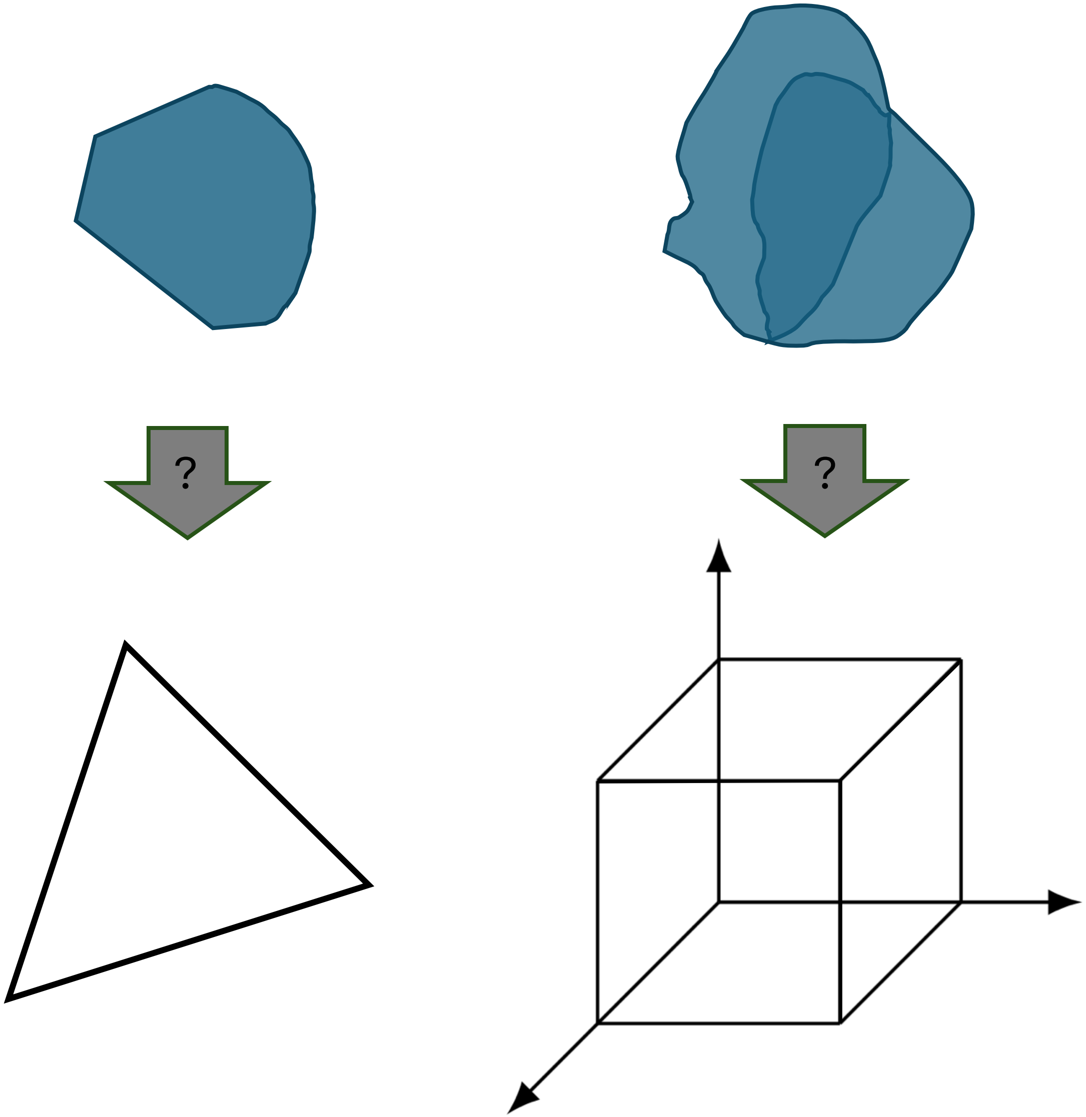

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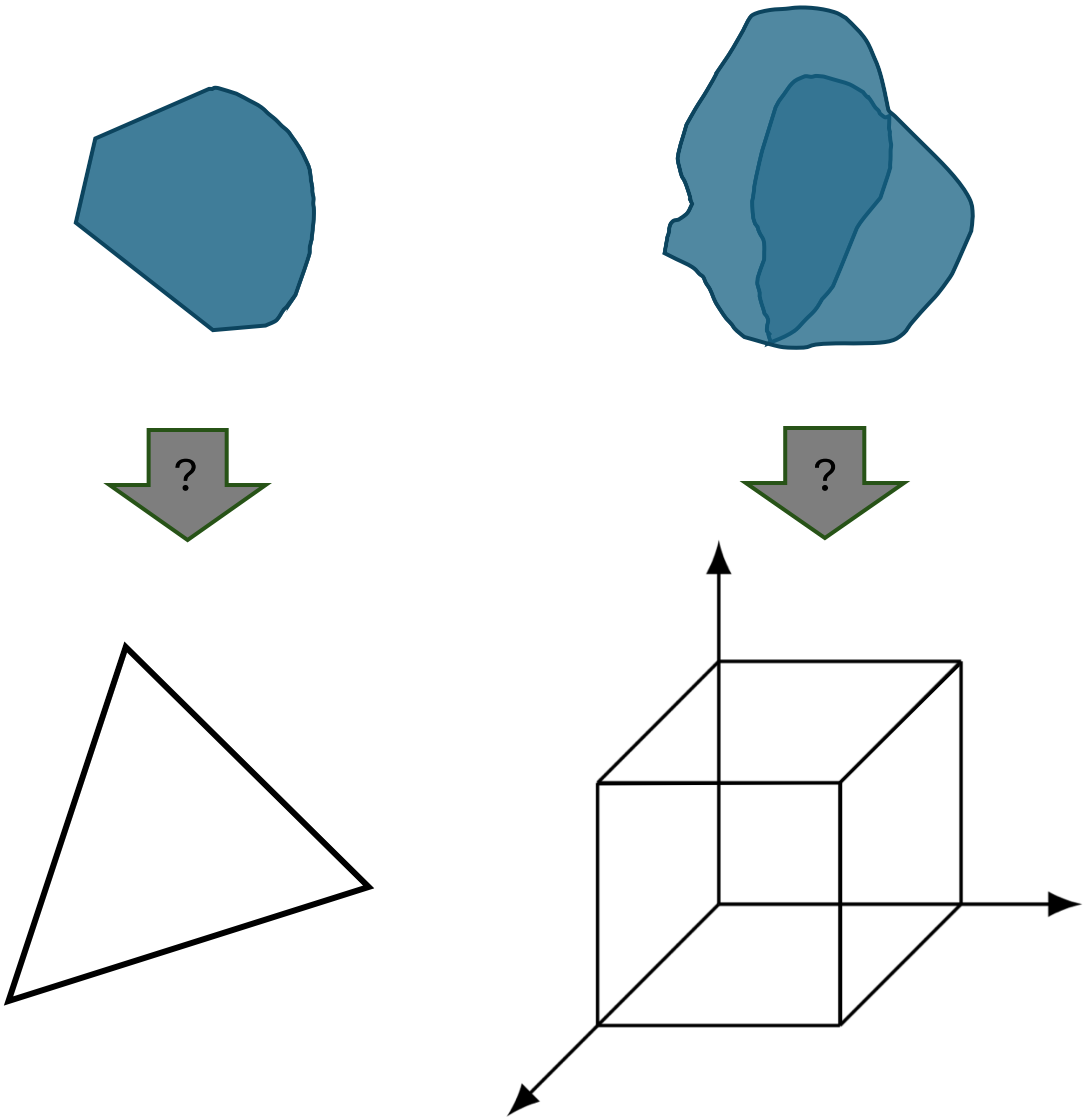
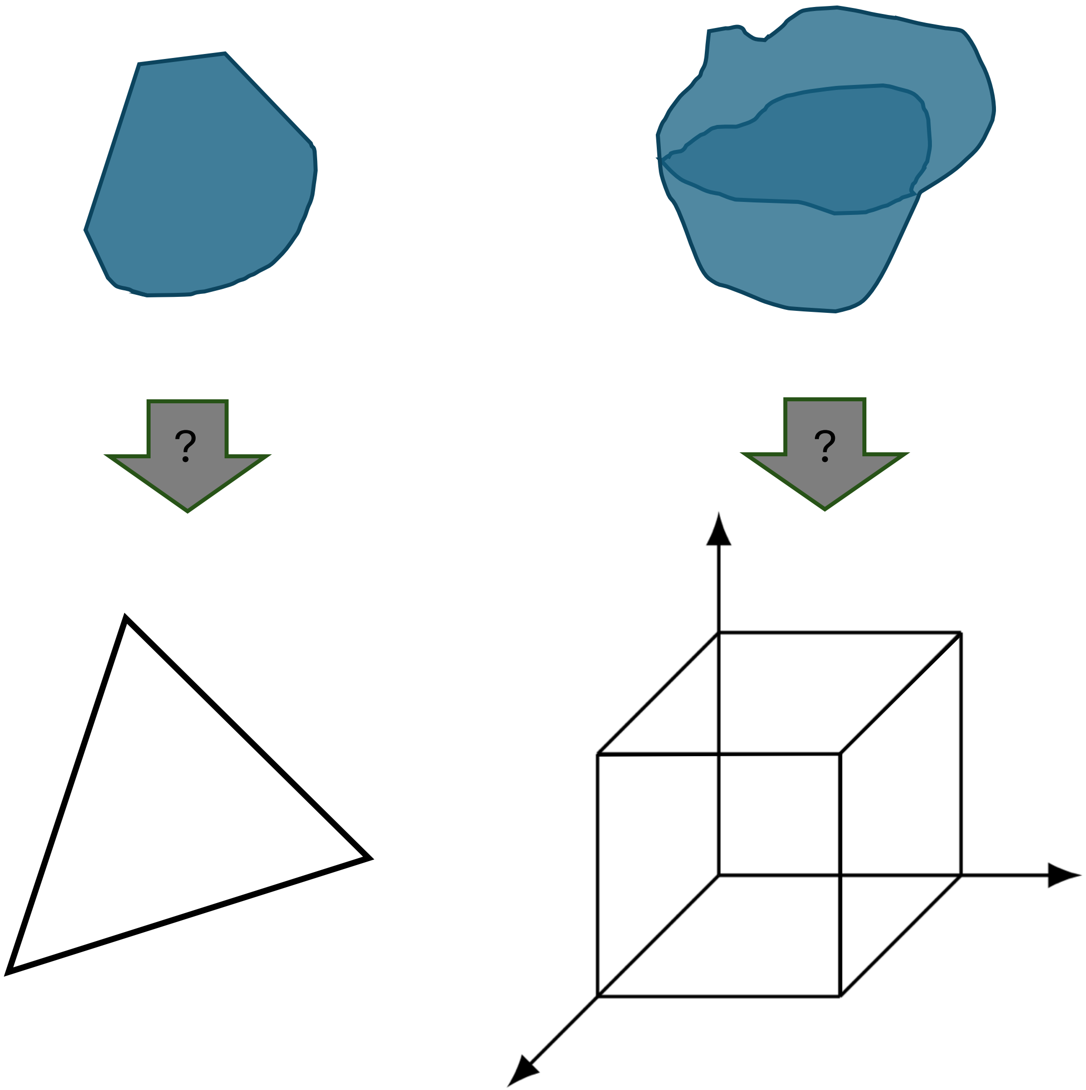


-may need to jointly reorient states and effects

Which GPTs/fragments are simplex-embeddable?



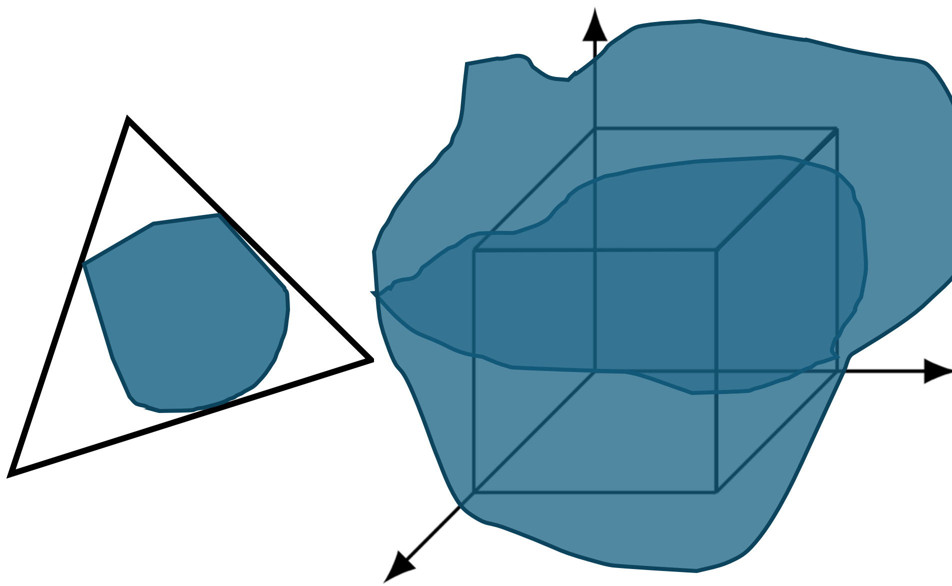
-may need to jointly reorient states and effects

(in fact any linear transformation can be applied to the states if the inverse is applied to the effects)

-dimension of simplex may be greater than GPT dimension!

Deciding if a GPT is simplex-embeddable is just a linear program!

It is clear geometrically that every GPT/fragment becomes classically explainable under sufficient depolarizing noise.



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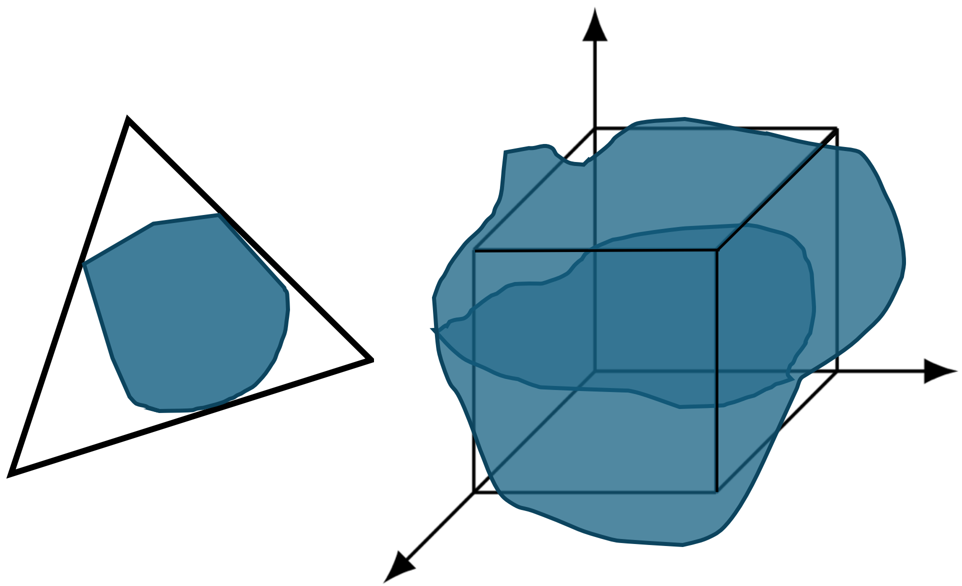

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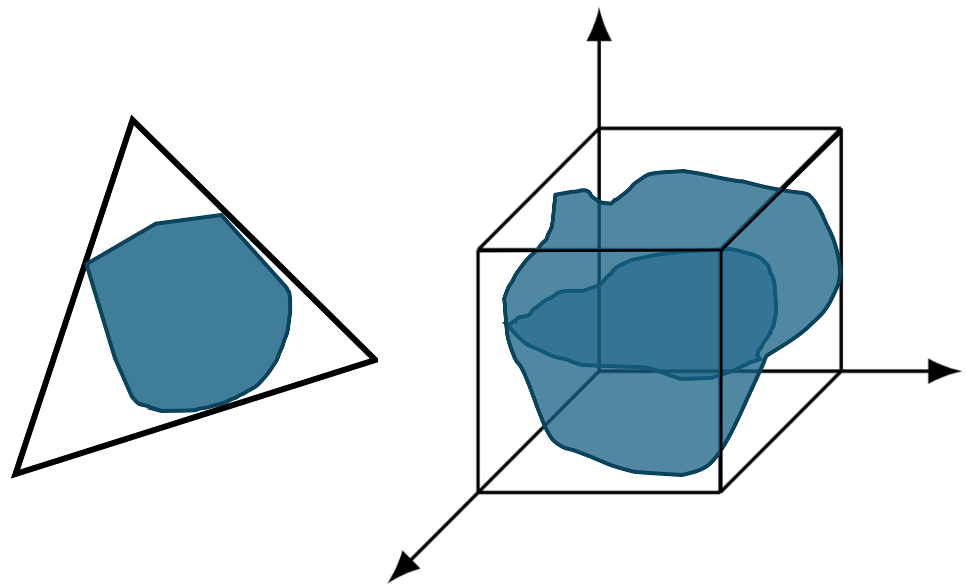
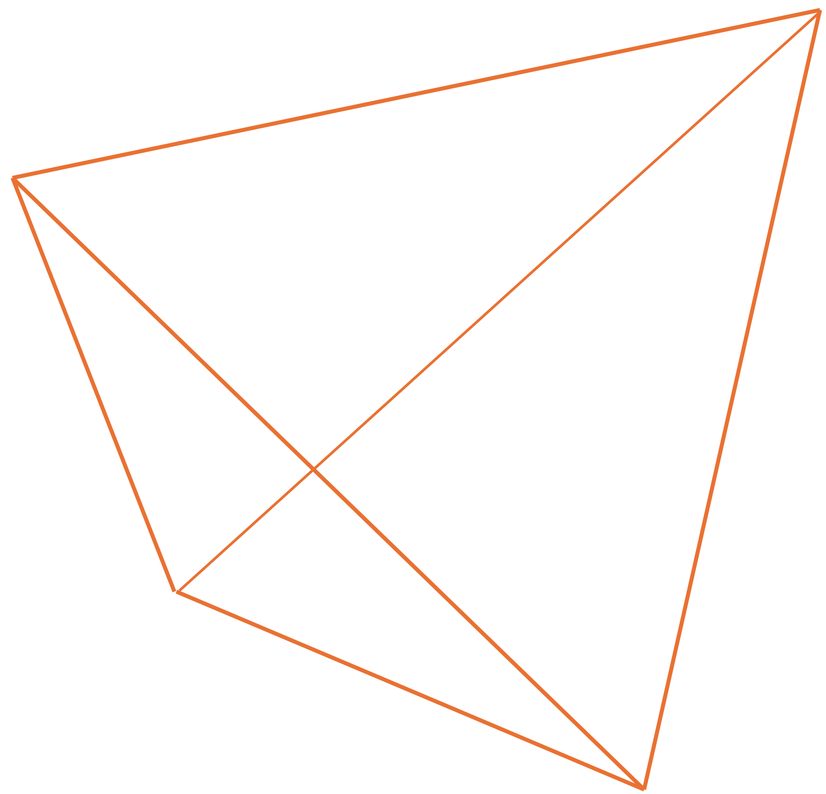


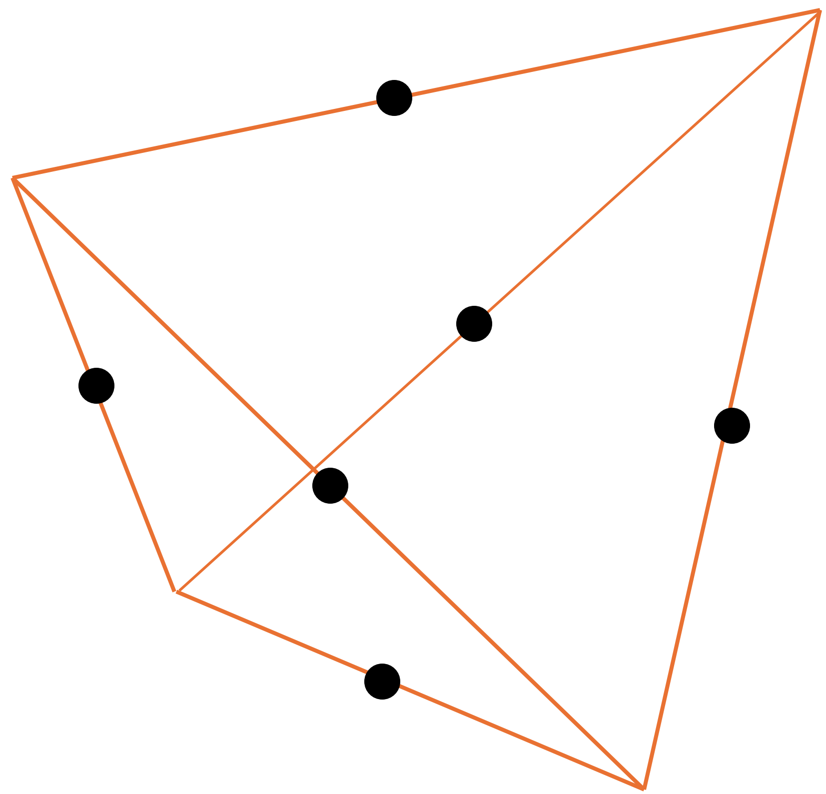
A measure of nonclassicality of a GPT:
how much depolarization can it undergo
before it becomes classically explainable?

