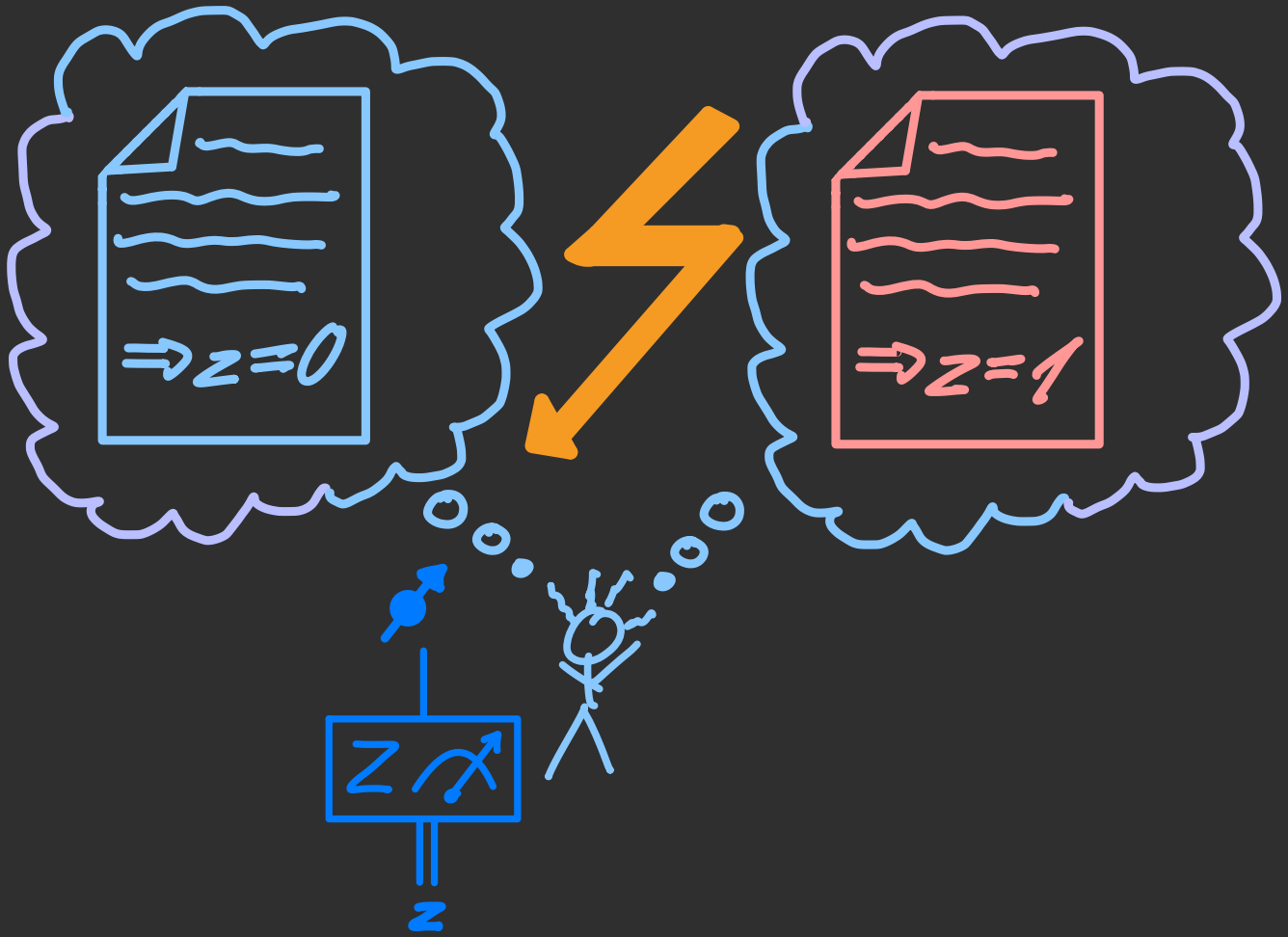