Example: Spekkens toy theory

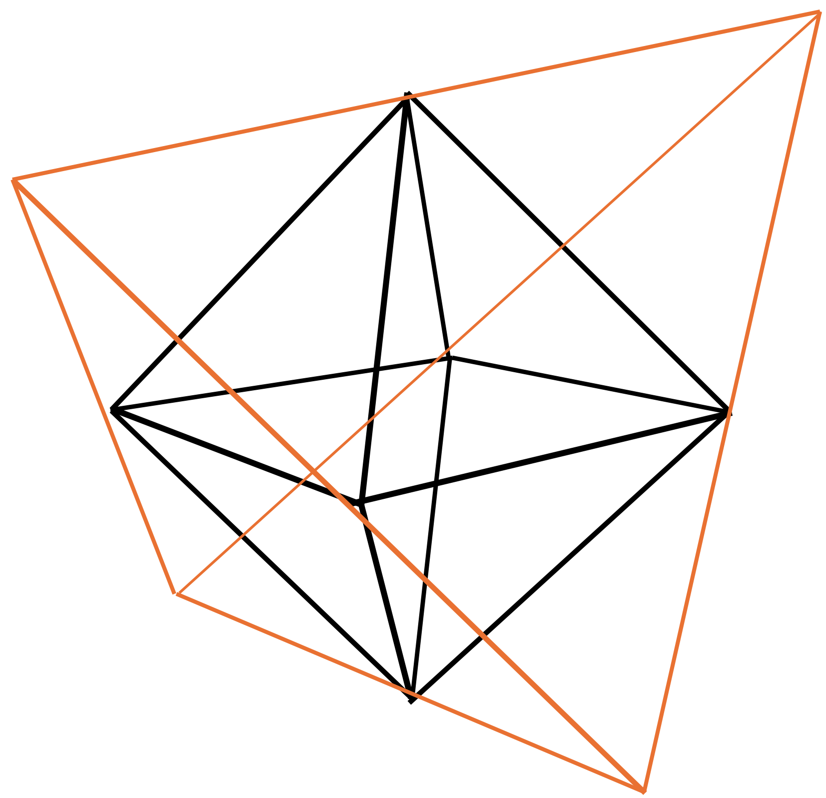
Consider a simplicial GPT with d = 4



Consider the midpoints of the 6 edges

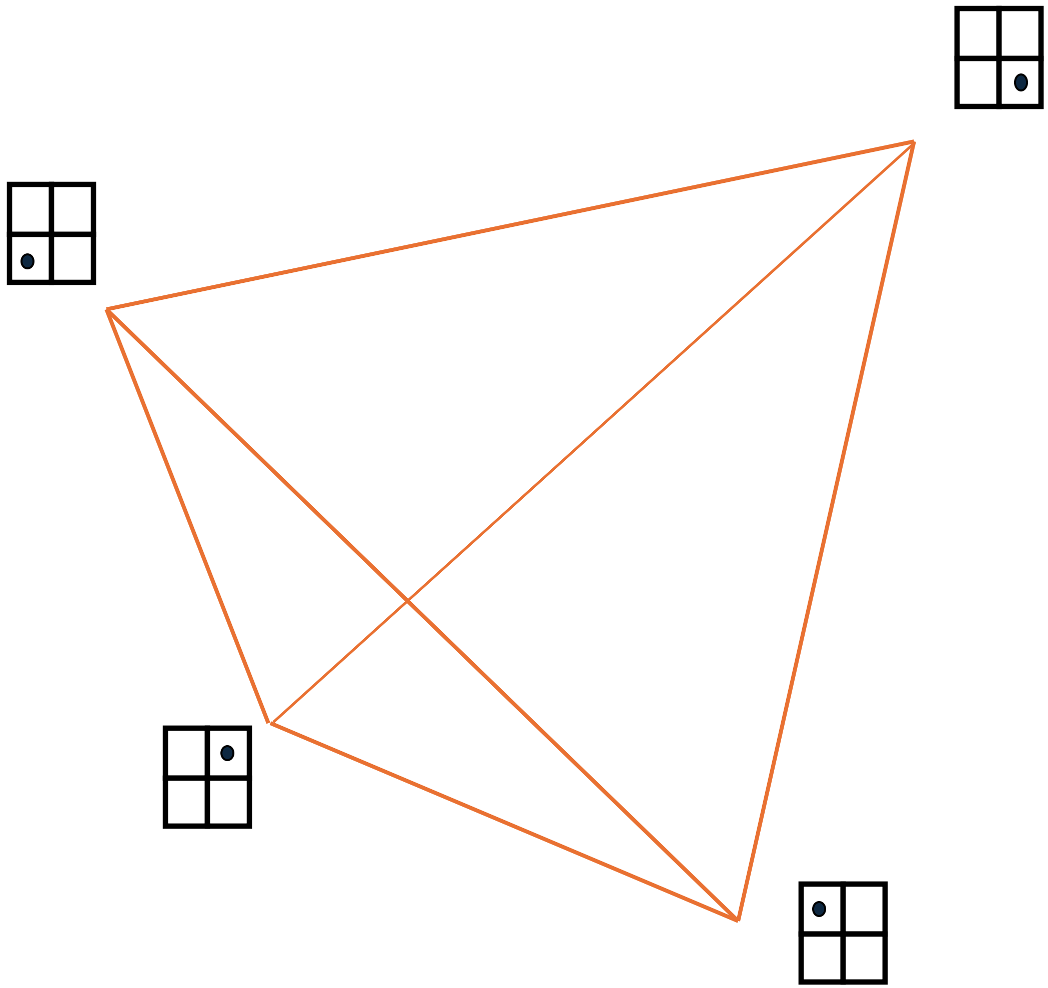


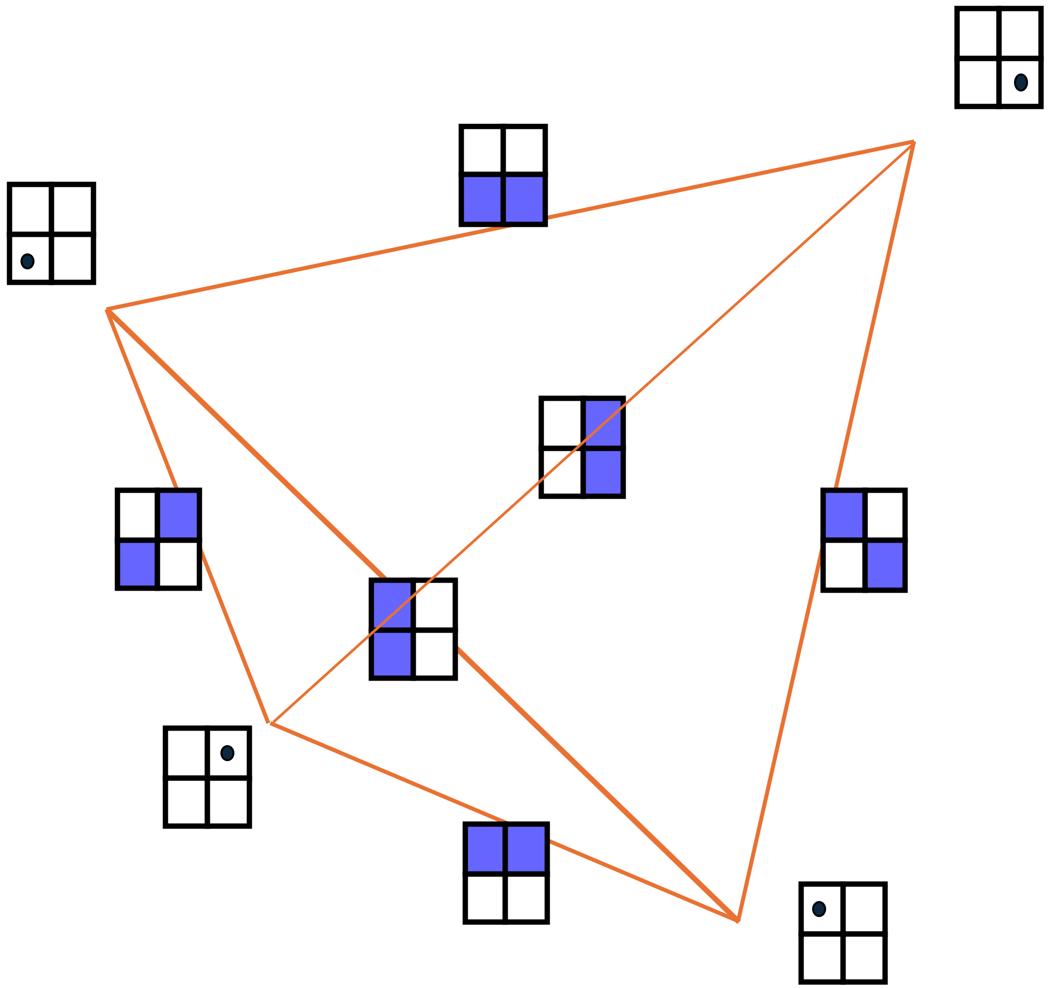
Take the convex hull



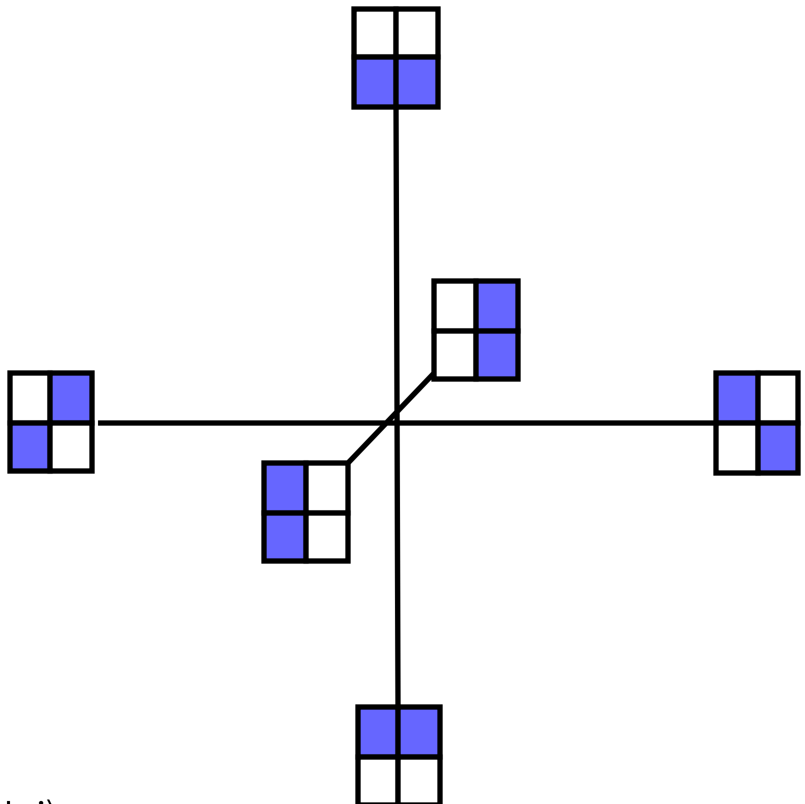
This is the state space of Spekkens toy theory!

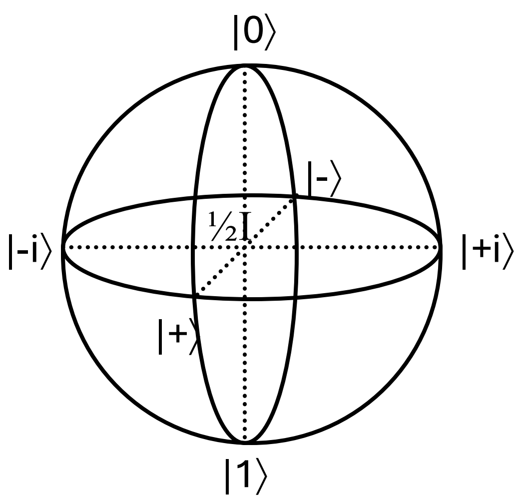
(the effect space is constructed similarly)





Spekkens toy theory: imagine that these are the state of maximal knowledge in the theory



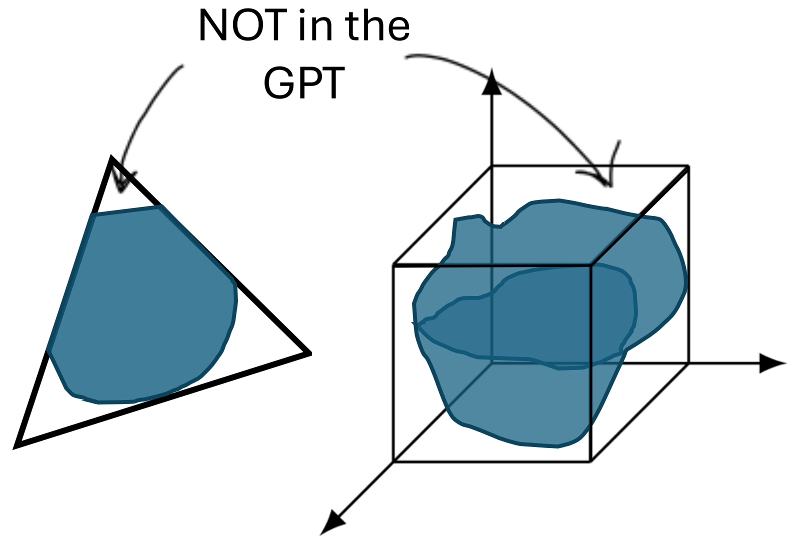


This theory exhibits:

-noncommutativity
-complementarity
-interference
-no-cloning
-teleportation
-dense coding
-entanglement
-remote steering
-quantum eraser
-mmts must disturb
-ambiguity of mixtures
-no perfect state discr.
...

Any simplex-embeddable GPT/fragment can be viewed similarly:

ruled out by an “epistemic restriction”



-one can always imagine that the vertices correspond to ontic states
-every GPT process is a stochastic process on the ontic states
-but you cannot perfectly prepare/measure/know the ontic state

Any simplex embedding gives an <u>ontological model</u> of the GPT.

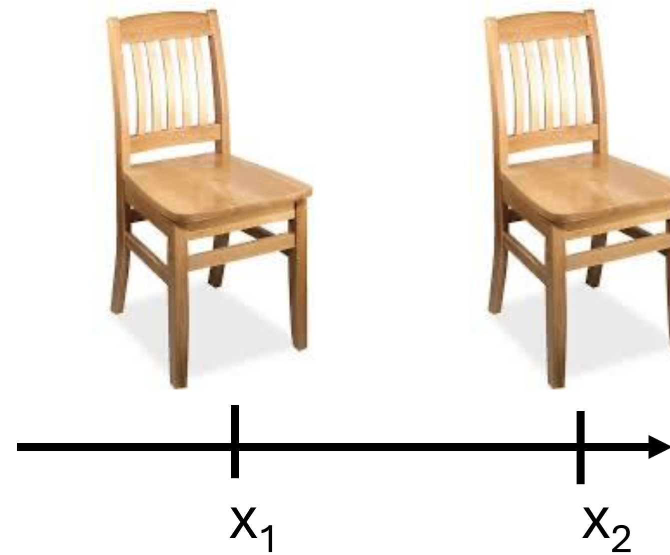
Simplicial GPT	Simplex-embeddable GPTs	Non-embeddable GPTs
simplex + dual	simplex + dual + restriction	all other GPTs
strictly classical	classically-explainable	nonclassical
no contextuality	no contextuality	contextuality
-all mmts compatible -unique decomposition of mixed states	-noncommutativity -complementarity -interference -no-cloning -teleportation -dense coding -entanglement -remote steering -quantum eraser -mmts must disturb -ambiguity of mixtures -no perfect state discr. ...	-contextuality -computational speedups -nonlocality other more nuanced phenomena

Examples of nonclassical phenomena and what we can learn from them

Quantum state discrimination

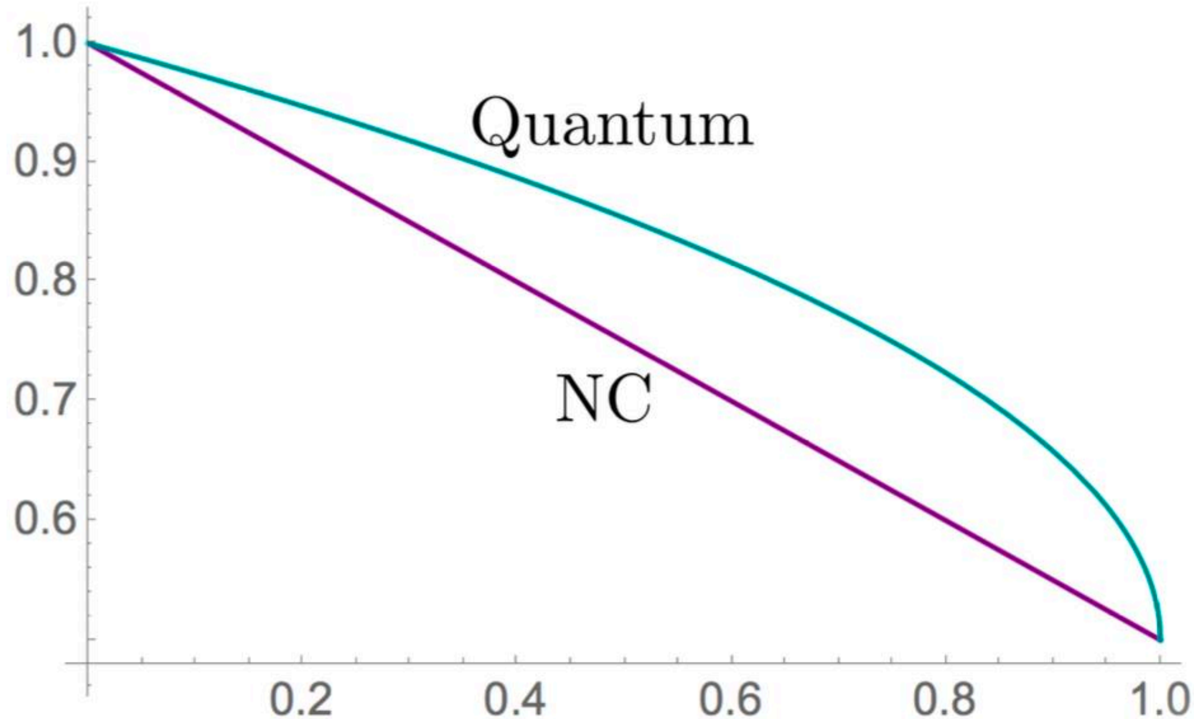
In quantum theory there is no perfect single-shot discrimination of non-orthogonal states.

Many have claimed this is evidence of nonclassicality.



But actually, it is <u>easier</u> to discriminate overlapping states in quantum theory than in any classically-explainable theory!

Quantum state discrimination

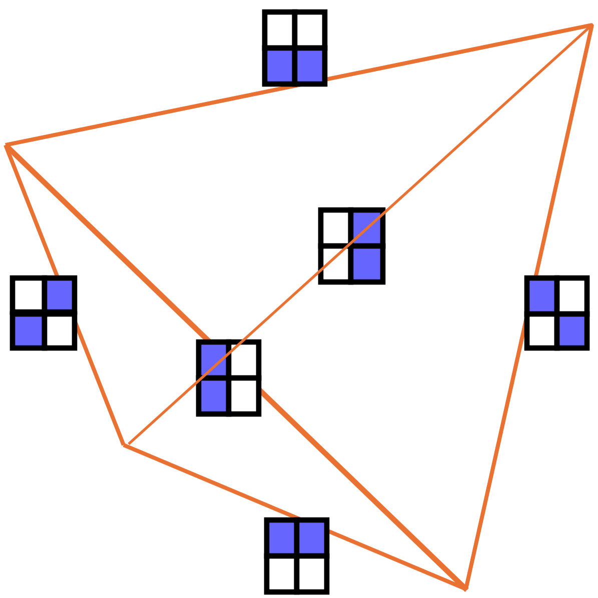


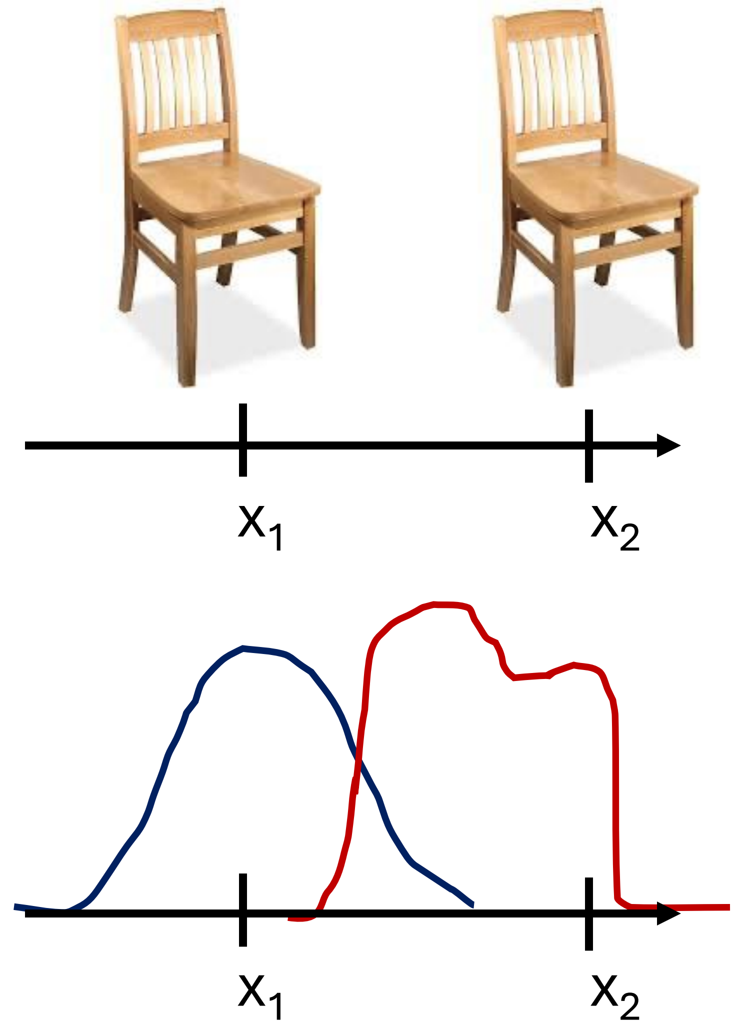
discrimination success

state overlap

Quantum state discrimination

The usual argument is
based on a bad analogy:





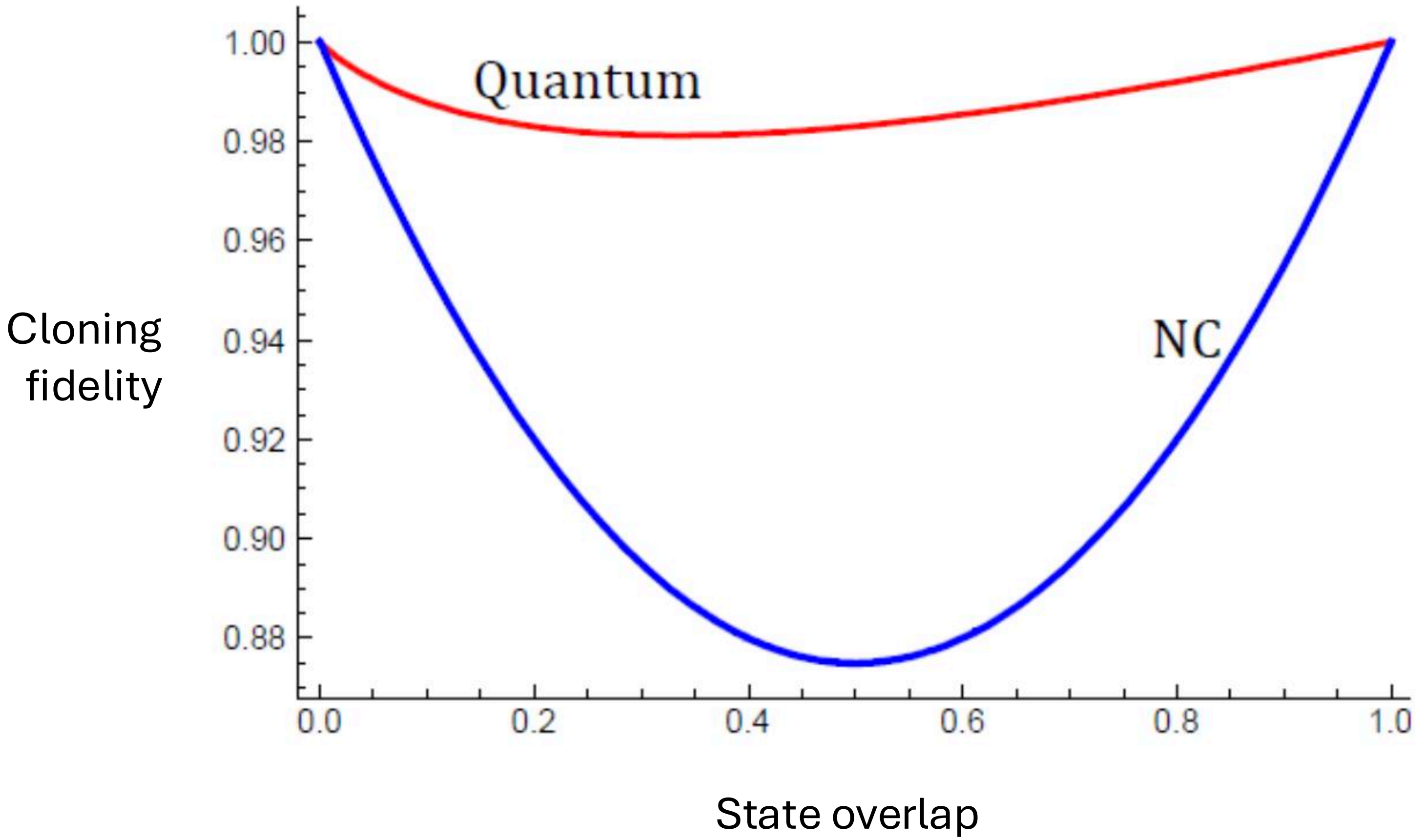
Cloning

Naïve take: no-cloning theorem = nonclassical

But, in *both* classical and quantum theories, a known state can always be cloned, and an unknown state cannot.

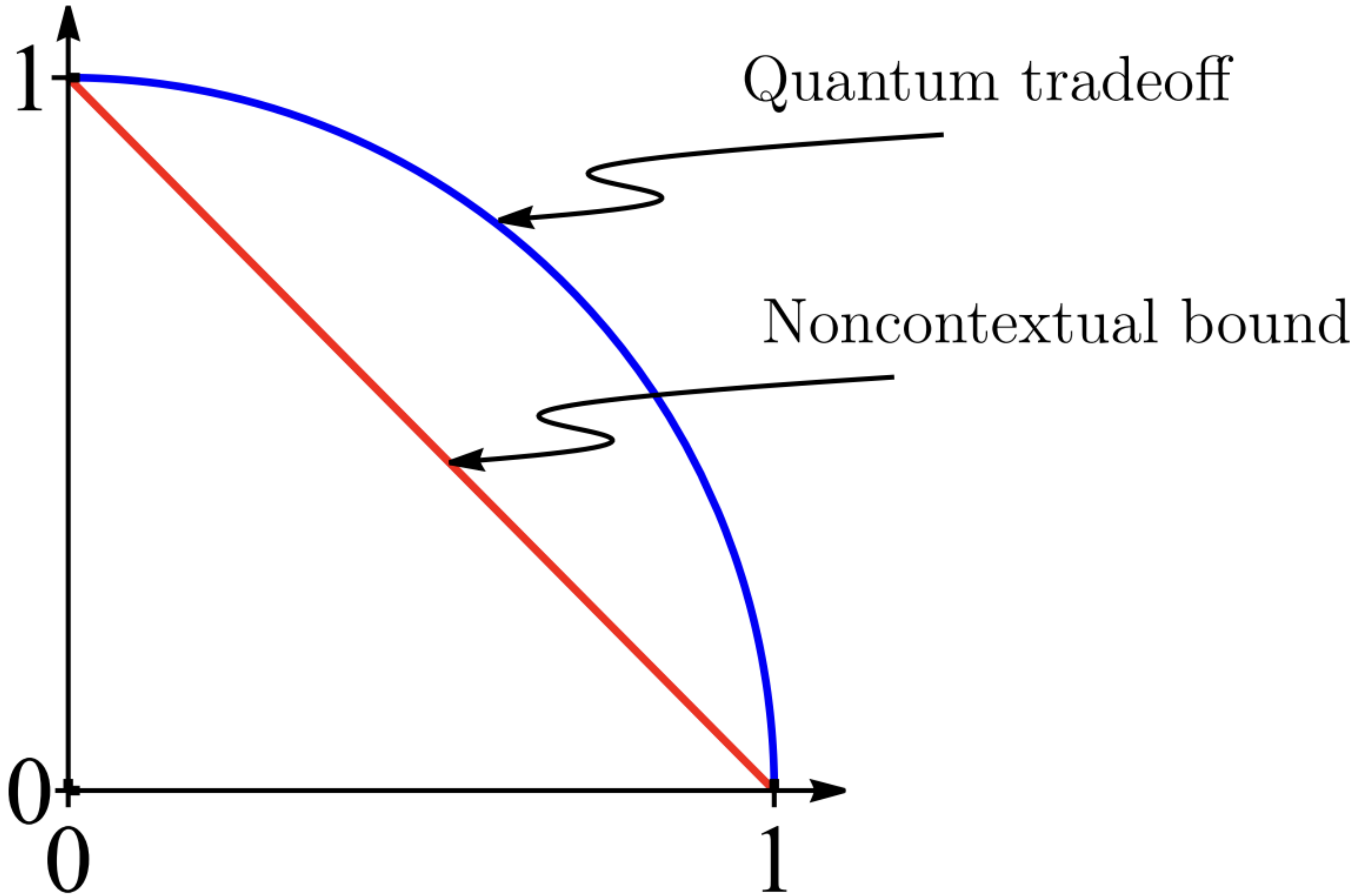
Unknown states are *easier* to clone in quantum theory than in any classical theory!

Cloning


Quantum
NC
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.0
0.2
0.4
0.6
0.8
1.0
Cloning fidelity
State overlap

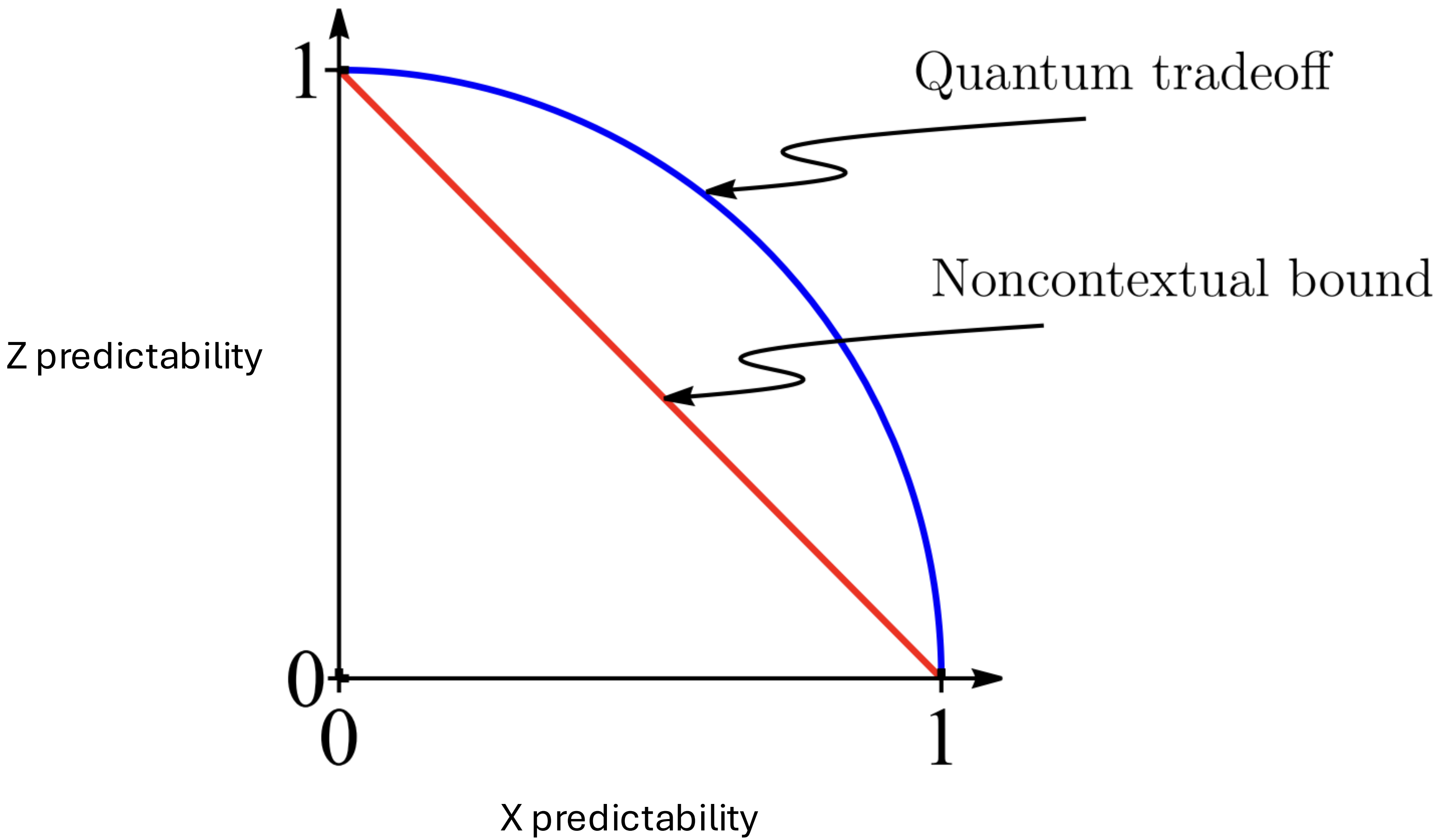
Interference

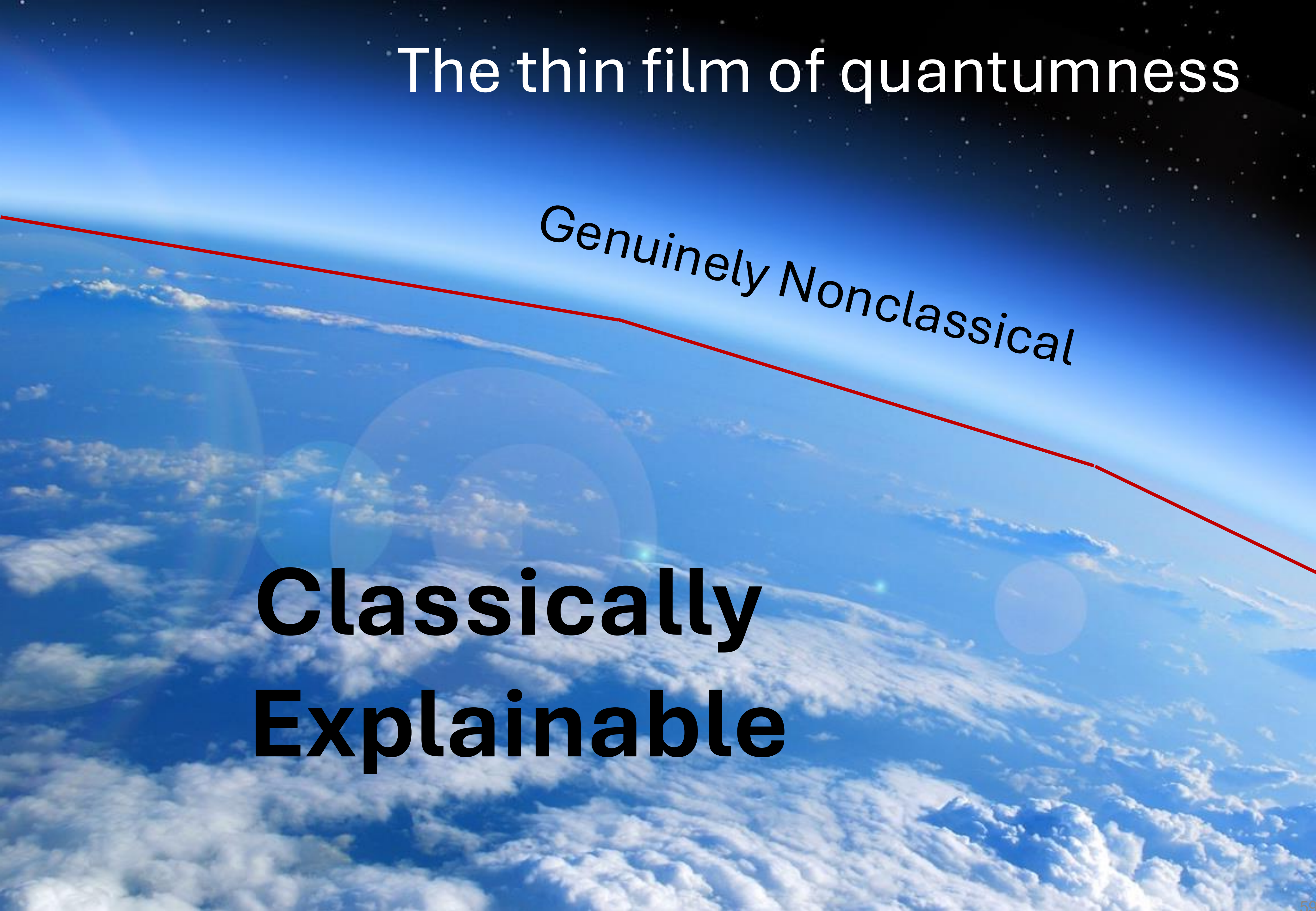
Path distinguishability



Fringe visibility

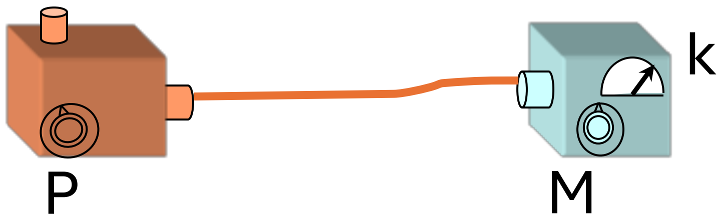
Uncertainty relations



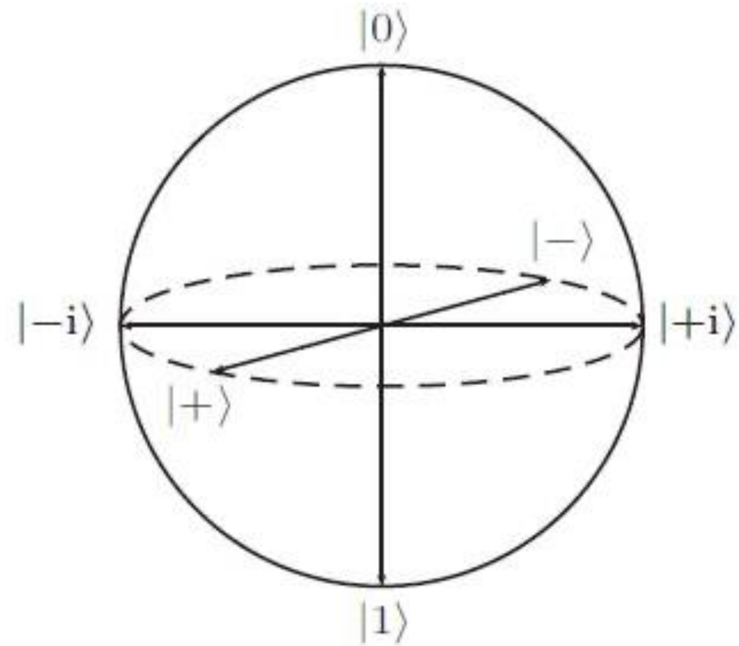

The thin film of quantumness
Genuinely Nonclassical
Classically
Explainable

Noncontextuality inequalities

choose circuit



find states/mmts/etc



Find the GPT identities these satisfy

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

...these imply constraints on any potential classical model

$$\frac{1}{2}\mu_{|0\rangle\langle 0|}(\lambda) + \frac{1}{2}\mu_{|1\rangle\langle 1|}(\lambda) = \frac{1}{2}\mu_{|+\rangle\langle +|}(\lambda) + \frac{1}{2}\mu_{|-\rangle\langle -|}(\lambda)$$

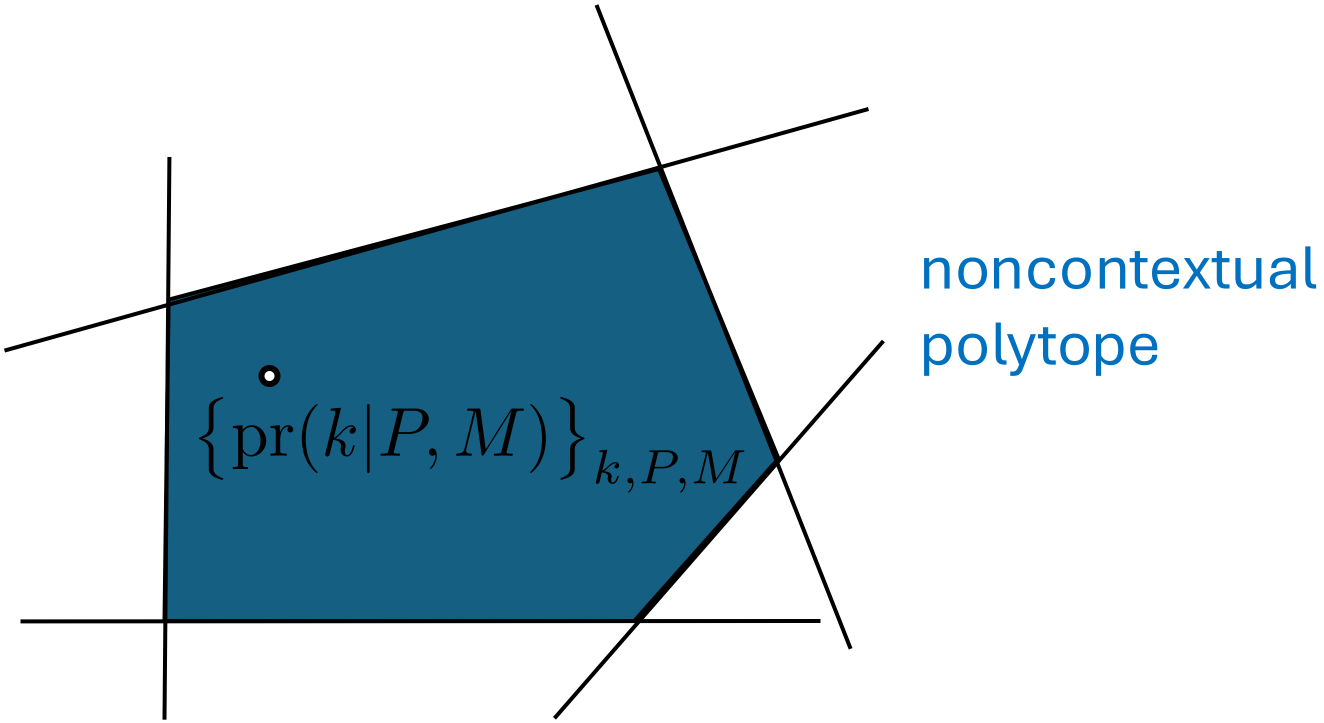
...which imply constraints on the observable data pr(k|M,P)

$$\sum_{P,k,M} \alpha_{P,k,M}\ \mathrm{pr}(k|P,M) \leq r$$

$$\{\mathring{\mathrm{pr}}(k|P,M)\}_{k,P,M}$$

noncontextuality inequalities

$$\sum_{P,k,M} \alpha_{P,k,M}\ \mathrm{pr}(k|P,M) \leq r$$

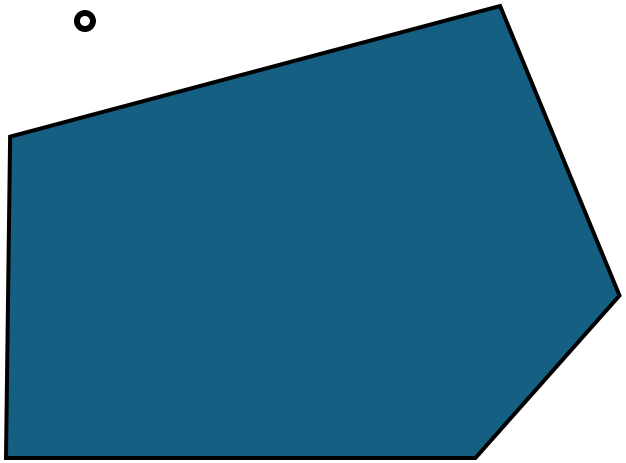


noncontextuality inequalities

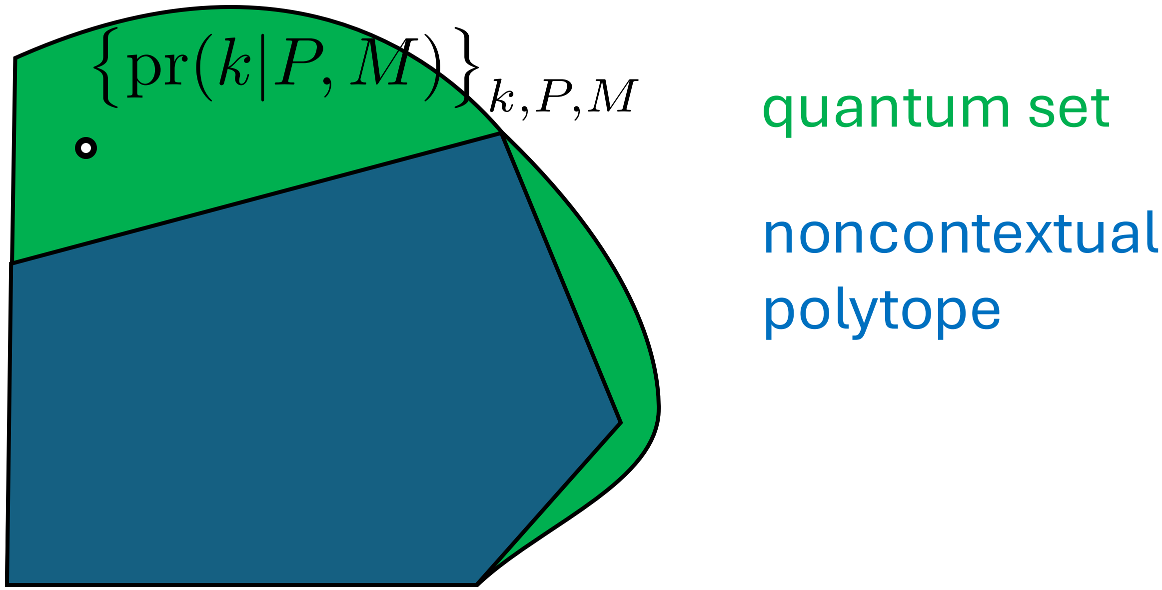
$$\sum_{P,k,M} \alpha_{P,k,M}\ \mathrm{pr}(k|P,M) \leq r$$

Proof of nonclassicality!

$$\left\{\mathrm{pr}(k|P,M)\right\}_{k,P,M}$$



noncontextual
polytope



Such proofs do not rely on the
correctness of quantum theory

Alternative method for testing for classical explainability:

1. do theory agnostic tomography to find the GPT fragment describing your experiment
2. check if that fragment is simplex-embeddable

Suggested references:

Basic definition of noncontextuality:
https://arxiv.org/abs/quant-ph/0406166

Noncontextuality in the GPT framework:
https://arxiv.org/pdf/1911.10386v2.pdf

NC beyond prepare and measure scenarios:
https://arxiv.org/pdf/2005.07161.pdf

Deriving all the noncontextuality inequalities:
https://arxiv.org/pdf/1710.08434.pdf

A linear program for testing simplex-embeddability:
https://arxiv.org/pdf/2204.11905

Experimental tests of noncontextuality:
https://arxiv.org/abs/1710.05948
https://arxiv.org/abs/1710.05948

Feedback encouraged!
davidschmid10@gmail.com

Youtube:
Noncontextuality, by David Schmid | Solstice of Foundations 2022
https://www.youtube.com/watch?v=M3qn3EHWdOg

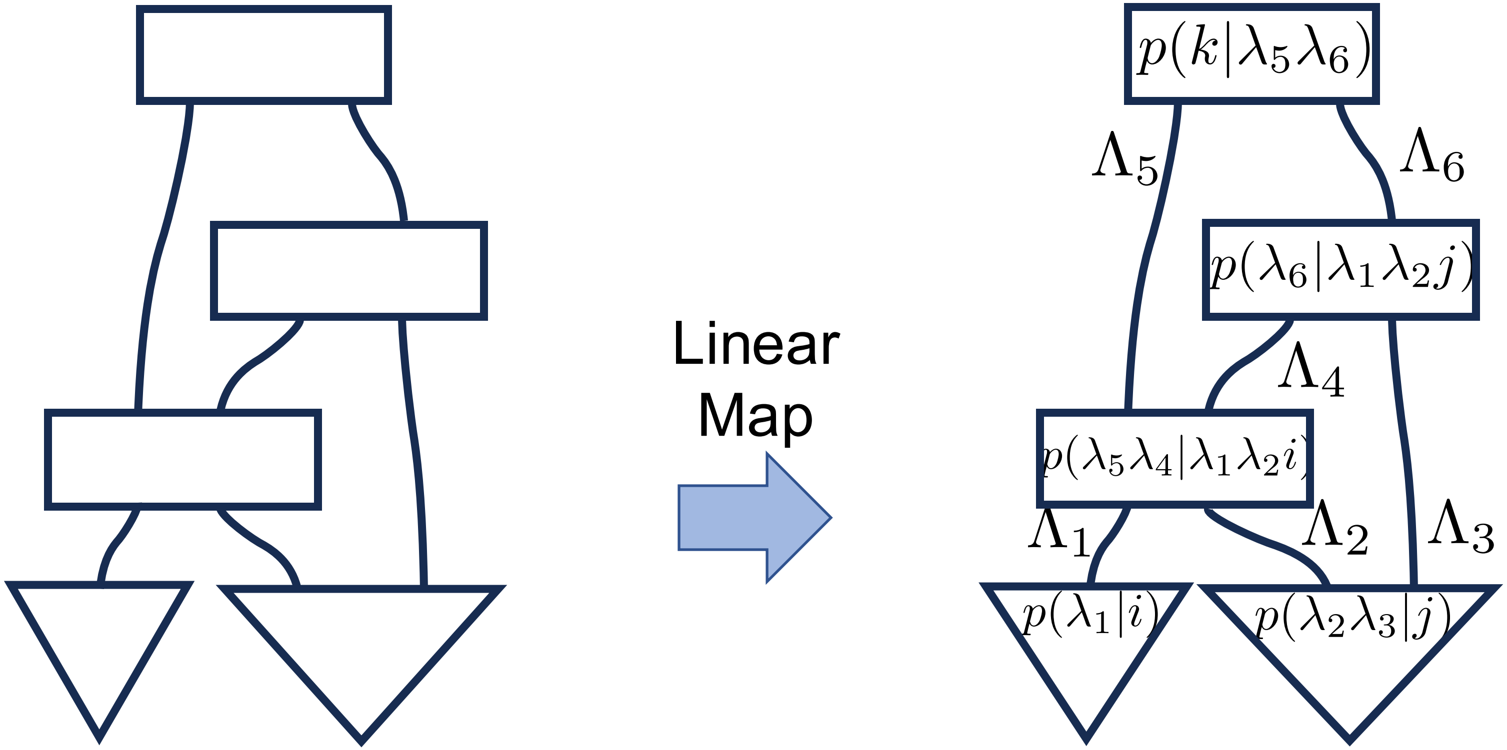
Nonclassicality in Bell scenarios

David Schmid

dschmid1@perimeterinstitute.ca

PERIMETER INSTITUTE

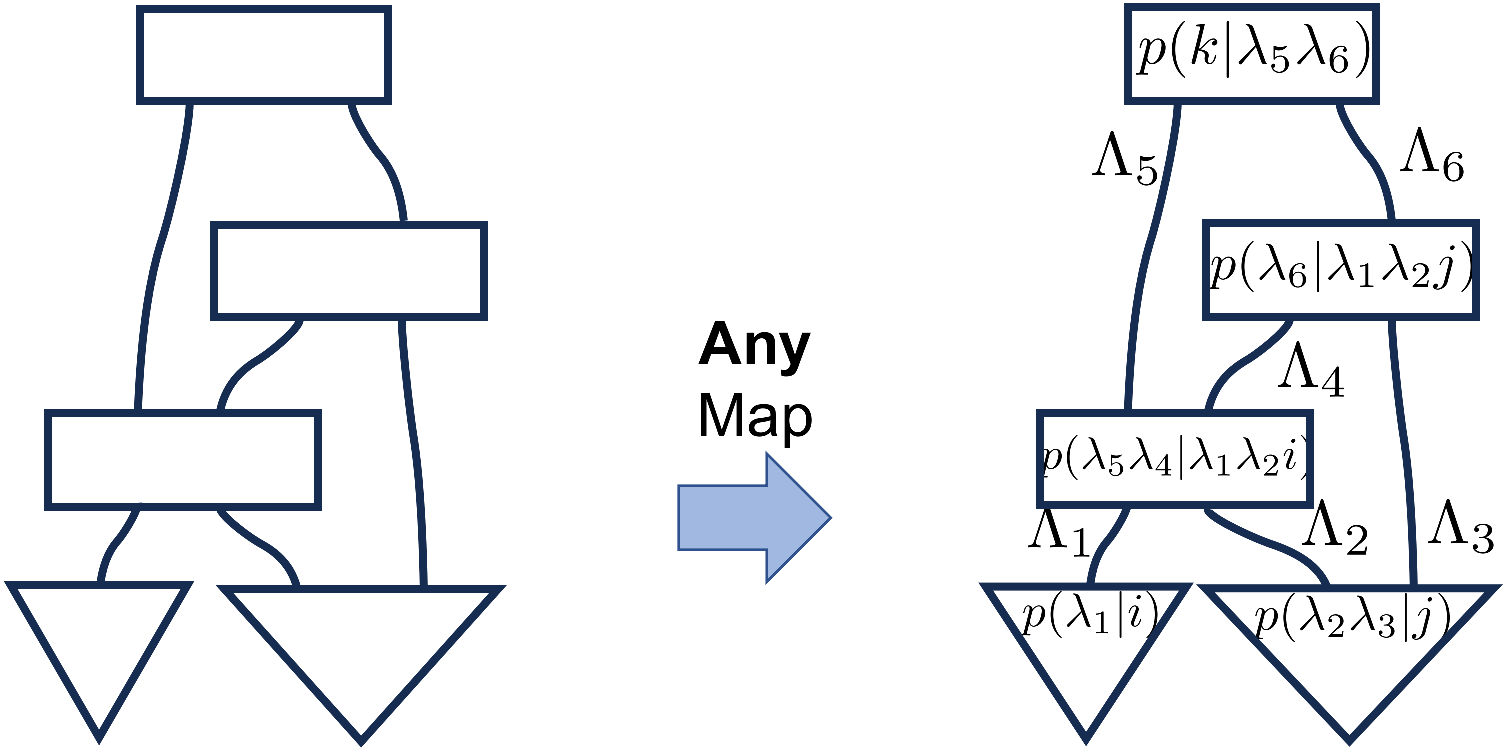
Classical explanation:



GPT systems
GPT processes

random variables
(sub)stochastic processes

Relax the assumption of linearity



GPT systems
GPT processes

random variables
(sub)stochastic processes

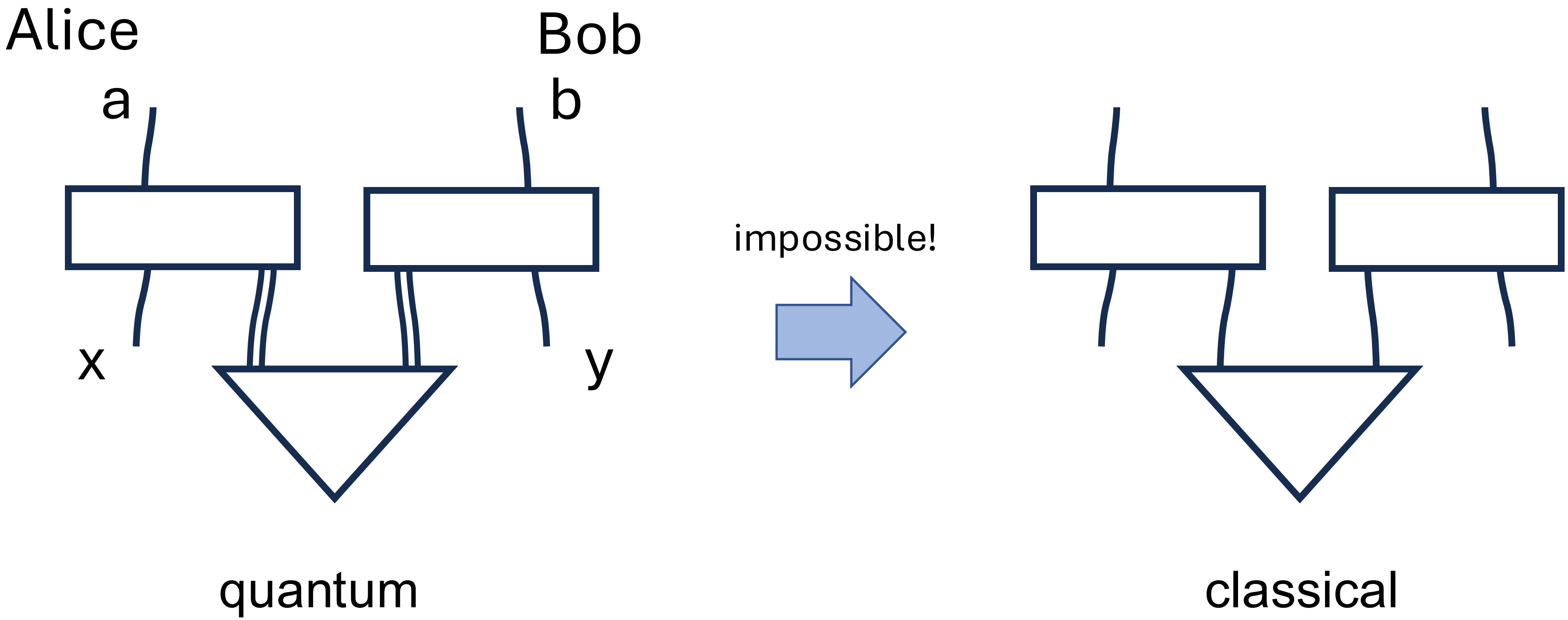
Much easier to construct such a representation…

But NOT a classical explanation in the usual sense
(doesn't explain the convex geometry of the theory)

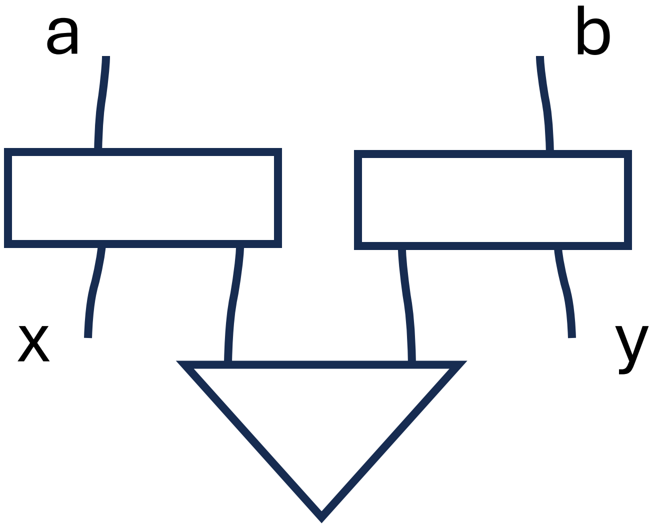
Really, we are interested in the case where
we can prove there is *still* no such model!