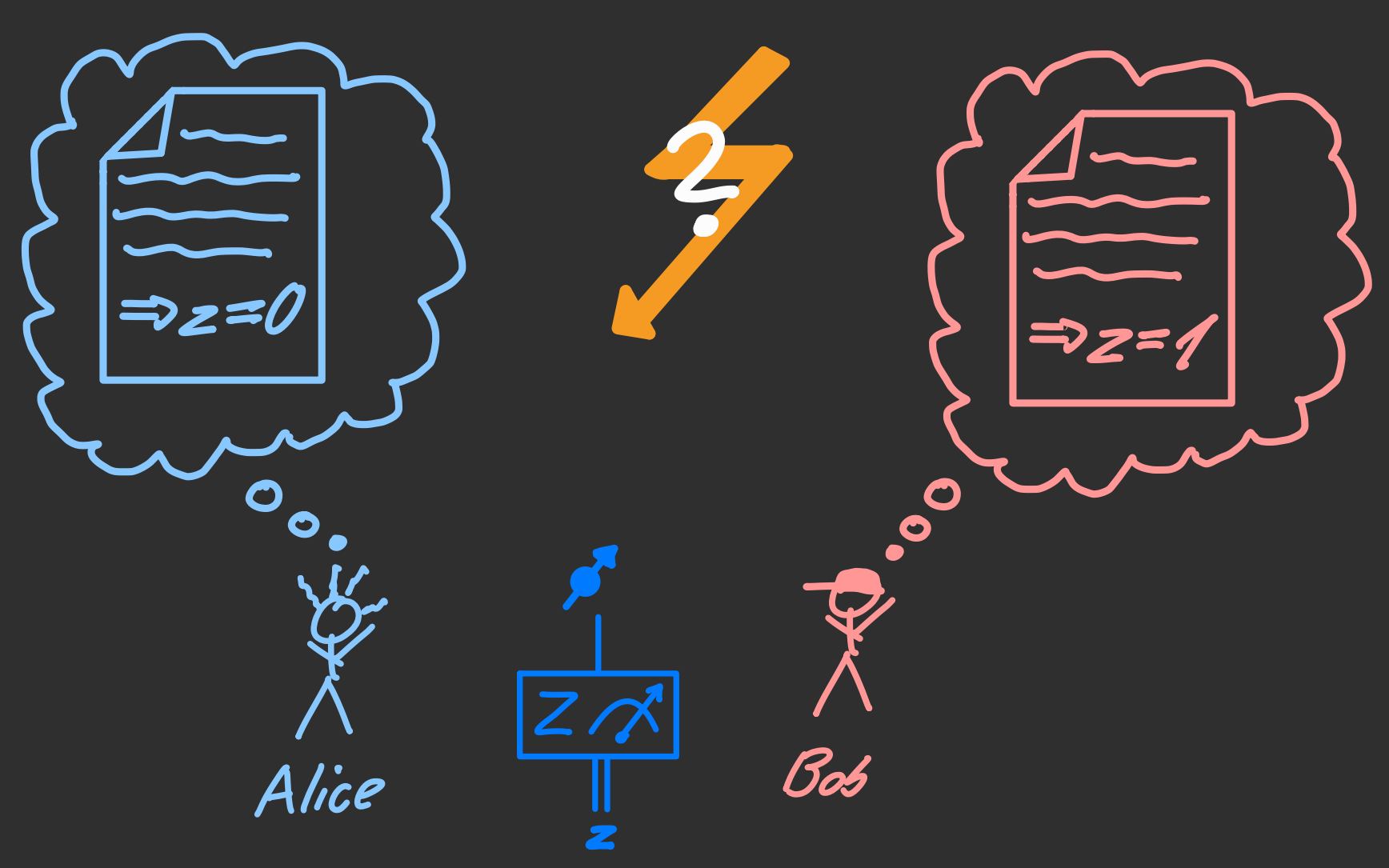