(strong) proofs of nonclassicality

Example: Bell’s theorem



Bell’s theorem

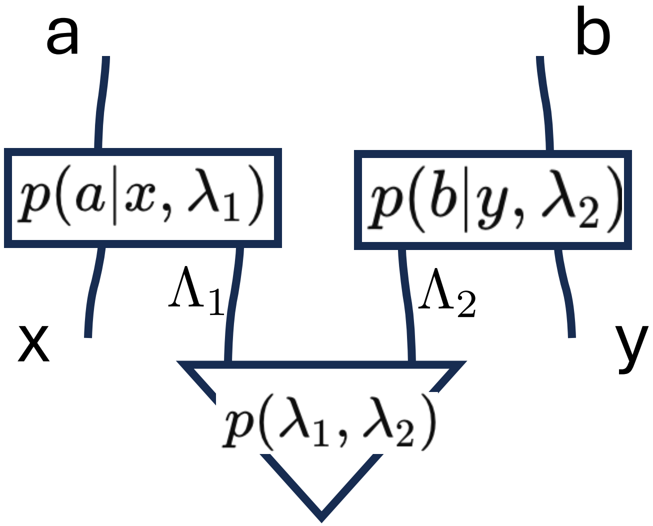


What kind of correlations can be observed?

(in classical theory?)

each party has two binary-outcome measurements

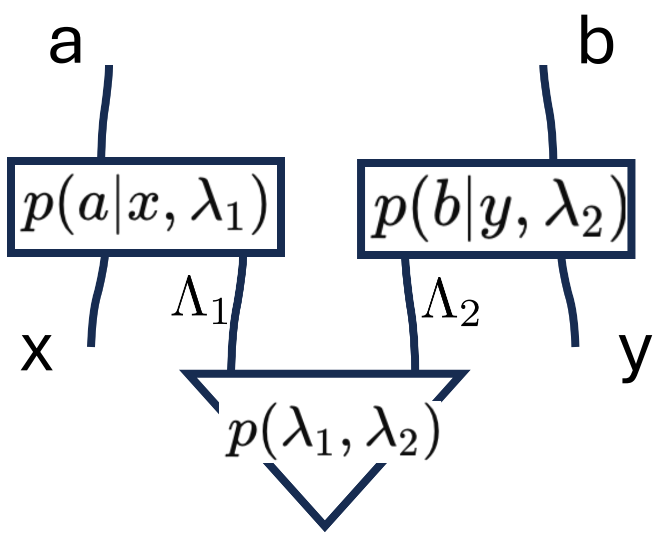
$x, y, a, b, \in \{0,1\}$



What kind of correlations can be observed?

(in classical theory?)

$$p(ab|xy)$$



What kind of correlations can be observed?

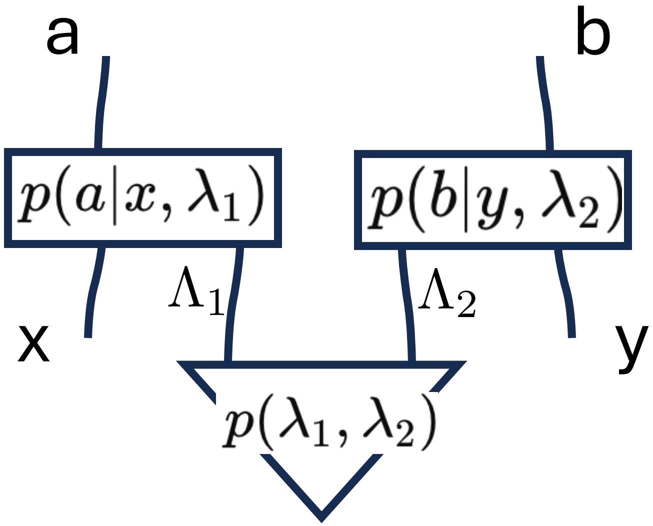
(in classical theory?)

$$p(ab|xy) = \sum_{\lambda_1,\lambda_2} p(a|x,\lambda_1)\, p(b|y,\lambda_2)\, p(\lambda_1,\lambda_2)$$

Any correlation of this form must satisfy some constraints:

1. p(b|xy)=p(b|y)

$$p(b|x,y) = \sum_a p(a,b|x,y) = \sum_{\lambda_2} p(b|y,\lambda_2)\, p(\lambda_2)$$



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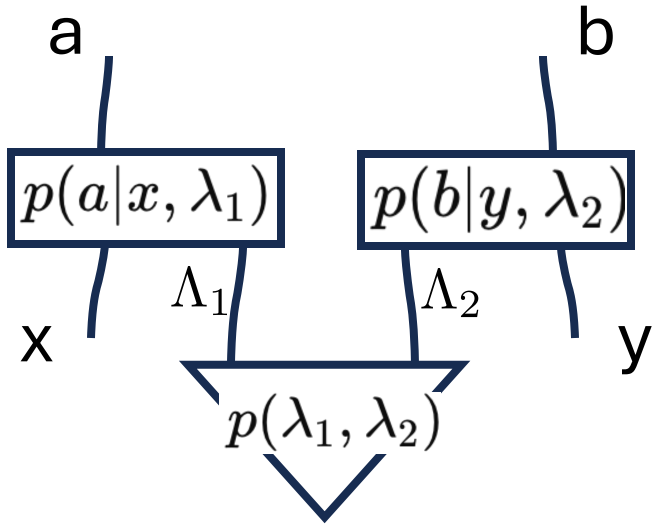
Any correlation of this form must satisfy some constraints:

1. p(b|xy)=p(b|y) No signaling



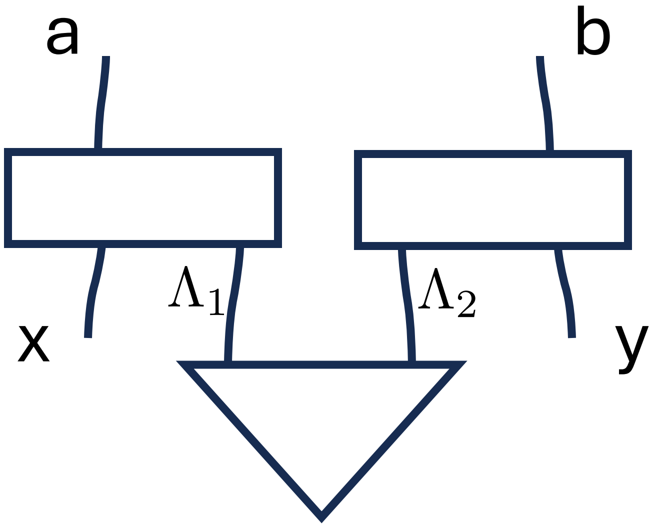
What kind of correlations can be observed?

(in classical theory?)

$$p(ab|xy) = \sum_{\lambda_1,\lambda_2} p(a|x,\lambda_1)\, p(b|y,\lambda_2)\, p(\lambda_1,\lambda_2)$$

Any correlation of this form must satisfy some constraints:

1. $p(b|xy)=p(b|y)$, $p(a|xy)=p(a|x)$ — No signaling
2. $p(a \oplus b=xy) \leq 3/4$ — Bell inequality

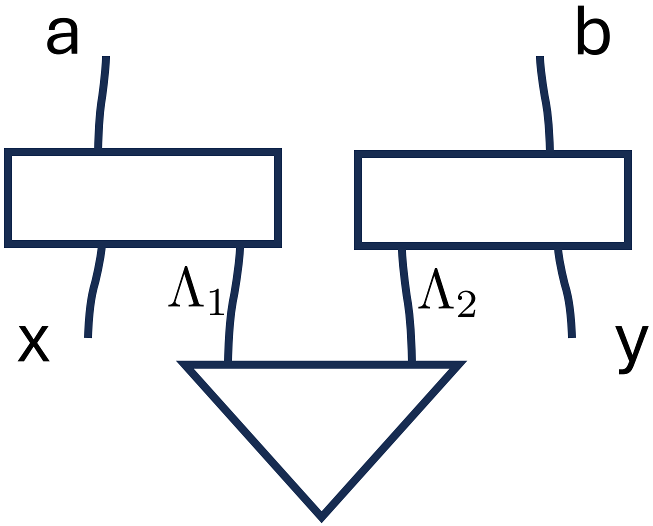


What kind of correlations can be observed?

(in classical theory?)

$$0 \oplus 0 = 0$$
$$0 \oplus 1 = 1$$
$$1 \oplus 0 = 1$$
$$1 \oplus 1 = 0$$

$p(a \oplus b = xy) \leq 3/4$



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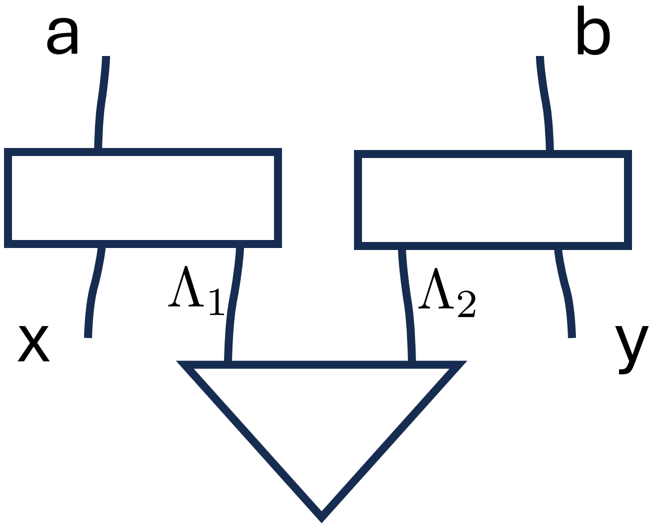
¼[p(same|x=0,y=0)



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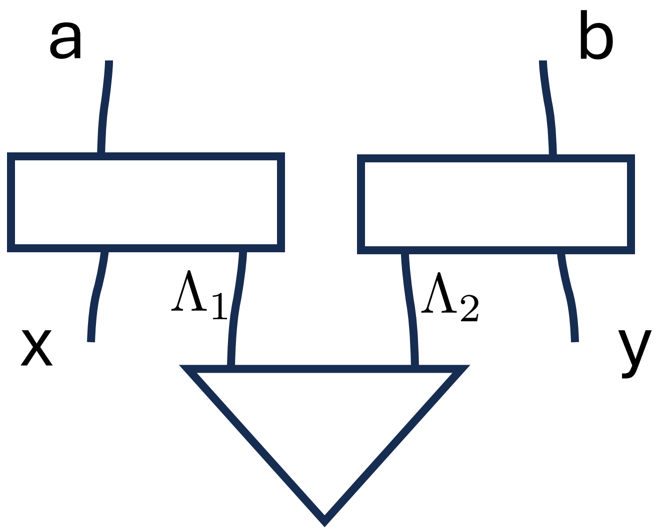
¼[p(same|x=0,y=0)+p(same|x=0,y=1)



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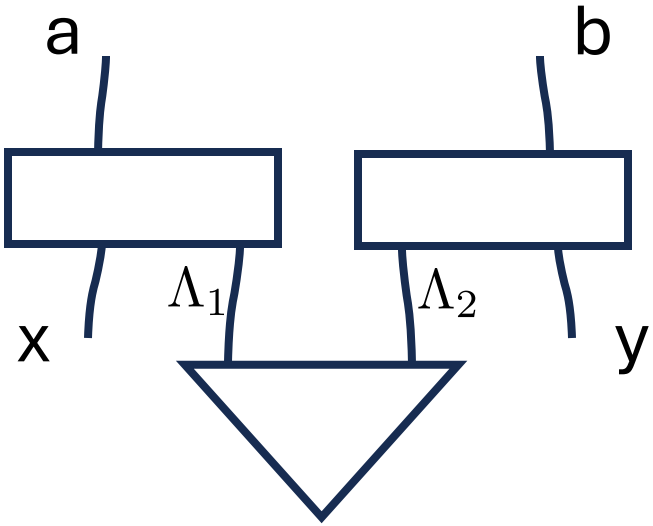
¼[p(same|x=0,y=0)+p(same|x=0,y=1)+p(same|x=1,y=0)



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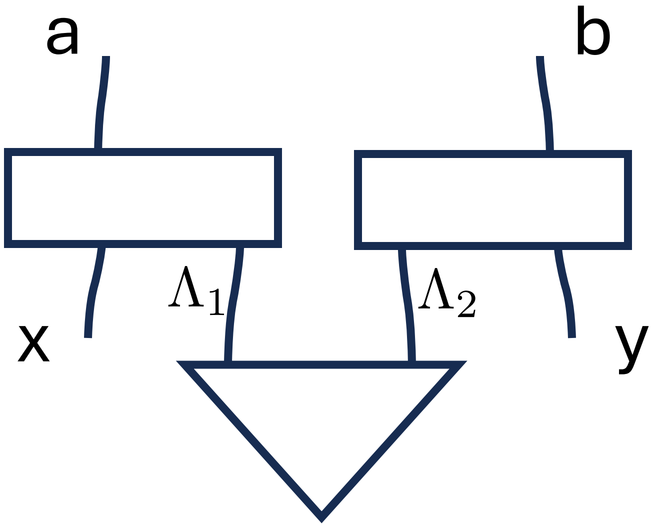
$$1 \oplus 0 = 1$$

$$1 \oplus 1 = 0$$

$p(a \oplus b = xy) \leq 3/4$

¼[p(same|x=0,y=0)+p(same|x=0,y=1)+p(same|x=1,y=0)+p(opposite|x=1,y=1)]



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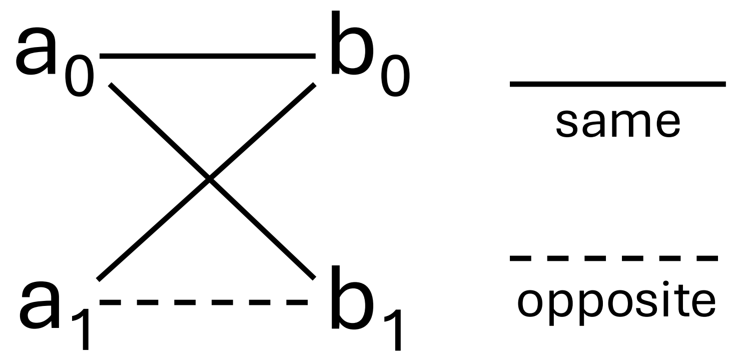
(in classical theory?)



$0 \oplus 0 = 0$

$0 \oplus 1 = 1$

$1 \oplus 0 = 1$

$1 \oplus 1 = 0$

a_x := value of a given x

b_y := value of b given y

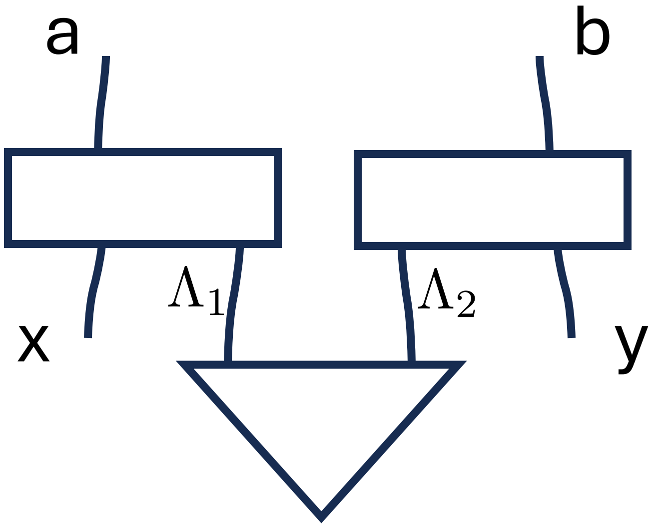
$p(a \oplus b = xy) \leq 3/4$

¼[p(same|x=0,y=0)+p(same|x=0,y=1)+p(same|x=1,y=0)+p(opposite|x=1,y=1)] ≤ 3/4

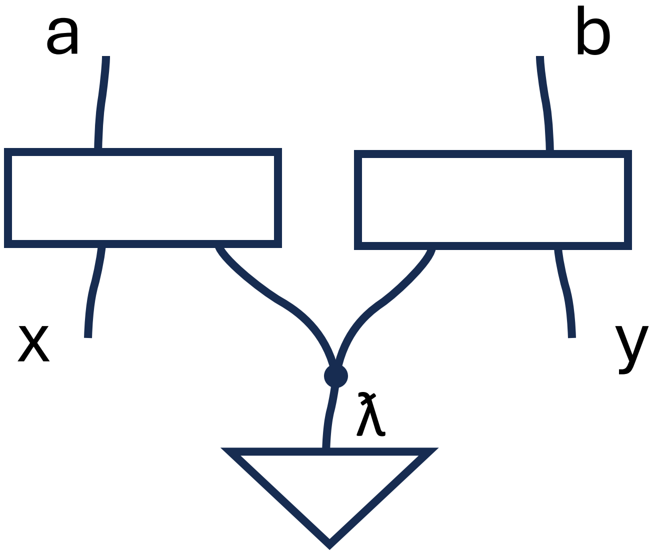
Note: we snuck in an assumption of determinism!

a_x := value of a given x
b_y := value of b given y

But adding randomness can't help you generate correlations, so we can drop this assumption.
(Fine's theorem)



$$p(ab|xy) = \sum_{\lambda_1,\lambda_2} p(a|x,\lambda_1)\, p(b|y,\lambda_2)\, p(\lambda_1,\lambda_2)$$



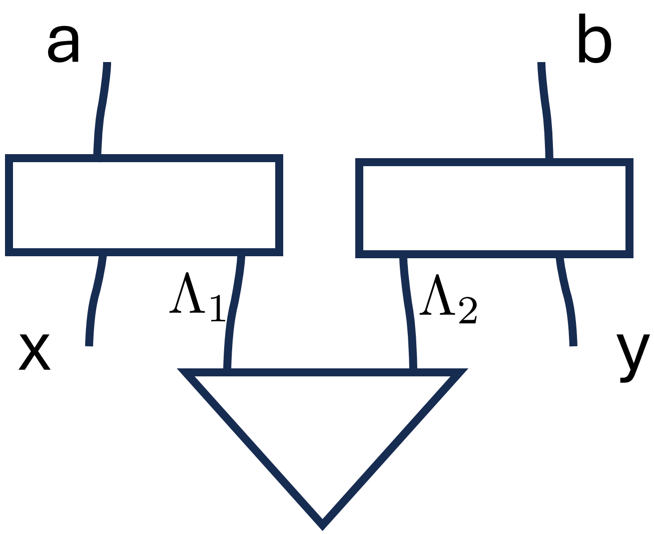
ƛ = “hidden variable”

$$p(ab|xy) = \sum_{\lambda_1,\lambda_2} p(a|x,\lambda_1)\, p(b|y,\lambda_2)\, p(\lambda_1,\lambda_2)$$

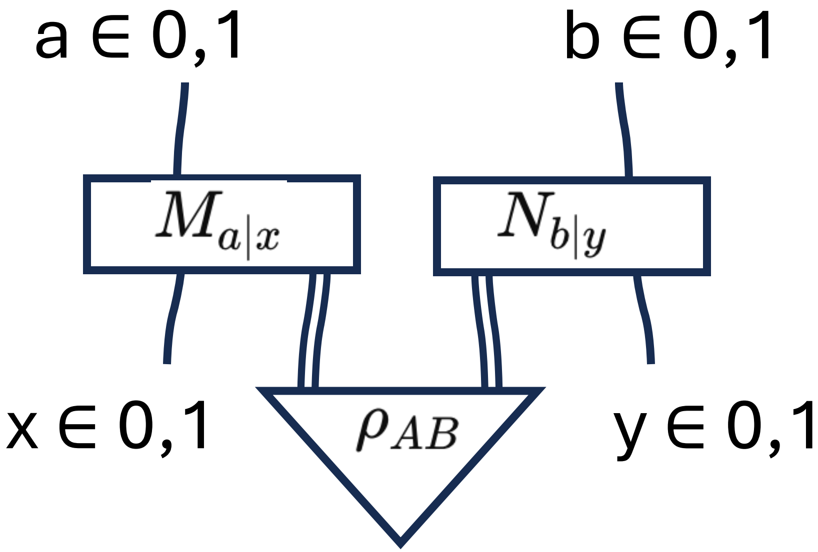
$$= \sum_{\lambda} p(a|x,\lambda)\, p(b|y,\lambda)\, p(\lambda)$$

“factorization condition”

So in a classical world, this causal structure implies that Bell inequalities must be satisfied

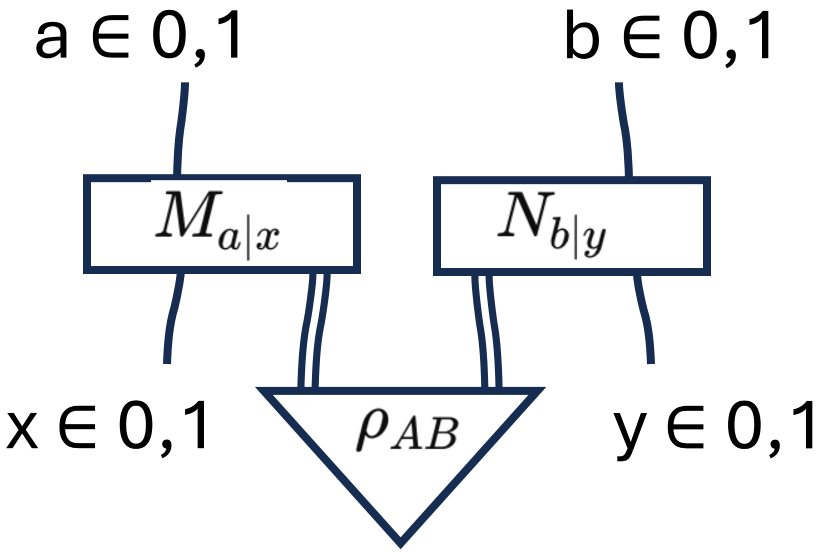


$\Rightarrow$ $p(a \oplus b = xy) \leq 3/4$



What correlations can be observed **in quantum theory**?

$$p(ab|xy) = \mathrm{Tr}\left[\left(M_{a|x} \otimes N_{b|y}\right) \rho_{AB}\right]$$



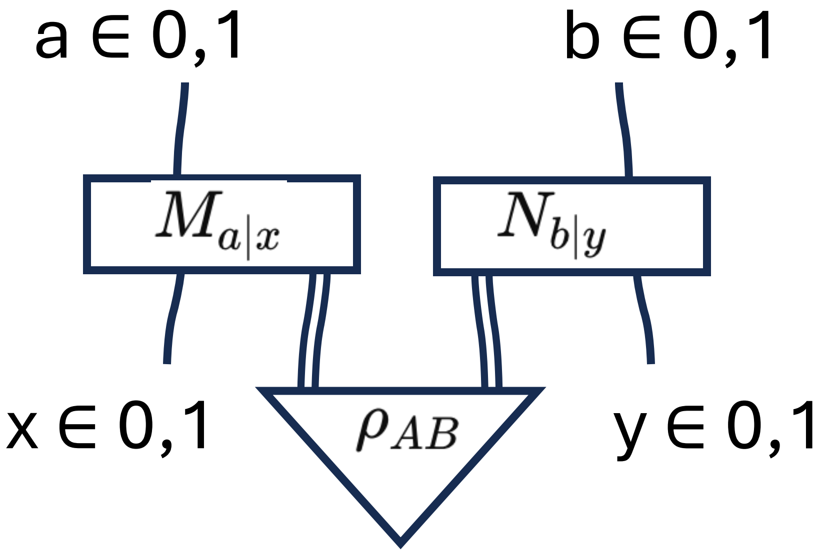
What correlations can be observed **in quantum theory**?

$$p(ab|xy) = \mathrm{Tr}\left[\left(M_{a|x} \otimes N_{b|y}\right) \rho_{AB}\right]$$

$$\begin{aligned} p(b|x,y) &= \sum_a p(a,b|x,y) \\ &= \mathrm{Tr}\left[\left(\mathbb{I} \otimes N_{b|y}\right) \rho_{AB}\right] \\ &= \mathrm{Tr}_B\left[N_{b|y}\left(\mathrm{Tr}_A\, \rho_{AB}\right)\right] \end{aligned}$$

Any correlation of this form must satisfy some constraints:

1. p(b|xy)=p(b|y), p(a|xy)=p(a|x) No signaling



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Any correlation of this form must satisfy some constraints:

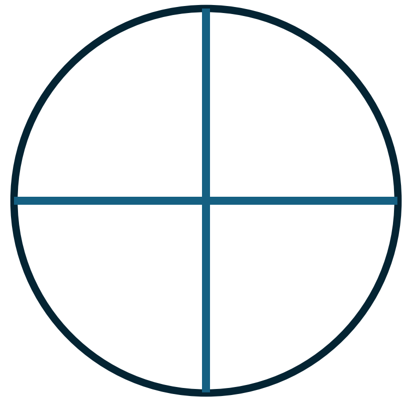
1. p(b|xy)=p(b|y), p(a|xy)=p(a|x) — No signaling
2. p(a⨁b=xy) ≤ .854 — Tsirelson inequality

Optimal quantum strategy: p(a⊕b=xy): ~0.85

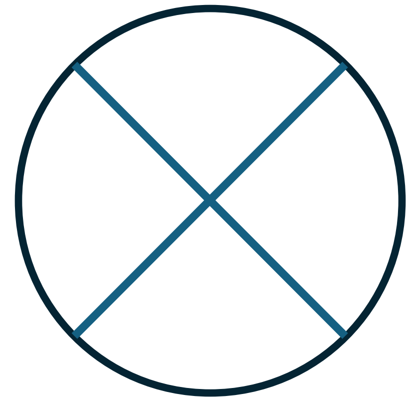
shared entanglement

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Alice's mmts

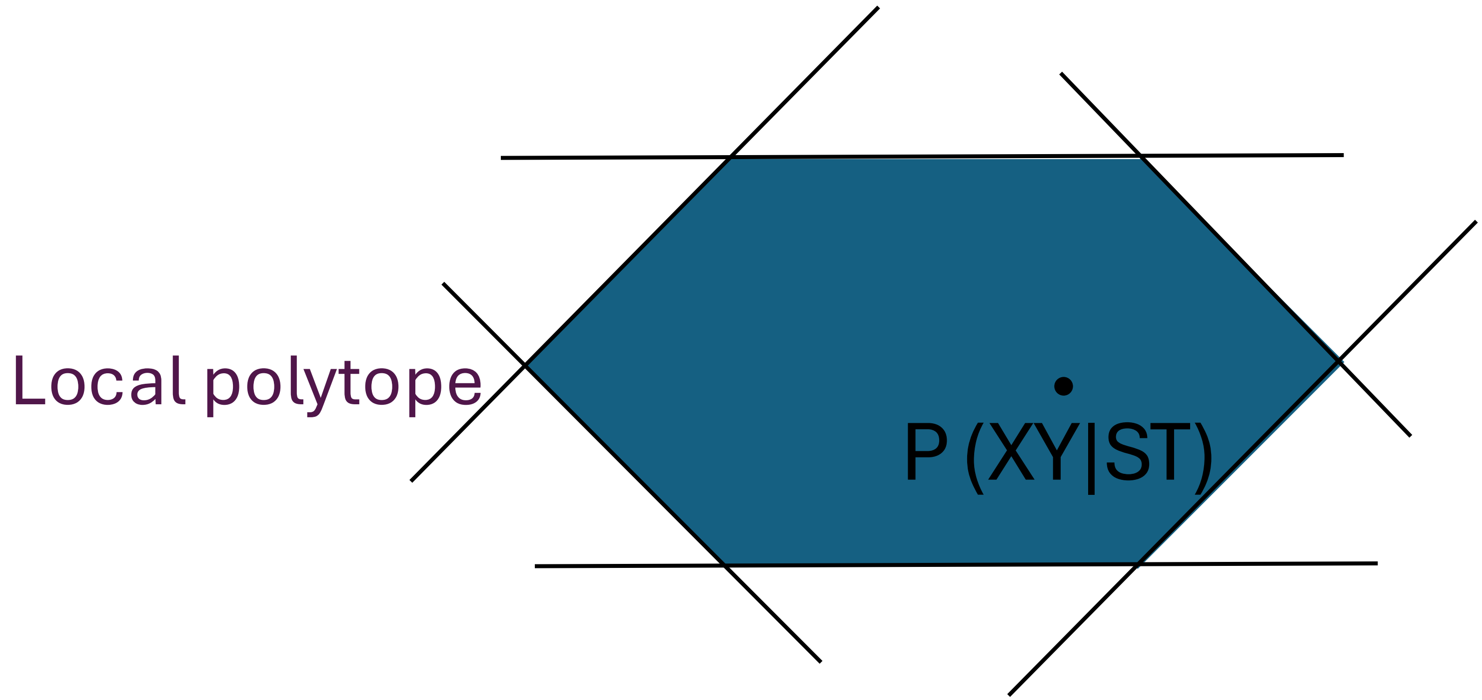


Bob's mmts



For more details, go to Youtube:
Non-locality, by Paul Skrzypczyk | Solstice of Foundations 2022

More generally, you can derive the whole set of Bell inequalities for a given scenario

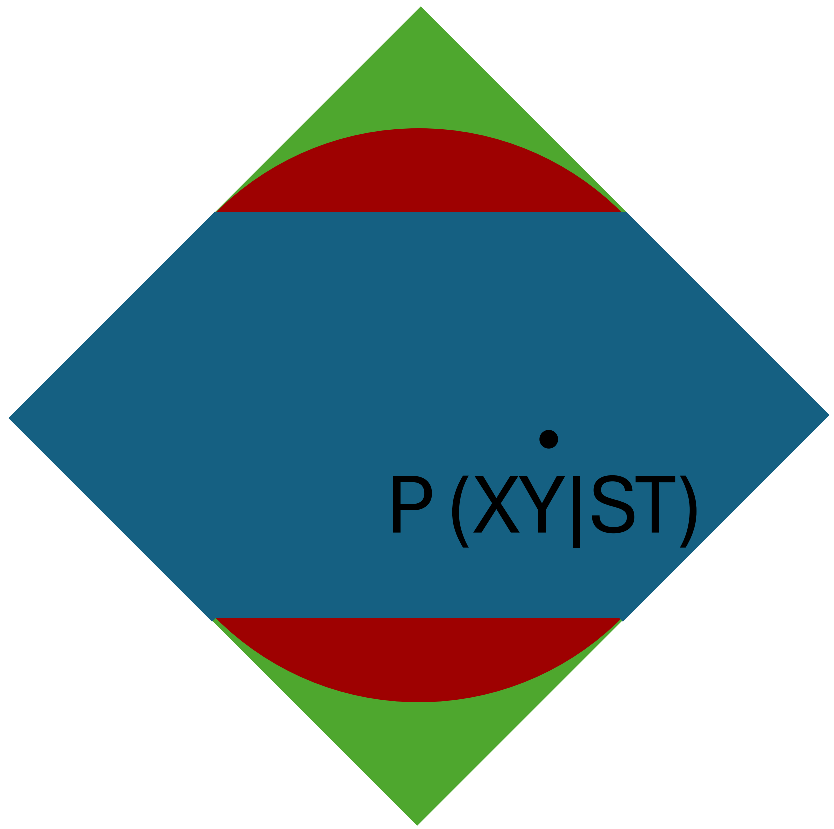


More generally, you can derive the whole set of Bell inequalities for a given scenario

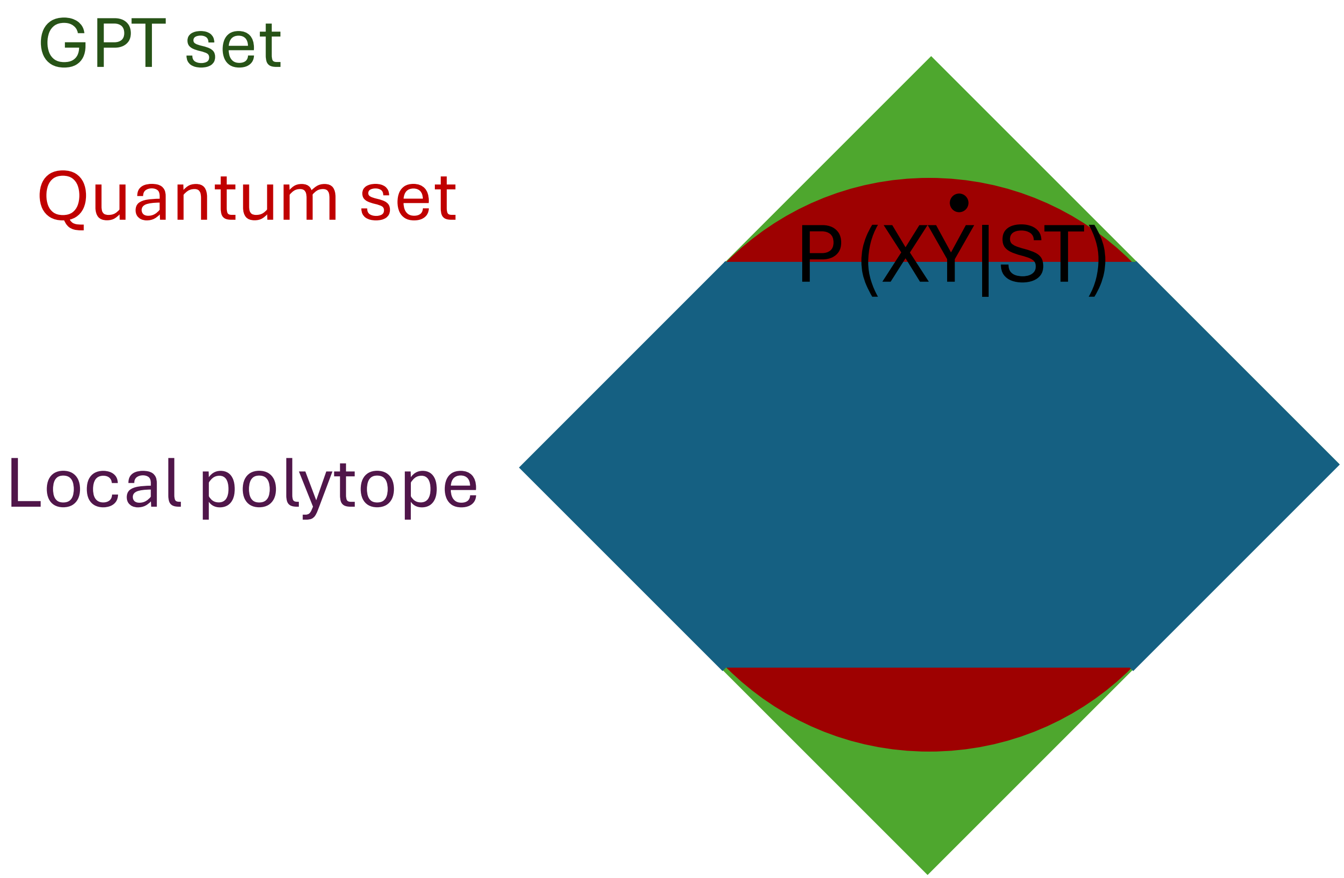
GPT set

Quantum set

Local polytope



More generally, you can derive the whole set of Bell inequalities for a given scenario



if $p(a \oplus b = xy) > \frac{3}{4}$:

quantum

classical

This is genuine nonclassicality in the sense of the previous lecture!

Theory-independent certification of nonclassicality!

The lesson of Bell's theorem

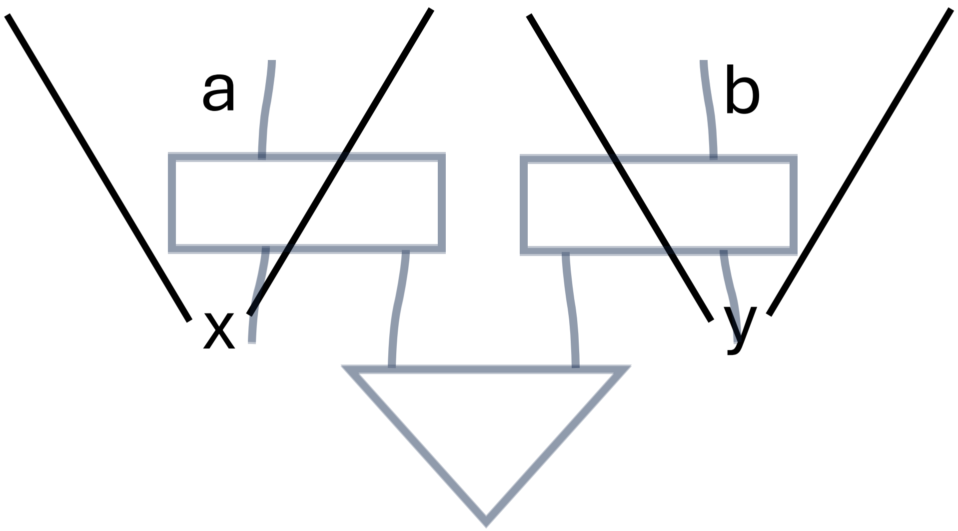
What do violations of Bell inequalities teach us?

<u>One of these must be false:</u>

-Bell causal structure — -justified by relativity theory

-classical GPT — -justified by traditional notion of realism

spacelike separation



<u>Alternative framing of assumptions</u>

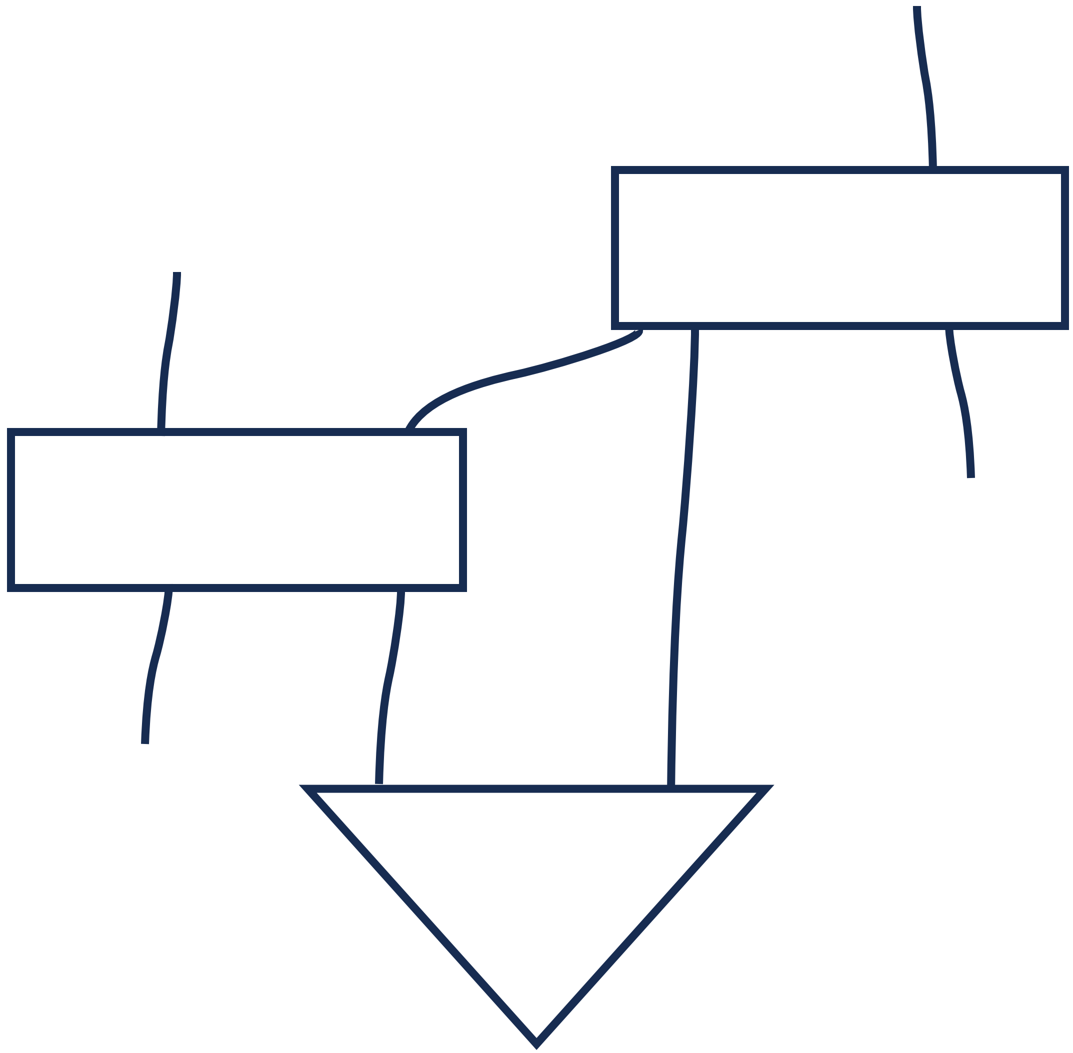
-locality

-no-retrocausality

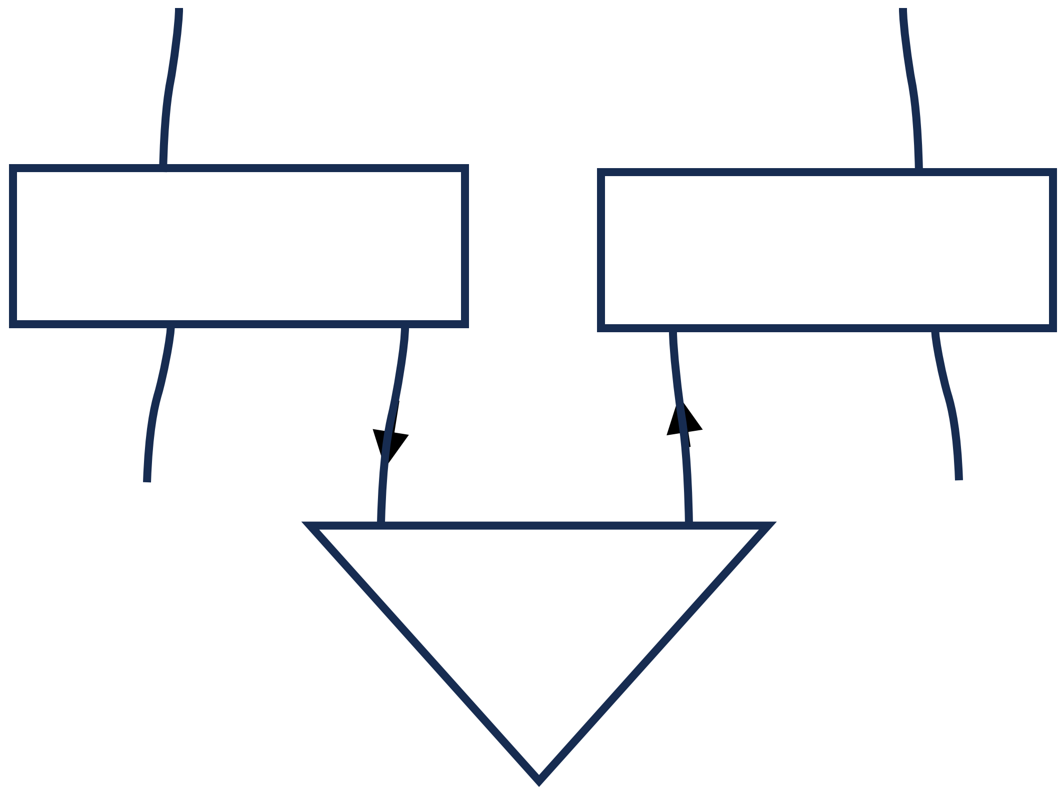
-no superdeterminism

-hidden variables

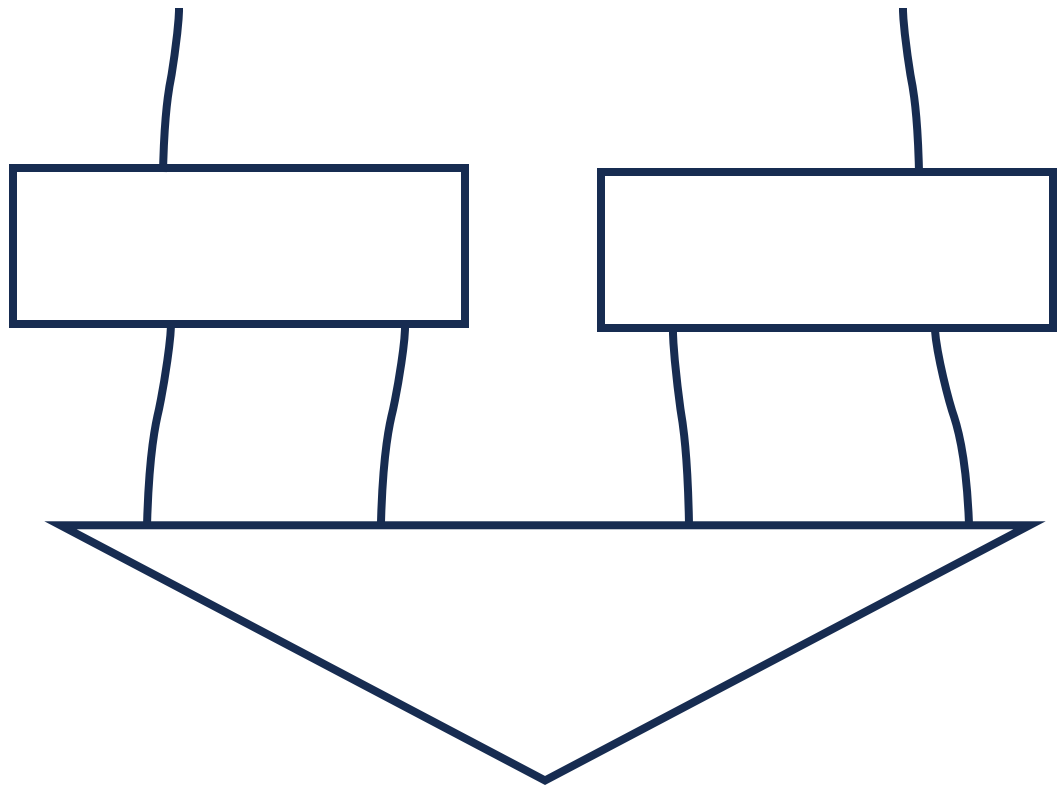
Superluminal causal influences?



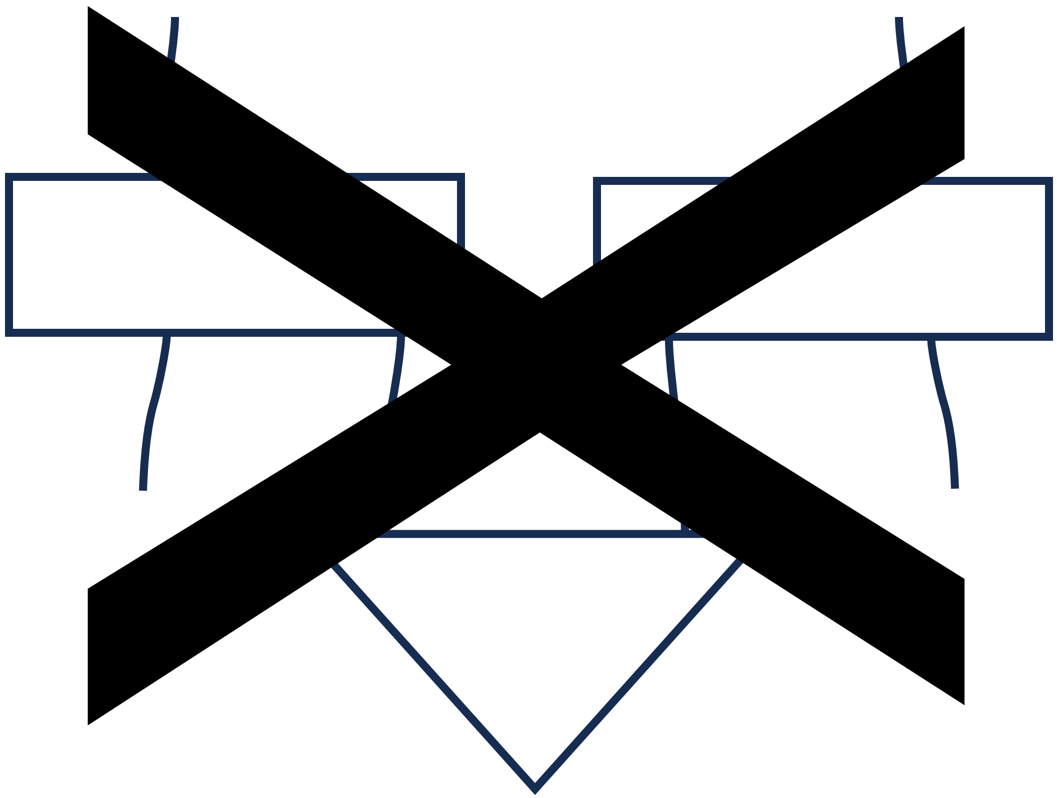
Retrocausal influences?



Superdeterminism?

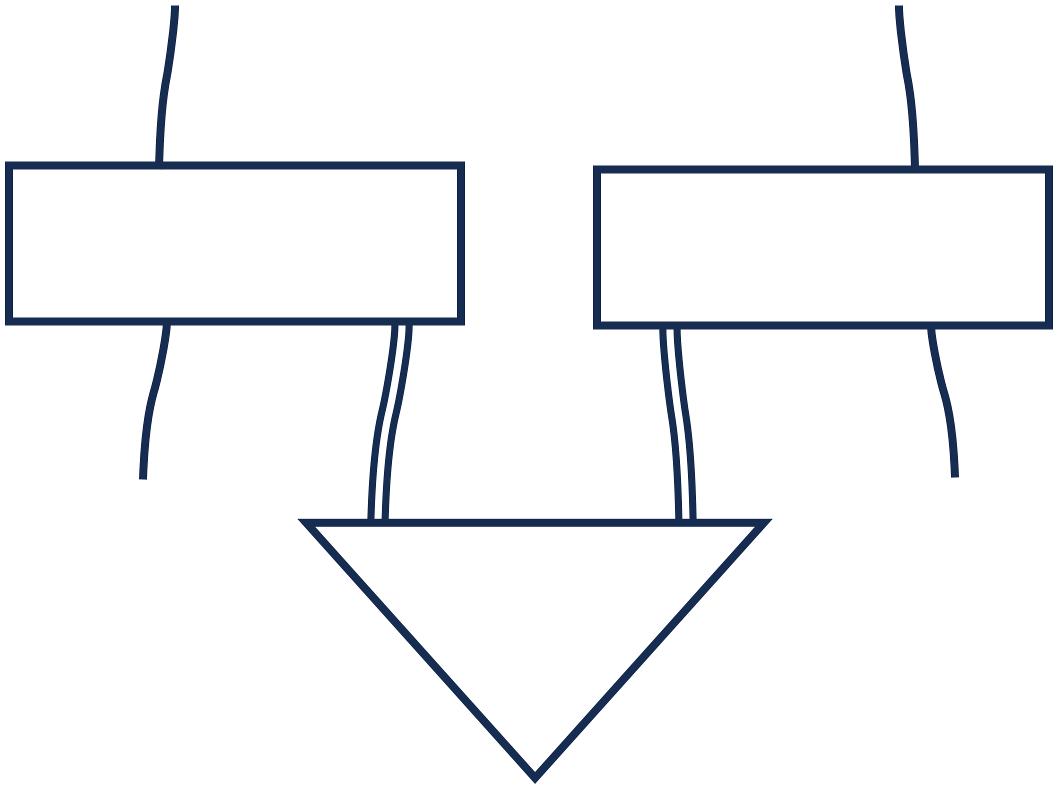


Give up on causal explanation (and realism) altogether?



Unperformed experiments have no results
-Asher Peres

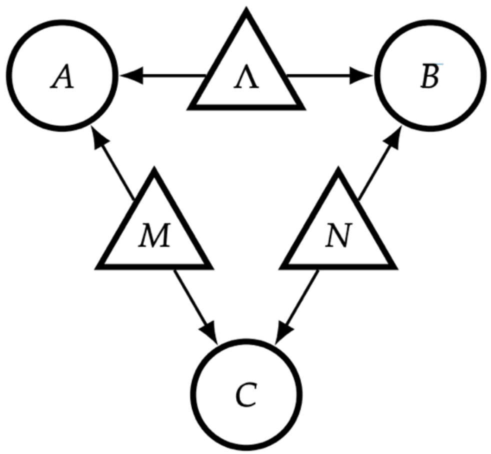
Give up on *classical* framework for causal explanation?



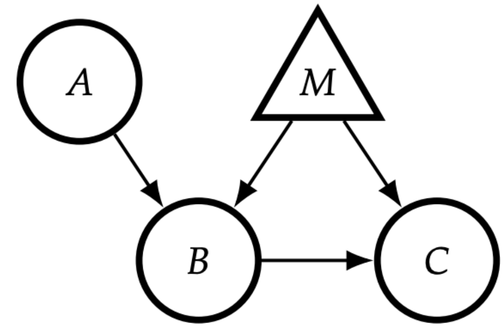
“nonclassical causal explanations”

General circuit structures

Triangle scenario

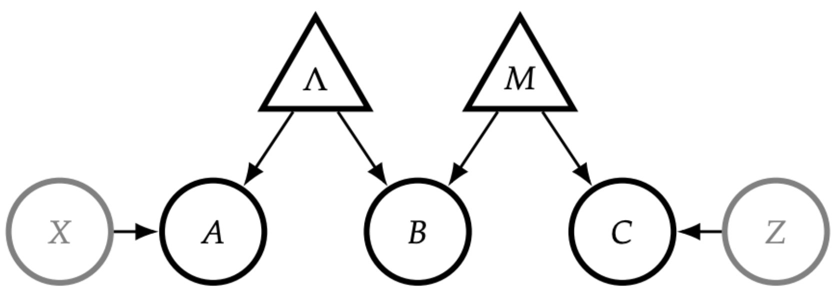


Instrumental scenario

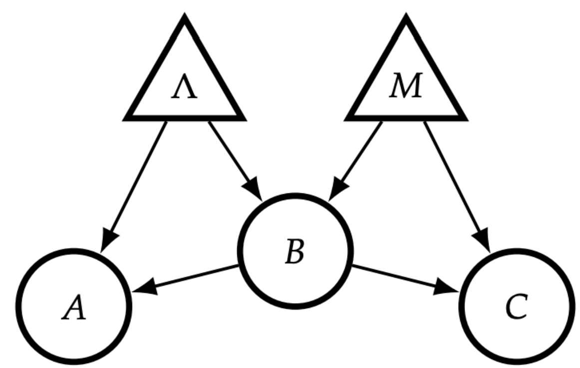


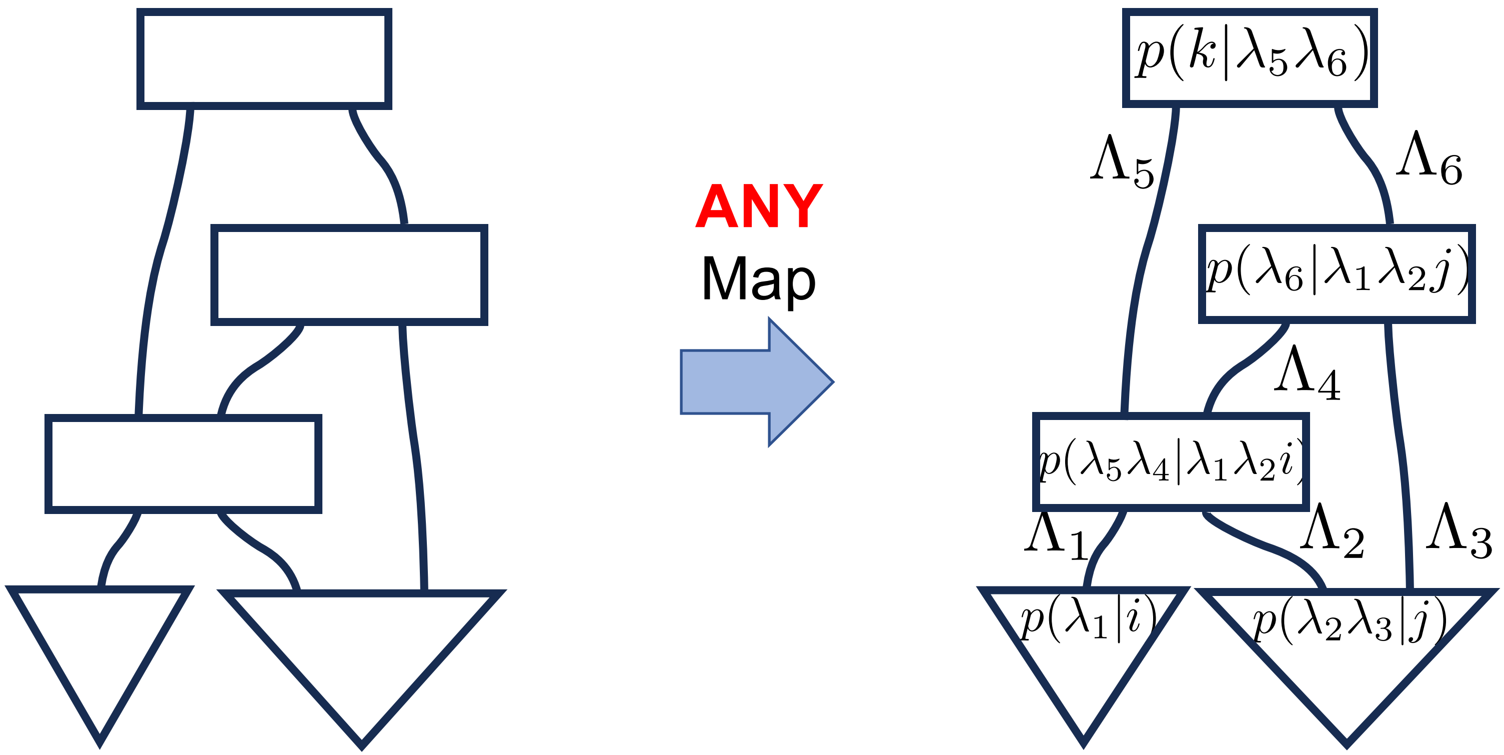
Quantum-classical gaps

Bilocality scenario



Evans scenario



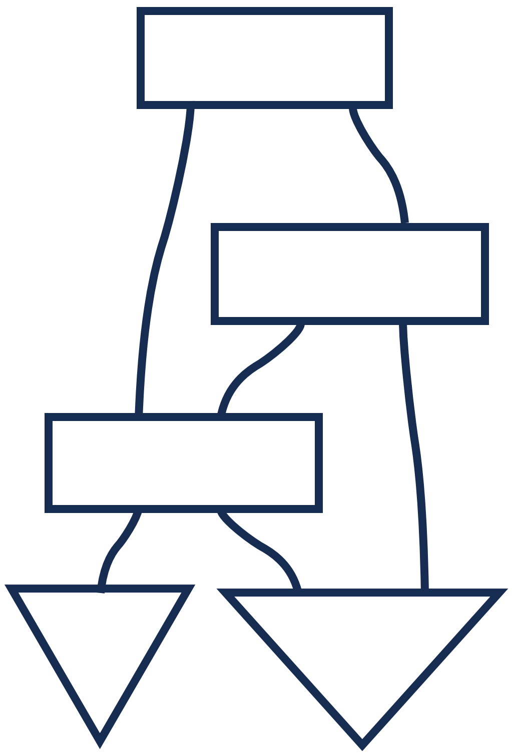


Quantum

Classical theory

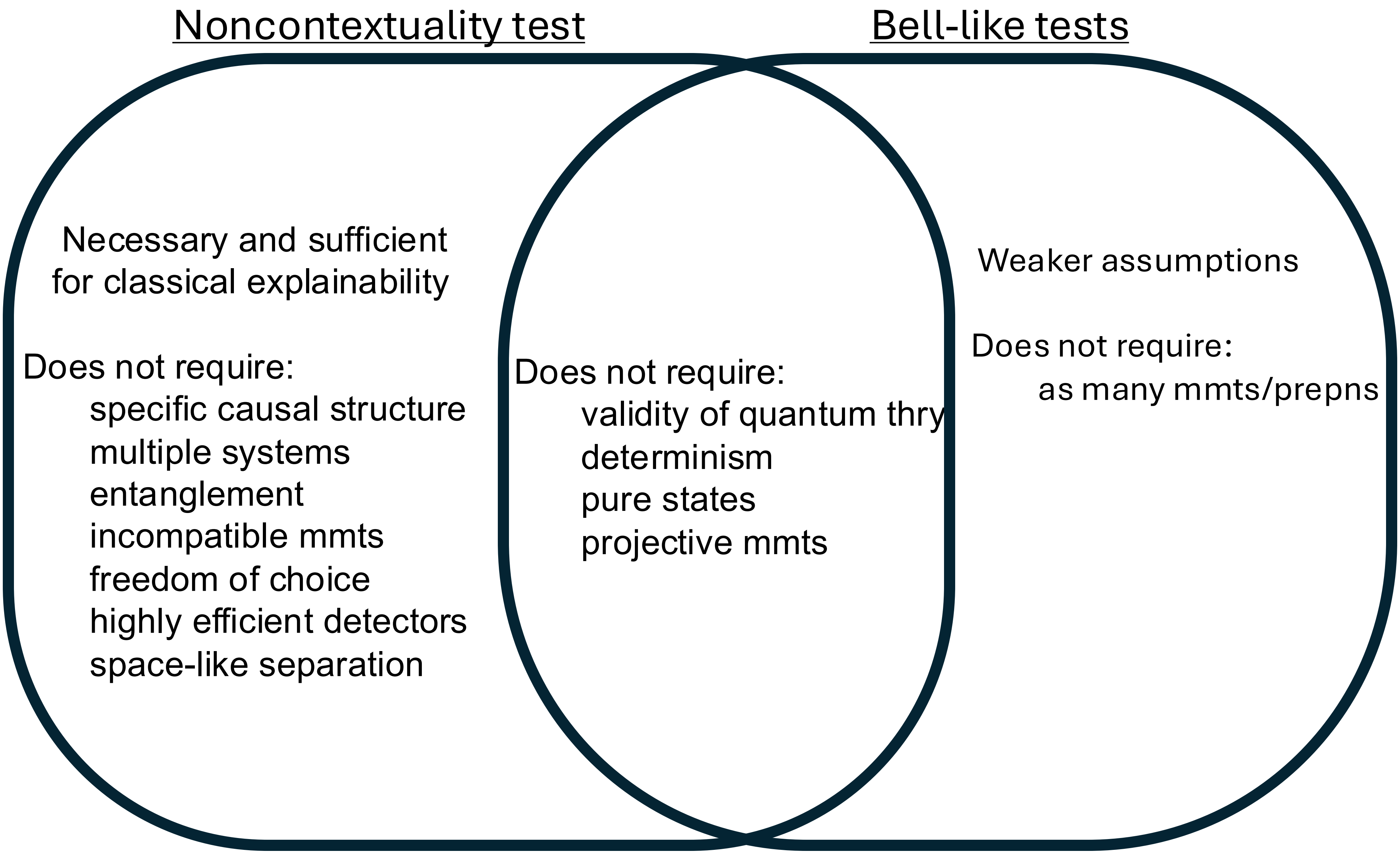
Necessary condition for classical explainability

If no such map exists $\Rightarrow$ strong manifestation of nonclassicality



Or more directly: is a given P(abc…|xyz…) consistent with the assumed causal structure and theory?

“Causal compatibility”


Noncontextuality test
Bell-like tests
Necessary and sufficient for classical explainability
Does not require:
specific causal structure
multiple systems
entanglement
incompatible mmts
freedom of choice
highly efficient detectors
space-like separation
Does not require:
validity of quantum thry
determinism
pure states
projective mmts
Weaker assumptions
Does not require:
as many mmts/prepns

Suggested references:

Causal modeling perspective on Bell's theorem:
https://arxiv.org/abs/1208.4119

Review article:
https://arxiv.org/pdf/1303.2849

Youtube:
Non-locality, by Paul Skrzypczyk | Solstice of Foundations 2022
https://www.youtube.com/watch?v=rYFIWIfW6mk

Feedback encouraged!
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Quantifying Nonclassicality in Bell scenarios

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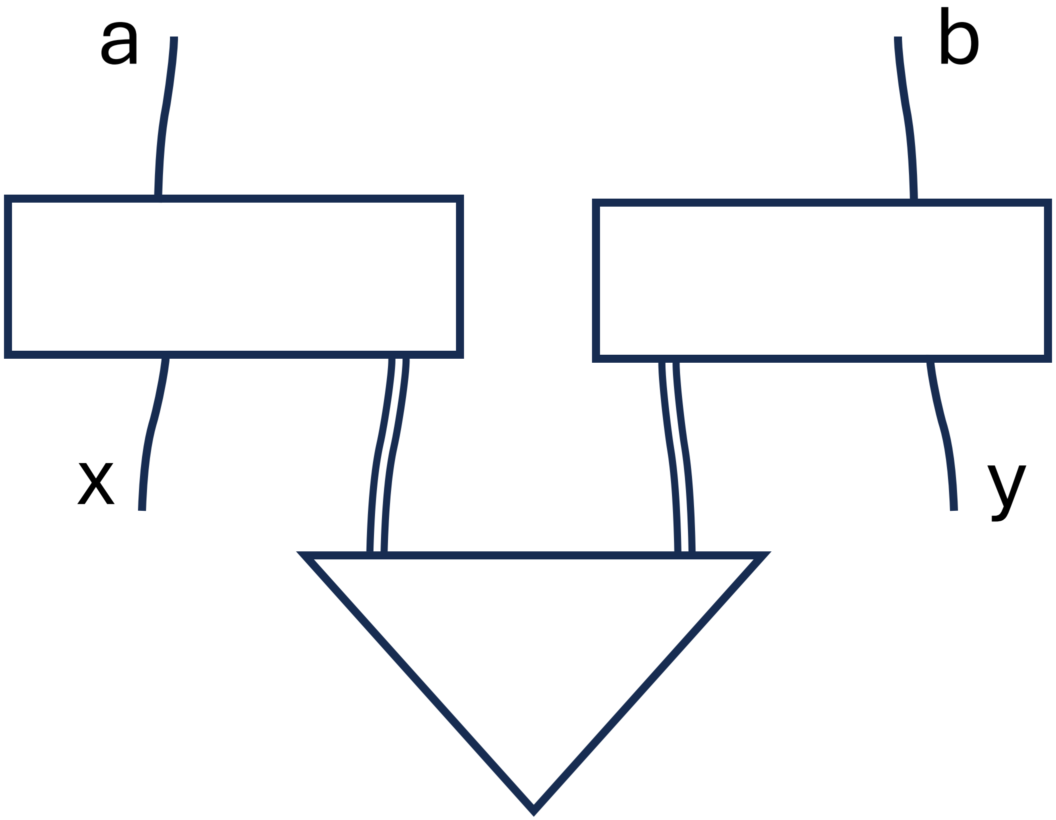
PERIMETER INSTITUTE



We can tell if the common cause is classical or nonclassical based on the observed correlations p(ab|xy)

Can we also tell *how nonclassical* it is?

Naïve answer: Just check how much a Bell inequality is violated...

We need a resource theory!

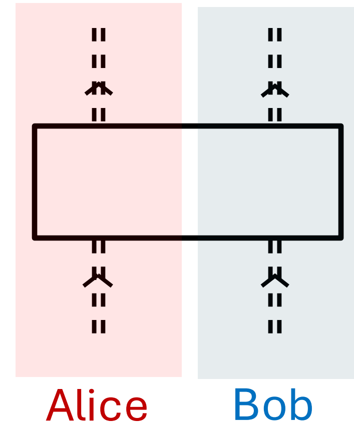
Resource theory of
Local Operations and Shared Randomness

Second motivation: unify and simplify a huge range of foundational concepts (in Bell scenarios)

Types of Resources

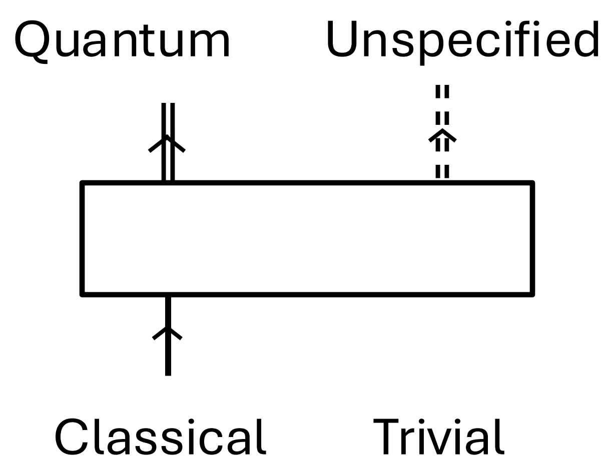
Resources:

no-signaling quantum channels
distributed among various parties
(focus on bipartite case for simplicity)

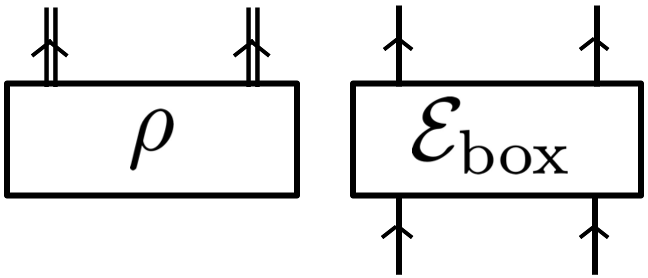


The **type** of a resource is determined by the nature of its input and output systems: quantum, classical, or trivial

Graphical notation:



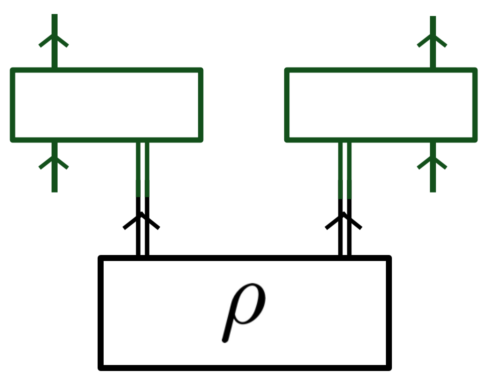
Resource type (examples)



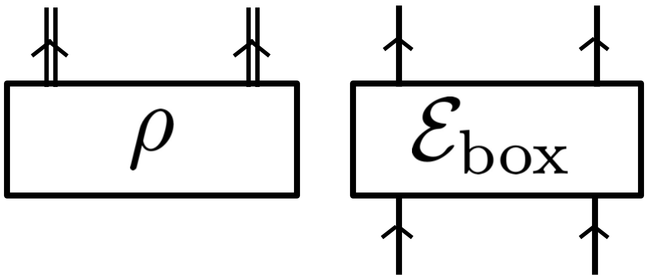
quantum state

no-signaling box

$$p(ab|xy) = \mathbf{Tr}\left[\left(M_{a|x} \otimes N_{b|y}\right) \rho_{AB}\right]$$



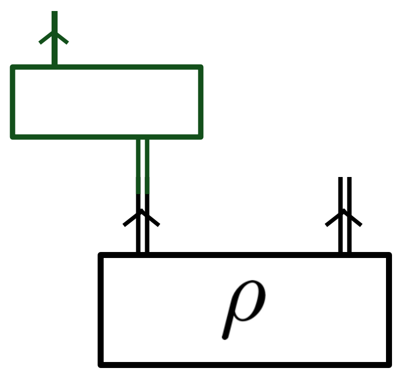
Resource type (examples)



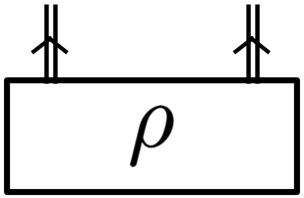
quantum state

no-signaling box

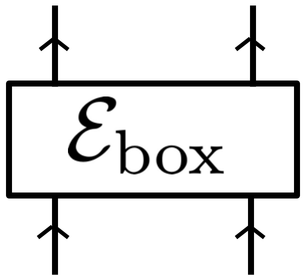
$$\{\sigma_a\}_a \text{ where } \sigma_a = \mathrm{Tr}_A[(M_a \otimes \mathbb{I}_B)\rho_{AB}]$$



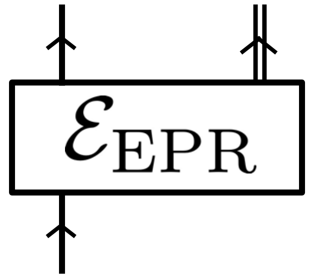
Resource type (examples)



quantum
state



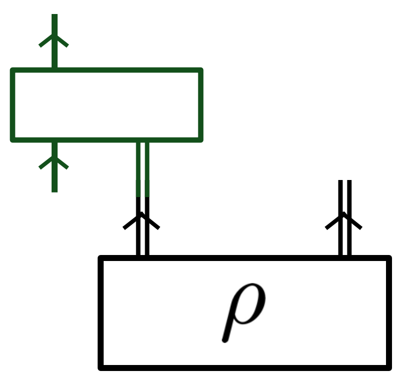
no-signaling
box



steering
assemblage

Named after Einstein,
Podolsky, and Rosen

$$\left\{\left\{\sigma_{a|x}\right\}_a\right\}_x \quad \text{where} \quad \sigma_{a|x} = \mathbf{Tr}_A\left[\left(M_{a|x} \otimes \mathbb{I}_B\right)\rho_{AB}\right]$$



example of steering

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

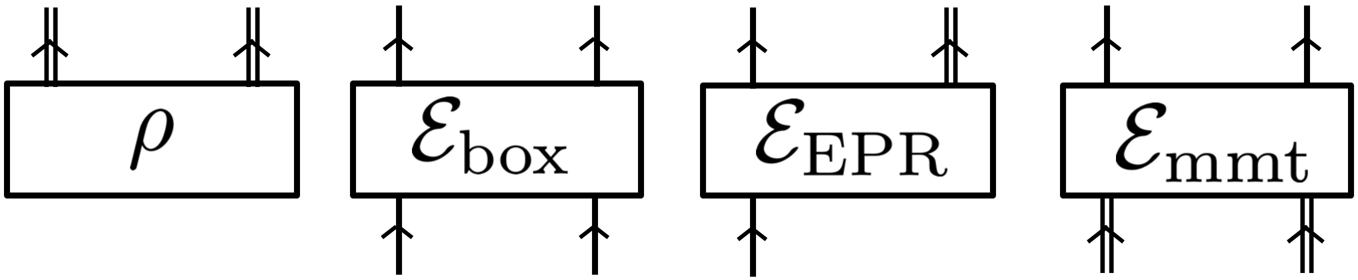
Alice measures 0/1 basis: updated state on Bob's side will be 1 or 0
Alice measures +/- basis: updated state on Bob's side will be - or +
Alice measures +i/-i basis: updated state on Bob's side will be -i or +i

$$\left\{ \begin{array}{l} \{\frac{1}{2}|1\rangle\langle 1|,\ \frac{1}{2}|0\rangle\langle 0|\} \\ \{\frac{1}{2}|-\rangle\langle -|,\ \frac{1}{2}|+\rangle\langle +|\} \\ \{\frac{1}{2}|-i\rangle\langle -i|,\ \frac{1}{2}|+i\rangle\langle +i|\} \end{array} \right\}$$

assemblage

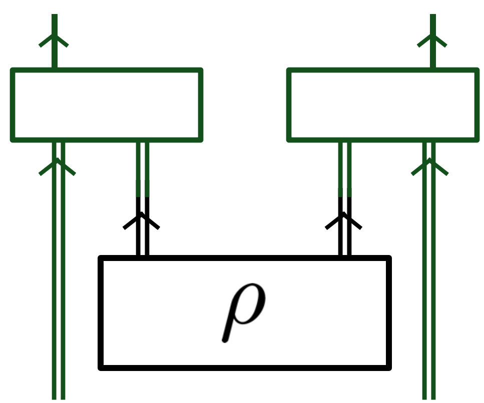
If you think the quantum state is ontic,
then this is already a proof of nonlocality!
-More sensible conclusion: quantum state is epistemic

Resource type (examples)

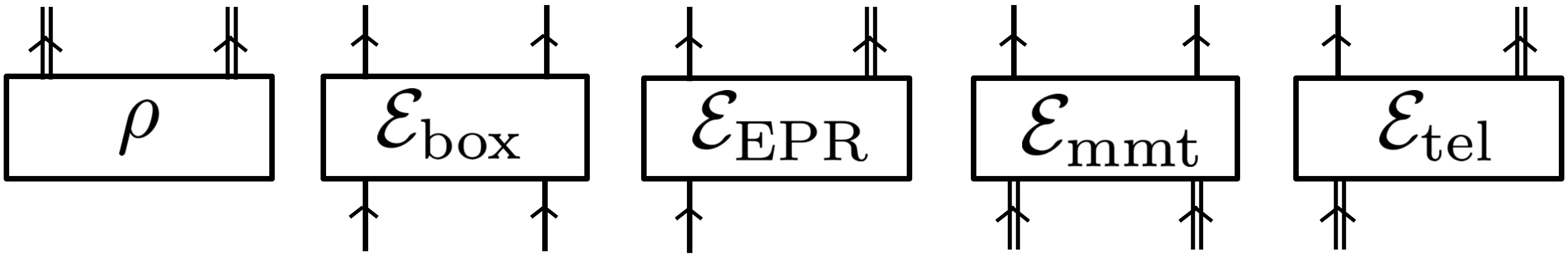


quantum state | no-signaling box | steering assemblage | distributed measurement

$$\left\{M_{ab}^{XY}\right\}_{a,b} \quad \text{where} \quad M_{ab}^{XY} = \mathrm{Tr}_{AA'}\left[\left(M_a^{XA} \otimes N_b^{YA'}\right)\left(\mathbb{I}_{XY} \otimes \rho_{AA'}\right)\right]$$



Resource type (examples)



quantum state

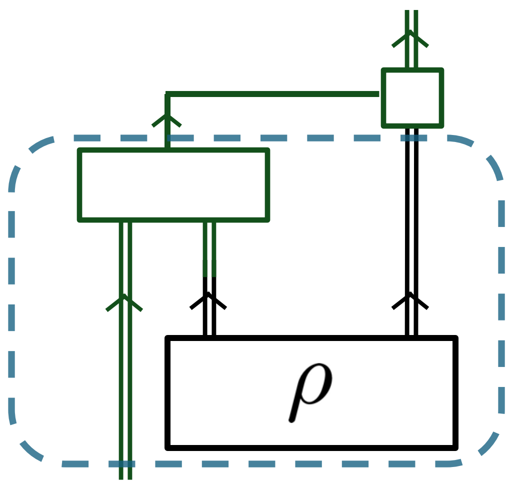
no-signaling box

steering assemblage

distributed measurement

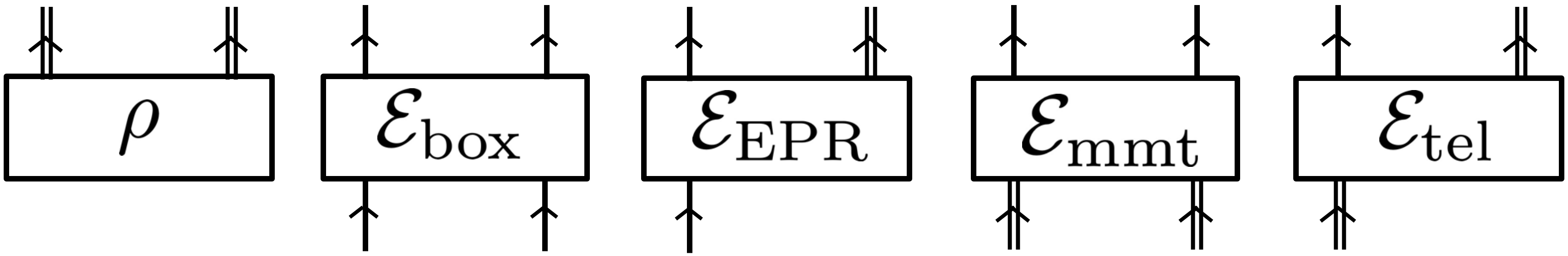
quantum teleportage

teleportation protocol:

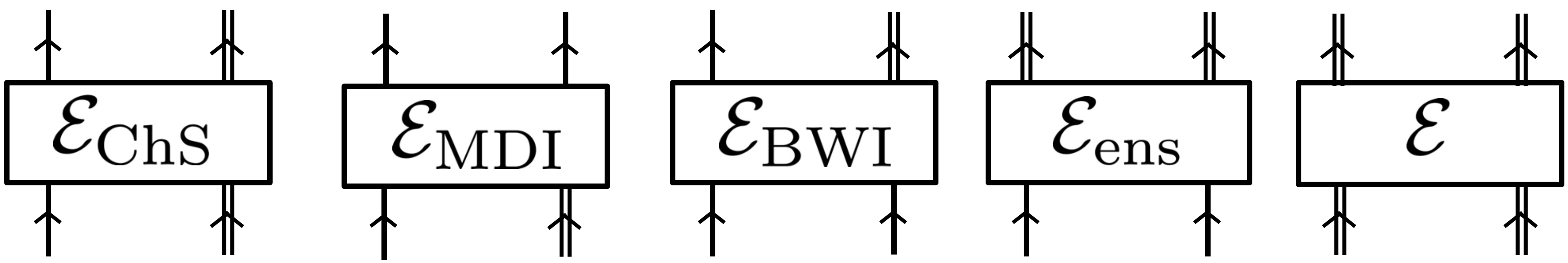


contains the relevant information for teleportation

Resource type (examples)

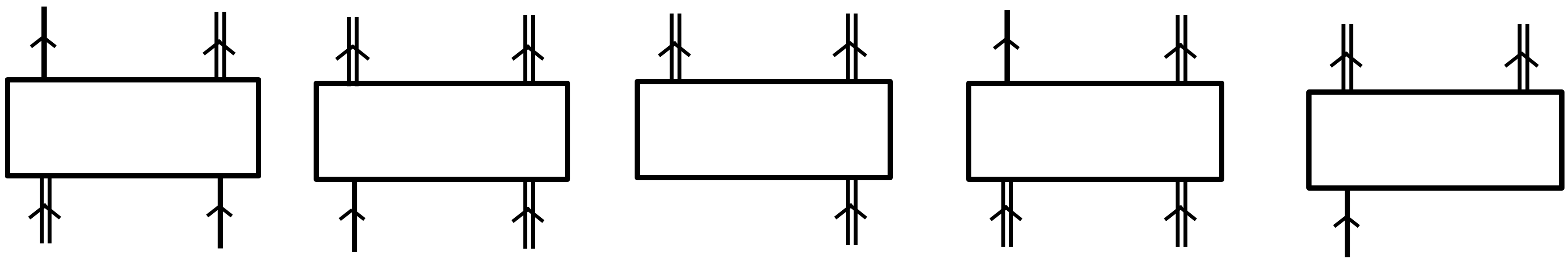


quantum state | no-signaling box | steering assemblage | distributed measurement | quantum teleportage



channel assemblage | measurement-device-independent steering | Bob-with-input assemblage | distributed ensemble-preparation | bipartite channel

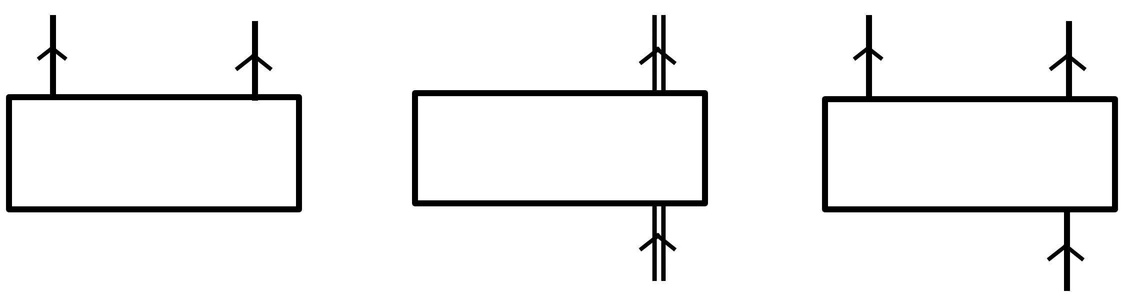
Five new nontrivial bipartite scenarios/resource types:



Open question: foundational or practical significance?

-five new manifestations of nonclassicality

Remaining types are all trivial:



etc

(Non)Free Resources

The KEY step in any resource theoretic research is identifying the relevant set of free operations.

<u>What are the physical restrictions in the scenario under study?</u>
-no cause-effect relations (no communication)
-locally unrestricted
-common causes are allowed

So, we allow local quantum operations and classical common causes. Then, anything nonfree requires a *nonclassical* common cause

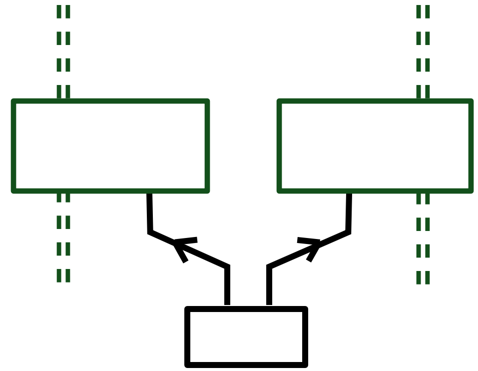
local operations and shared randomness (LOSR)

Free LOSR resources:

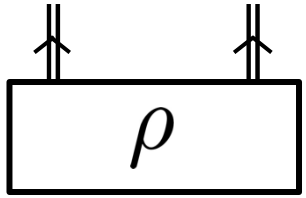
those simulable by

-local operations

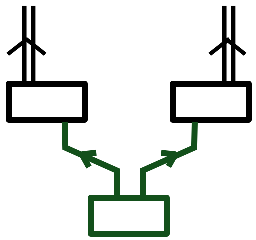
-shared randomness



a bipartite density operator



is LOSR-free if it decomposes as



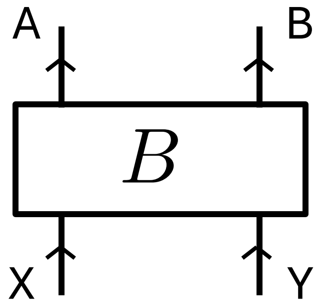
$$\sum_i p_i \rho_i \otimes \sigma_i$$

separable

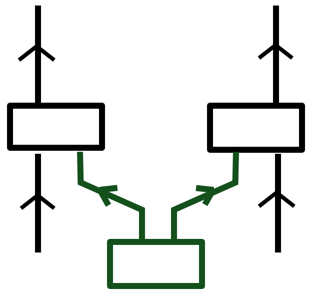
otherwise, it a nonfree resource

entangled

a bipartite correlation (or “box”)



is LOSR-free if it decomposes as



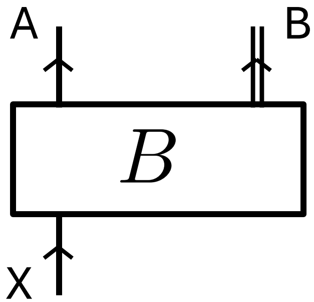
$$\sum_{\lambda} p(a|x,\lambda)\, p(b|y,\lambda)\, p(\lambda)$$

local

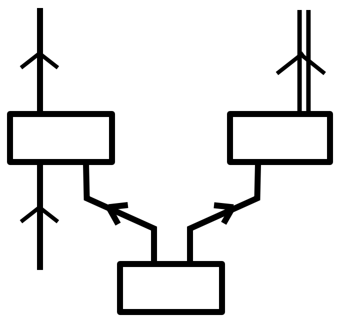
otherwise, it is a nonfree resource

nonlocal

a bipartite steering assemblage



is LOSR-free if it decomposes as



$$\sigma_{a|x} = \sum_{\lambda} p(\lambda)\, p(a|x,\lambda)\, \rho_{\lambda}$$

unsteerable!

otherwise, it is a nonfree resource

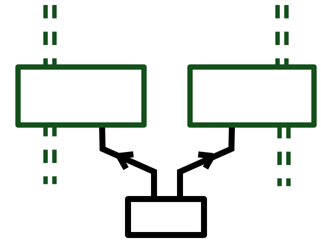
steerable

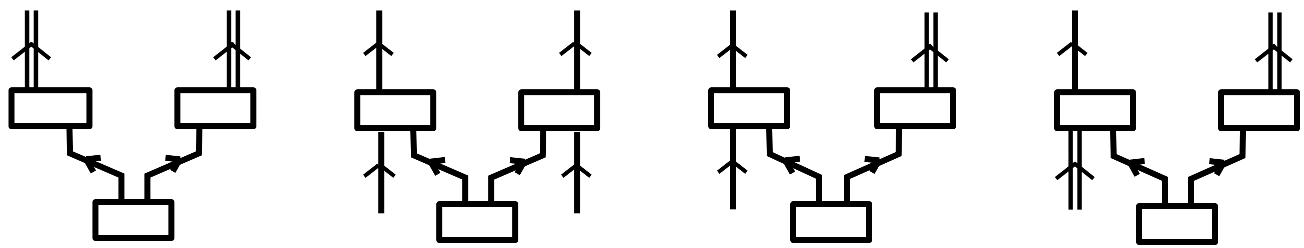
Free LOSR resources:

those simulable by

-local operations

-shared randomness





separable state

local box

unsteerable assemblage

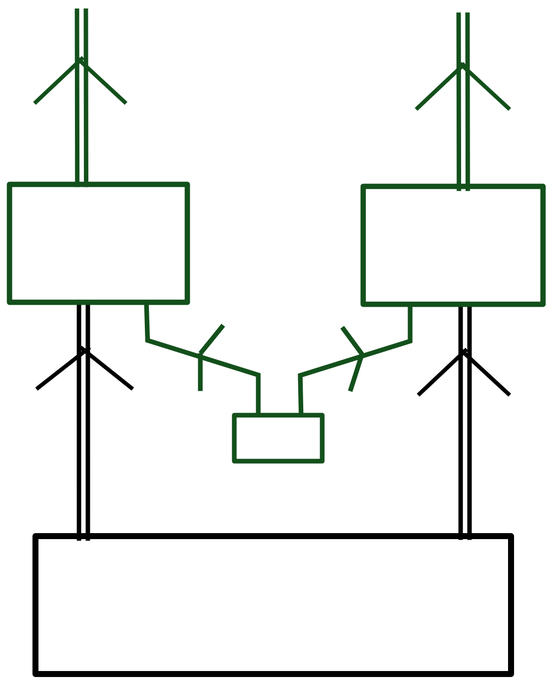
classical teleportage

etc

In every case, the ` useless' set is the LOSR free set!

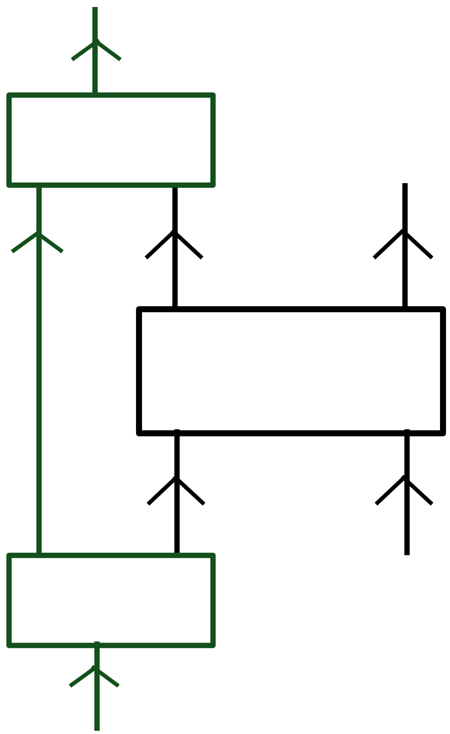
Resource Transformations

State-to-State conversions

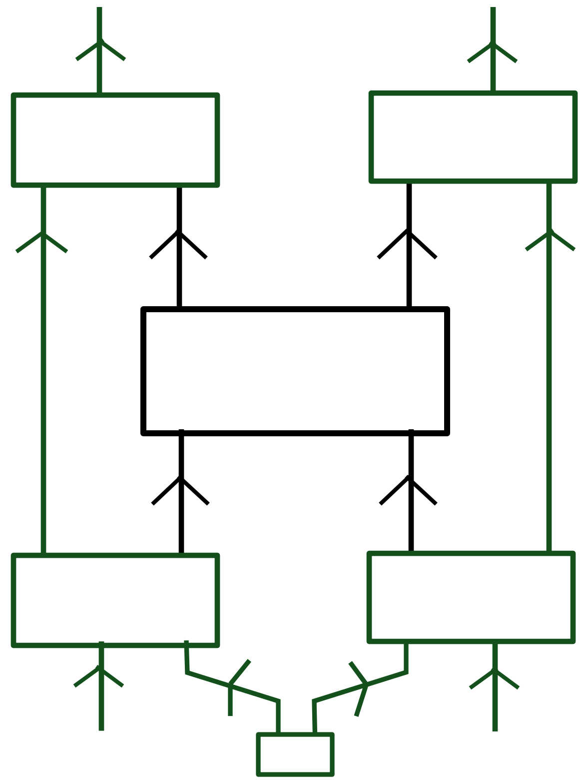


-arbitrary local channels
-correlated by shared randomness

Box-to-Box conversions

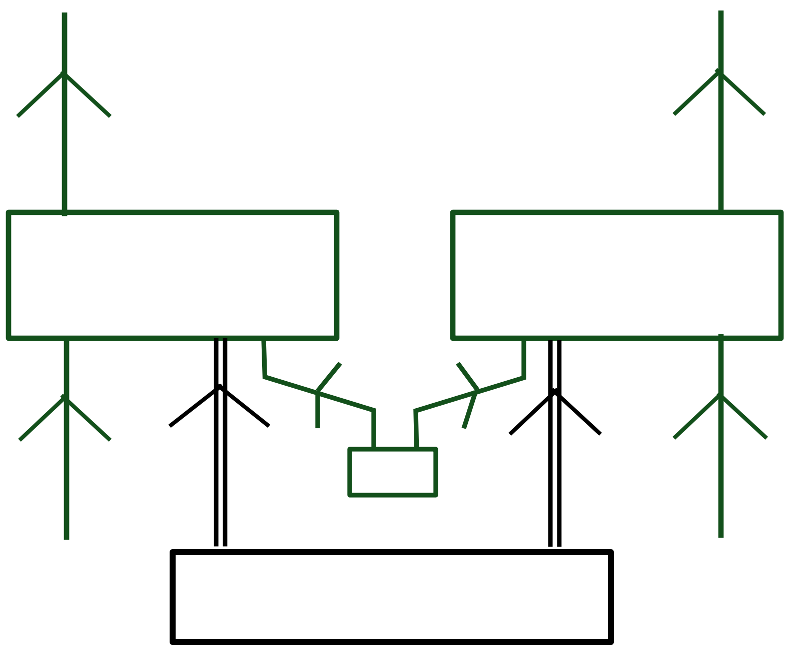


Box-to-Box conversions

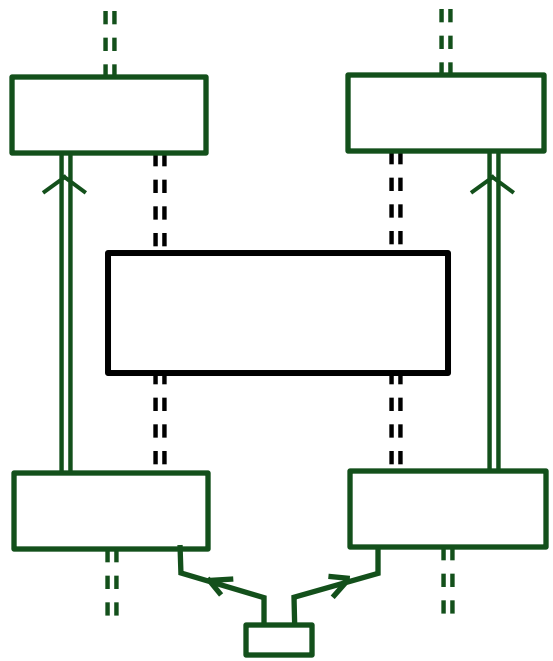


-arbitrary local pre-and-post processings
-correlated by shared randomness

State-to-Box transformations

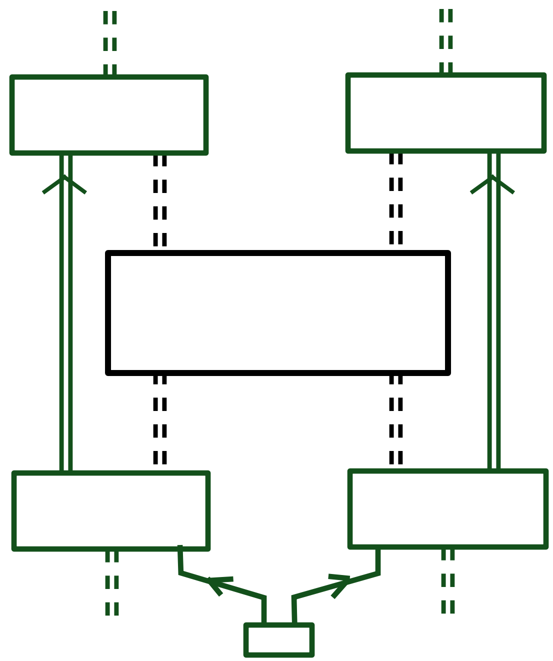


General procedure:



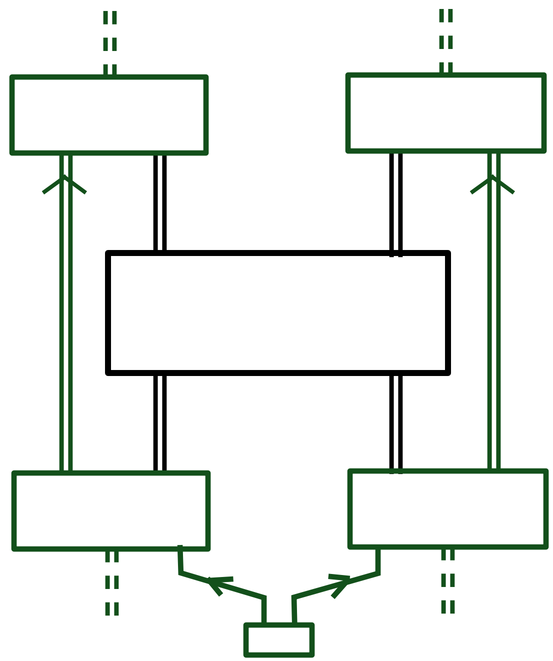
1. Draw this figure
2. Specialize system types

Ex: Channel-to-Assemblage transformations



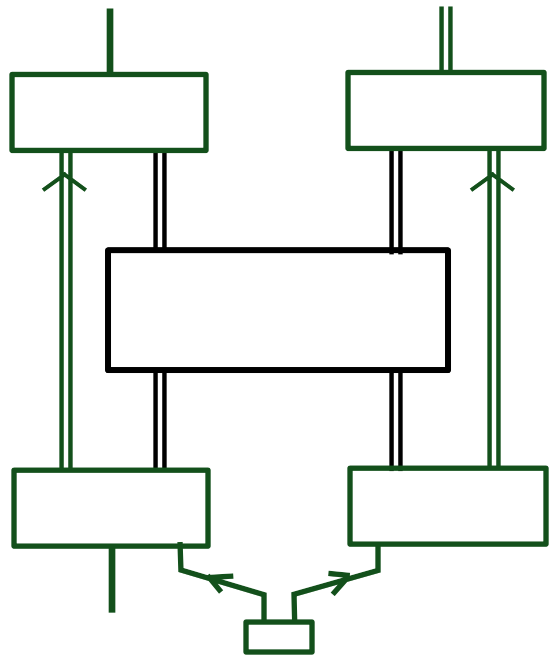
1. Draw this figure
2. Specialize system types

Channel-to-Assemblage transformations



1. Draw this figure
2. Specialize system types

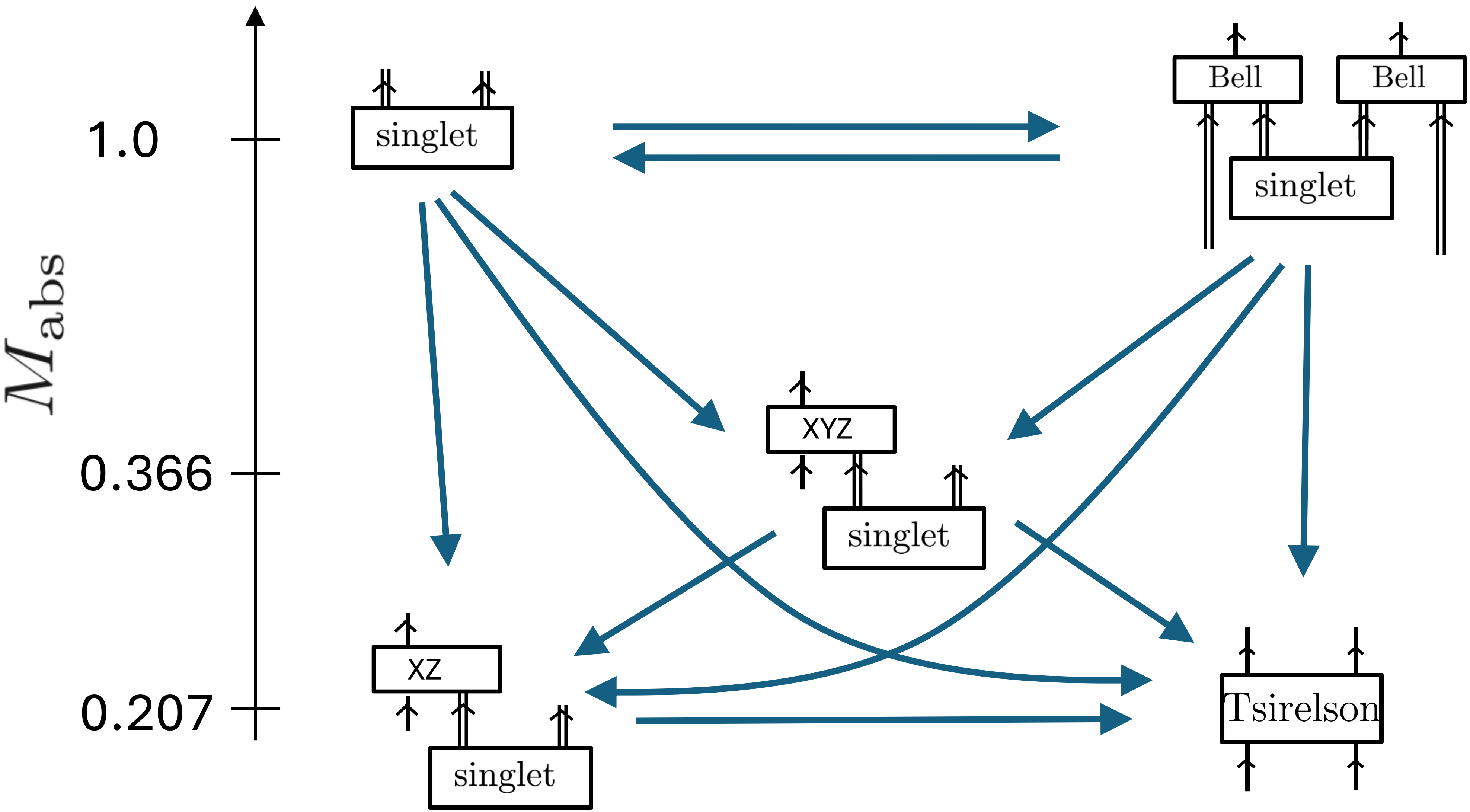
Channel-to-Assemblage transformations



1. Draw this figure
2. Specialize system types

Quantifying nonclassicality of common cause

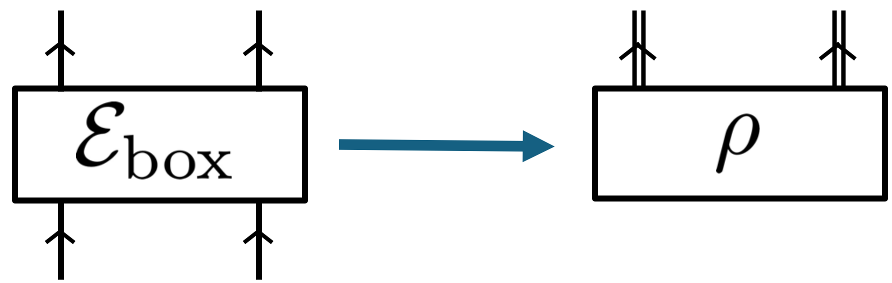
Preorder of resources (of arbitrary types!)



Transformations among types which **necessarily** destroy all nonclassicality

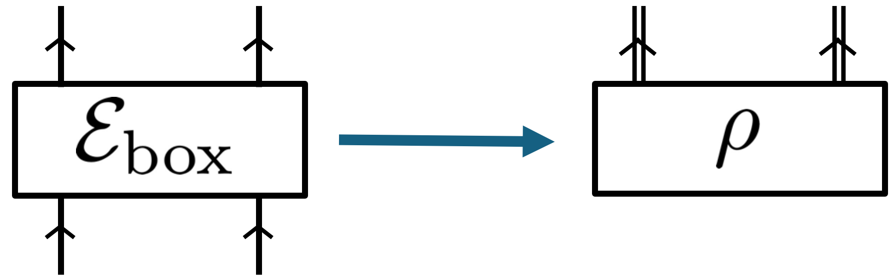
Nonclassicality degrading type changes

box to state

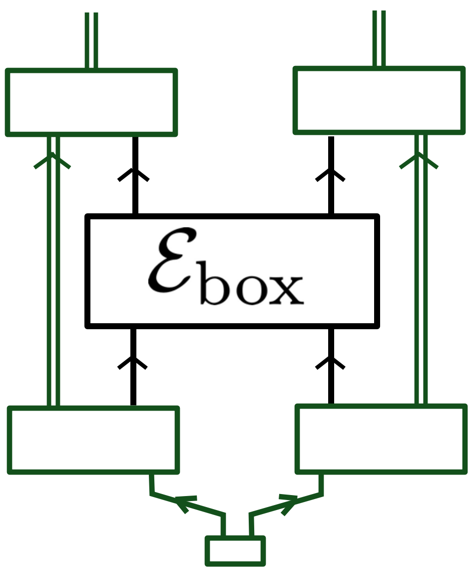


Nonclassicality degrading type changes

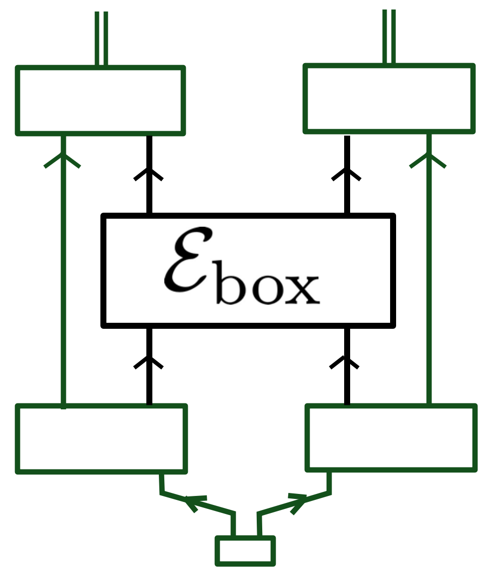
box to state



proof

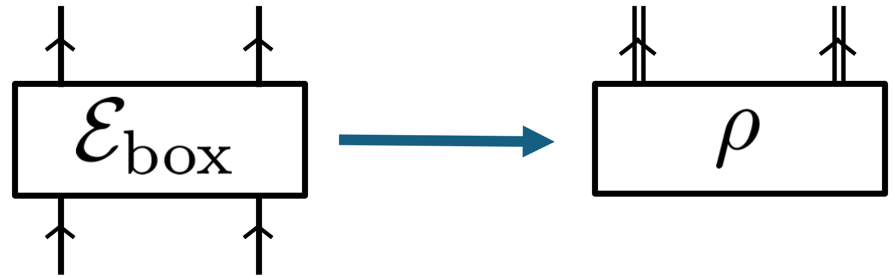


wlog:

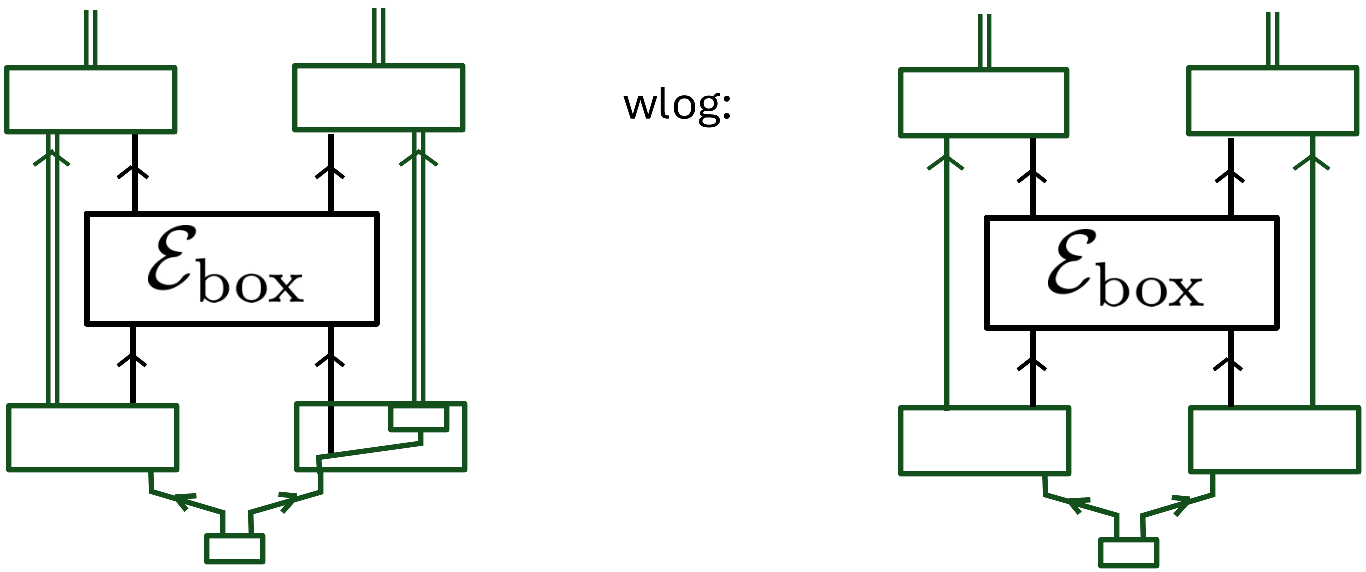


Nonclassicality degrading type changes

box to state

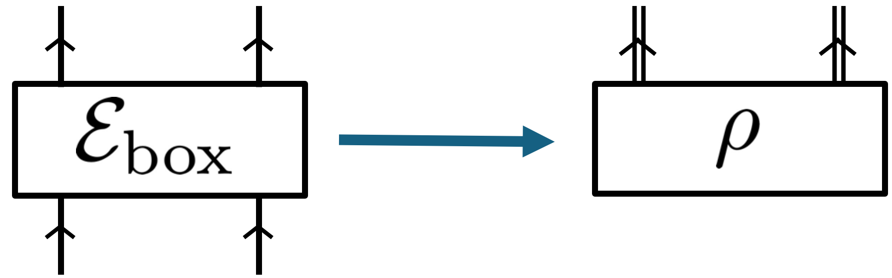


proof

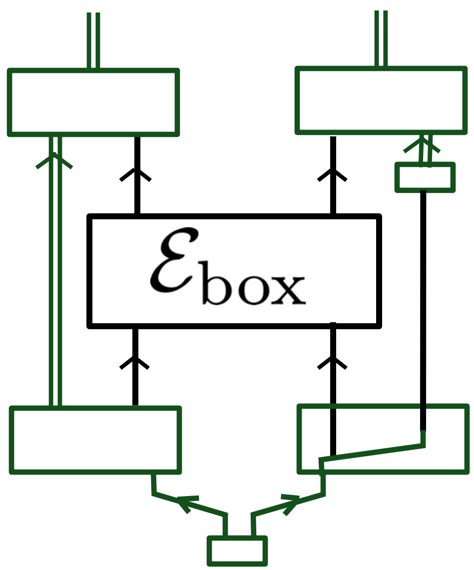


Nonclassicality degrading type changes

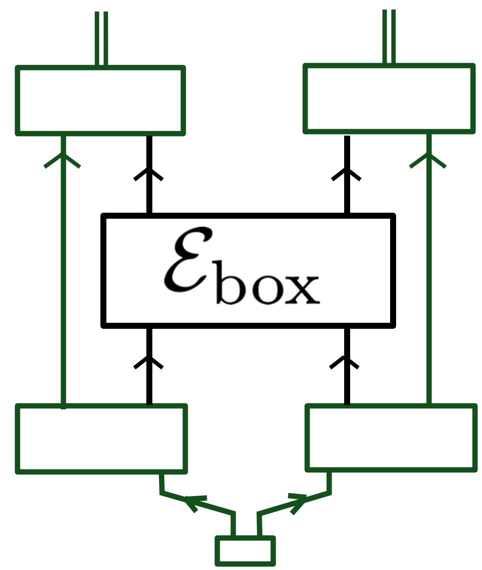
box to state



proof

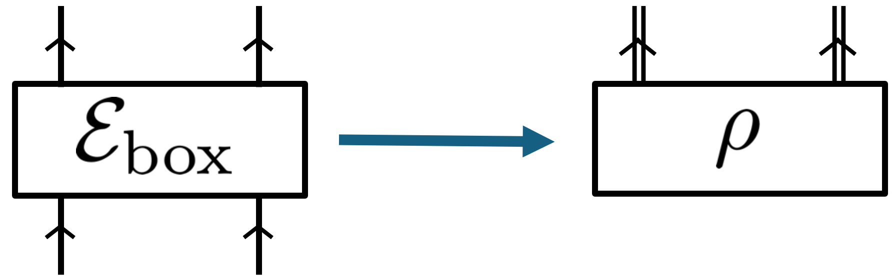


wlog:



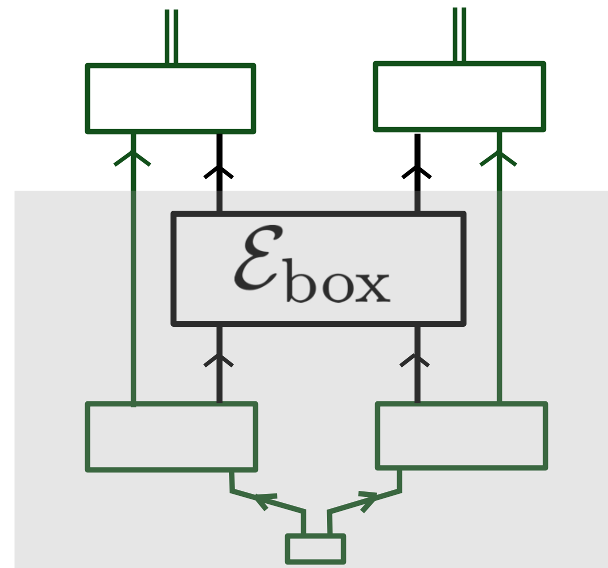
Nonclassicality degrading type changes

box to state



proof

wlog:

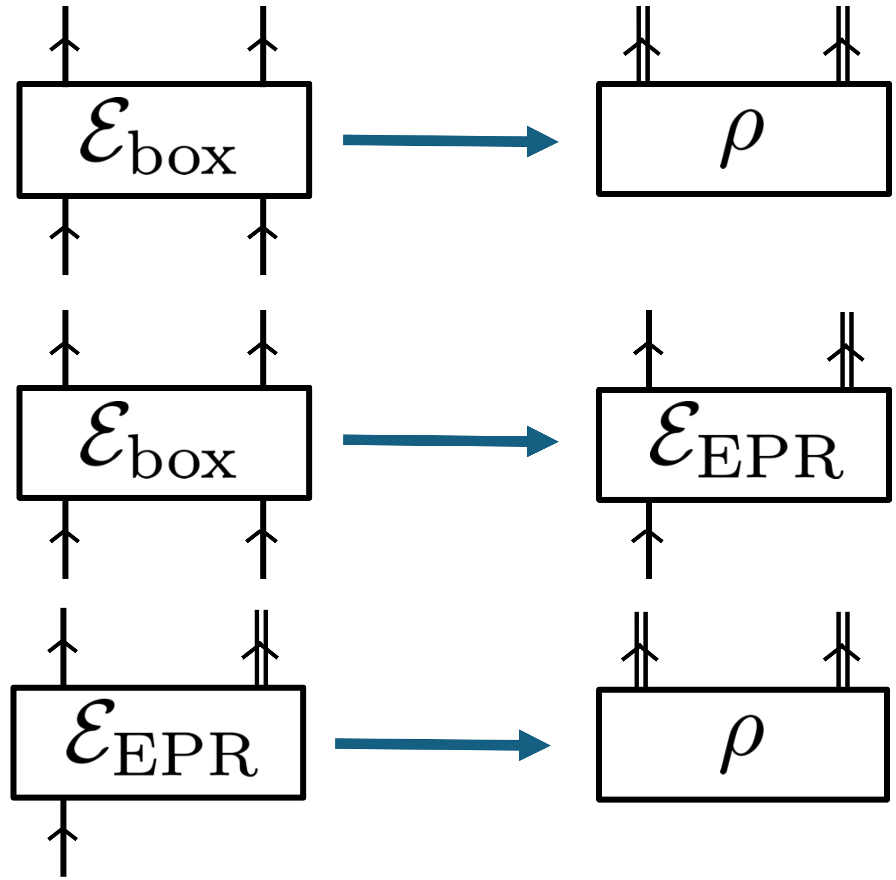


Nonclassicality degrading type changes

box to state

box to assemblage

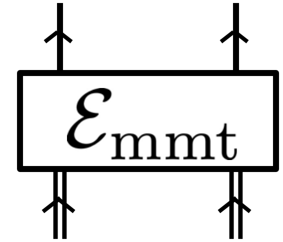
assemblage to state



Proofs are similar

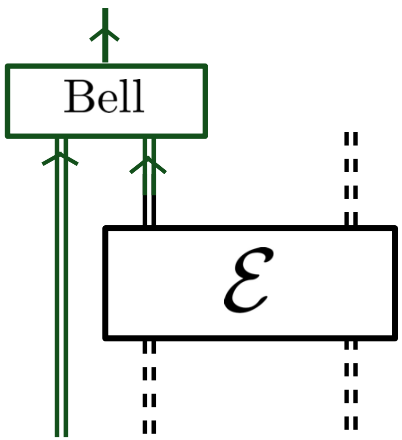
Transformations among types which **preserve** all nonclassicality

The `distributed measurement' type encodes the nonclassicality of all other types.

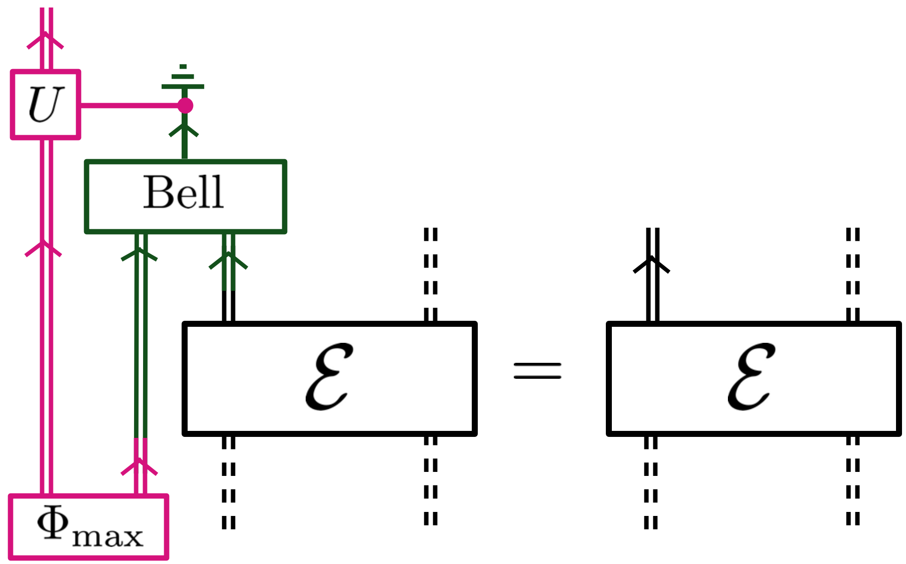


-*every* resource can be converted to one with classical outputs and quantum inputs, *without* degrading its LOSR nonclassicality

Proof:

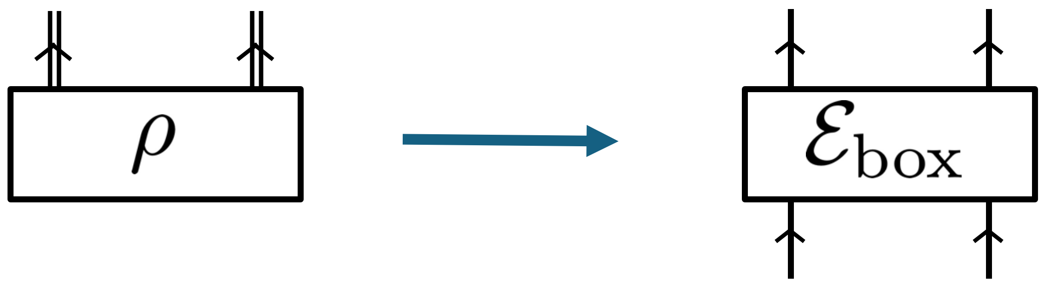


same equivalence class as $\mathcal{E}$



(same for all other quantum outputs)

Can all nonfree states be transformed into *some* box that is nonfree?



No! "Werner states cannot violate any Bell inequality"

LOSR-entanglement vs LOCC-entanglement

A pure state is entangled if it is not a tensor product of two components —Schrodinger

A mixed state is entangled if it is not separable (a mixture of product states)

Entanglement is a **resource** for quantum communication tasks (teleportation, quantum Shannon theory, etc)

To study entanglement as a resource for *nonclassical* communication, *Classical* Communication was considered free (as were Local Operations)

Over time, entanglement came to be understood as "the resource which cannot be generated by LOCC operations".

ρ_1 is at least as entangled as ρ_2 iff

$$\rho_1 \to \rho_2$$

using LOCC operations

ρ_1 is at least as entangled as ρ_2 iff

$$\rho_1 \to \rho_2$$

using LOSR operations

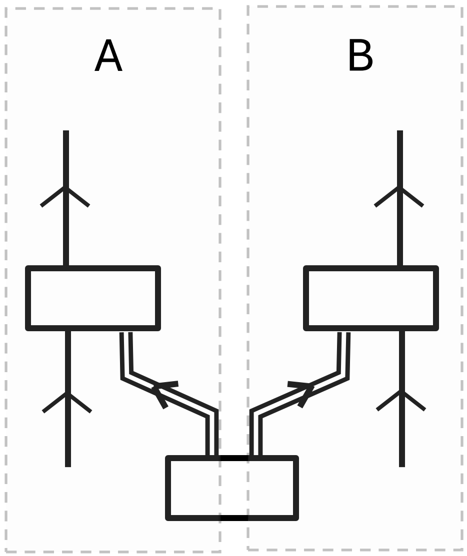
Quantitatively a very different notion of entanglement!

So, are the relevant free operations for studying entanglement LOCC or LOSR?

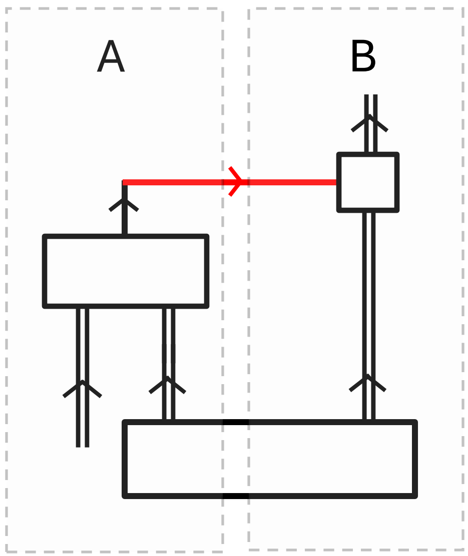
It depends on the situation!

common-cause scenario

cause-effect scenario



Bell scenario:
LOSR



Quantum communication:
LOCC

Resolving the Anomalies of Nonlocality

The “anomaly”:
sometimes, having *more* entanglement means one *cannot* generate as much nonlocality!

[20] A. A. Methot and V. Scarani, “An Anomaly of Nonlocality,” Quantum Info. Comput. **7**, 157 (2007).

[21] N. Brunner, N. Gisin, and V. Scarani, “Entanglement and non-locality are different resources,” New J. Phys. **7**, 88 (2005).

[22] N. Brunner, N. Gisin, S. Popescu, and V. Scarani, “Simulation of partial entanglement with nonsignaling resources,” Phys. Rev. A **78**, 052111 (2008).

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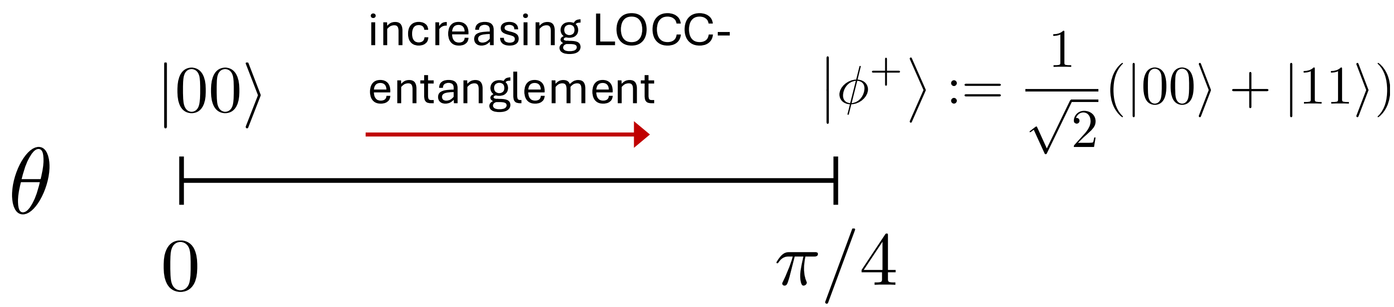
There exist measures of nonlocality which can be maximized by a partially entangled state, but *not* by a maximally entangled state.

Four instances of the anomaly

Consider the family of states given by $\cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$

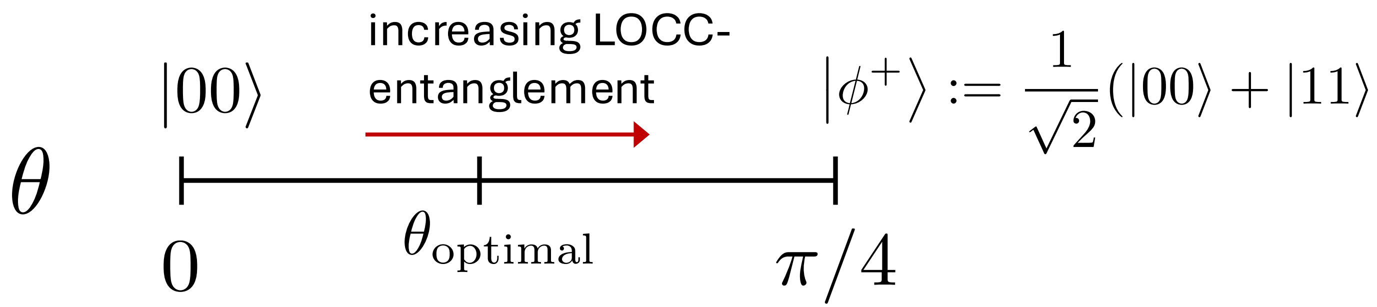


naively, having strictly more entanglement can't hurt for generating nonlocality

Four instances of the anomaly

Consider using these states to generate nonlocality, as measured by:

1. probability of running a Hardy proof of nonlocality
2. violation of a tilted Bell inequality
3. extractable secret key rate
4. relative entropy distance from the local set



The optimum is different in each case

(a resource-theoretic spin on the anomaly)

Entanglement theory says

LOCC $|\phi^+\rangle \to |\psi_{\text{Hardy}}\rangle$

By definition

LO $|\psi_{\text{Hardy}}\rangle \to B_{\text{Hardy}}$

But

LO $|\phi^+\rangle \not\to B_{\text{Hardy}}$

Apparent inconsistency

But we have argued that one must take all three of these relative to LOSR

Under LOSR operations

$$|\phi^+\rangle \not\to |\psi_{\text{Hardy}}\rangle$$

Under LOSR operations

$$|\psi_{\text{Hardy}}\rangle \to B_{\text{Hardy}}$$

Under LOSR operations

$$|\phi^+\rangle \not\to B_{\text{Hardy}}$$

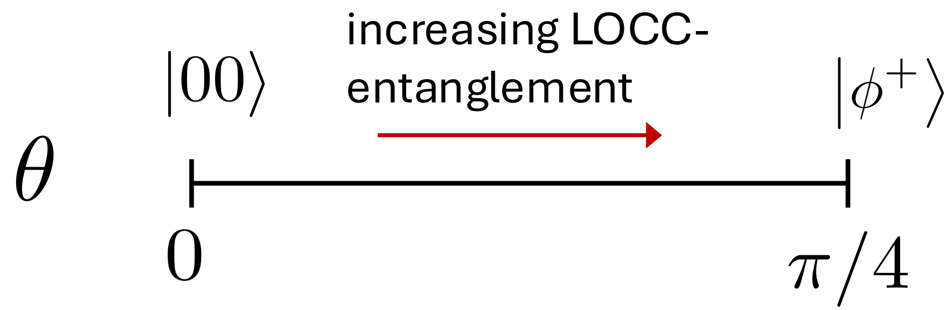
Consistent!

Relative to LOSR

$|\phi^+\rangle$ incomparable to $|\psi_{\mathrm{Hardy}}\rangle$

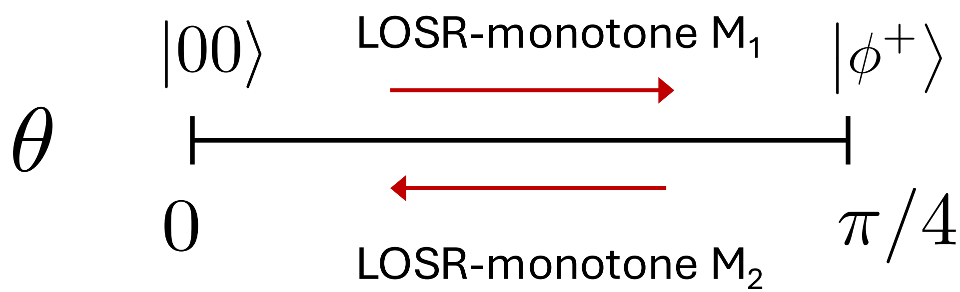
Hence the terms "maximally entangled" and "partially entangled" are *not appropriate* for LOSR-entanglement

Consider again the family of states given by $\cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$

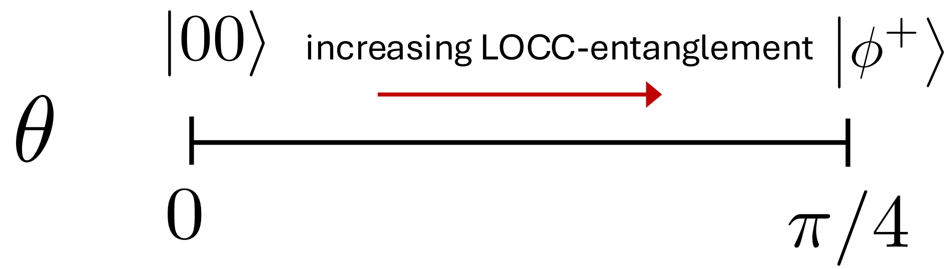


ALL of these states are LOSR-incomparable!
So there is no *single* measure of LOSR-entanglement.

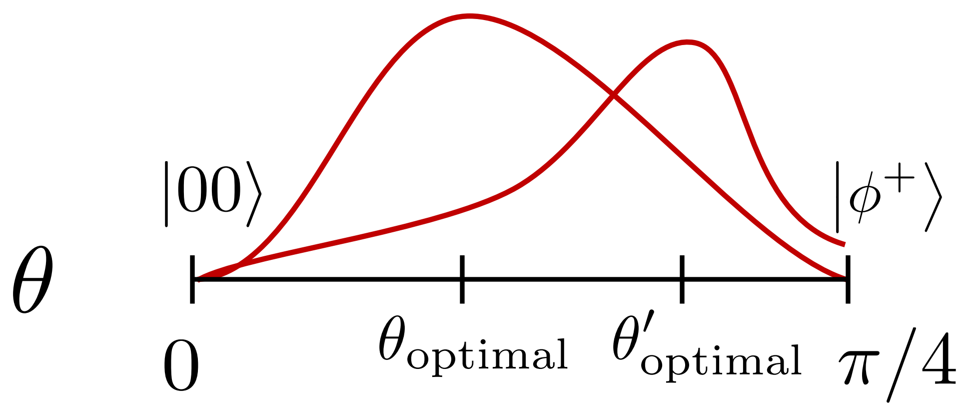
e.g.



Consider again the family of states given by $\cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$



For each anomaly, the associated task (generating a Hardy paradox, generating a secret key, etc) has its own optimal state, and defines a monotone which is peaked at that state!



Standard conclusion from the anomalies:
Nonlocality and entanglement are “different resources”

Better conclusion: Nonlocality and LOSR-entanglement *are* manifestations of the same resource (nonclassicality of common cause)

More nonclassicality is always better if you measure it correctly

Suggested references:

LOSR entanglement and nonlocality
arXiv:2004.09194

LOSR resources of all types
arXiv:1909.04065

Resource theory of nonlocality
arXiv:1903.06311

PIRSA:
https://pirsa.org/20040095
https://pirsa.org/19110120

Feedback encouraged!
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