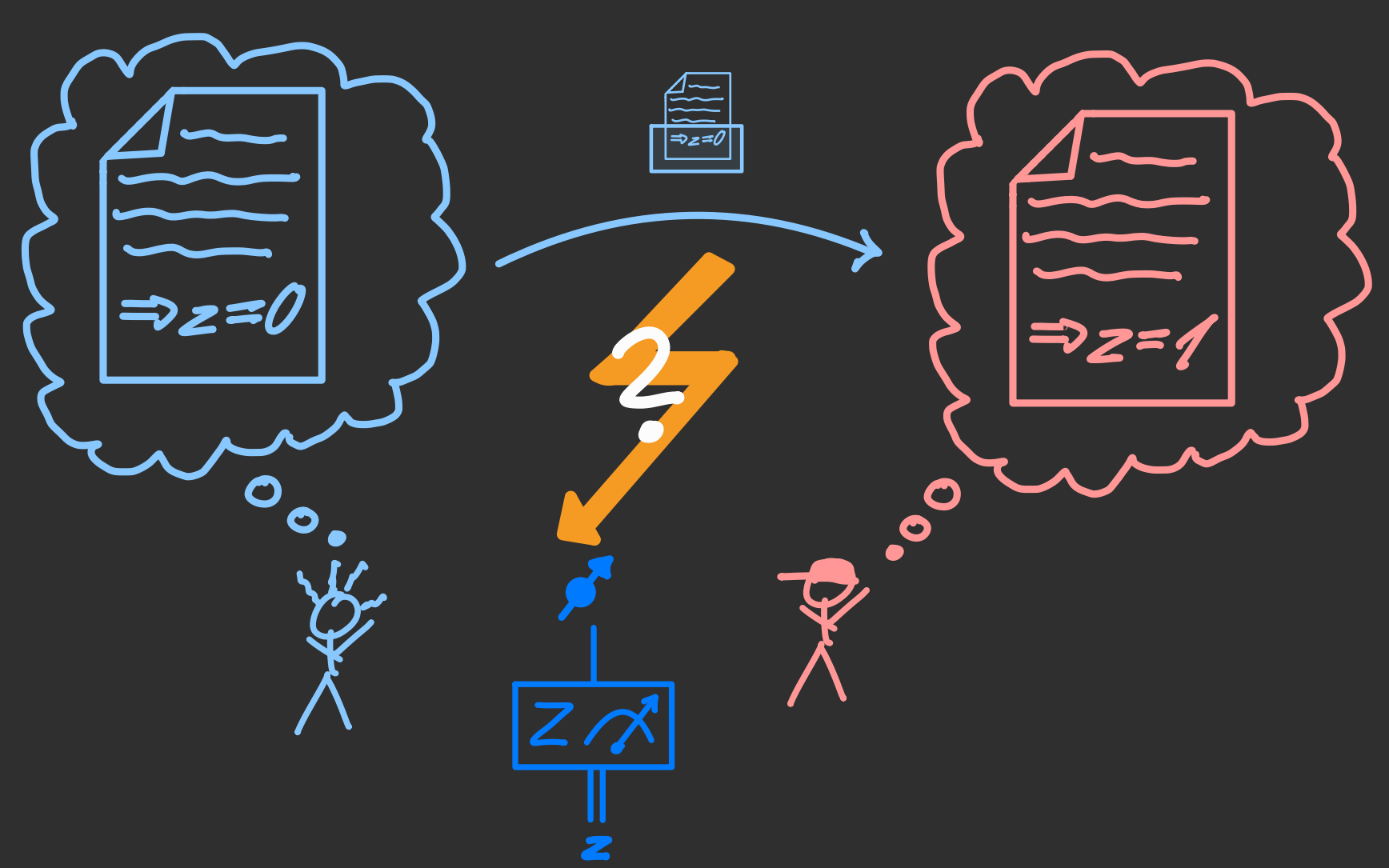
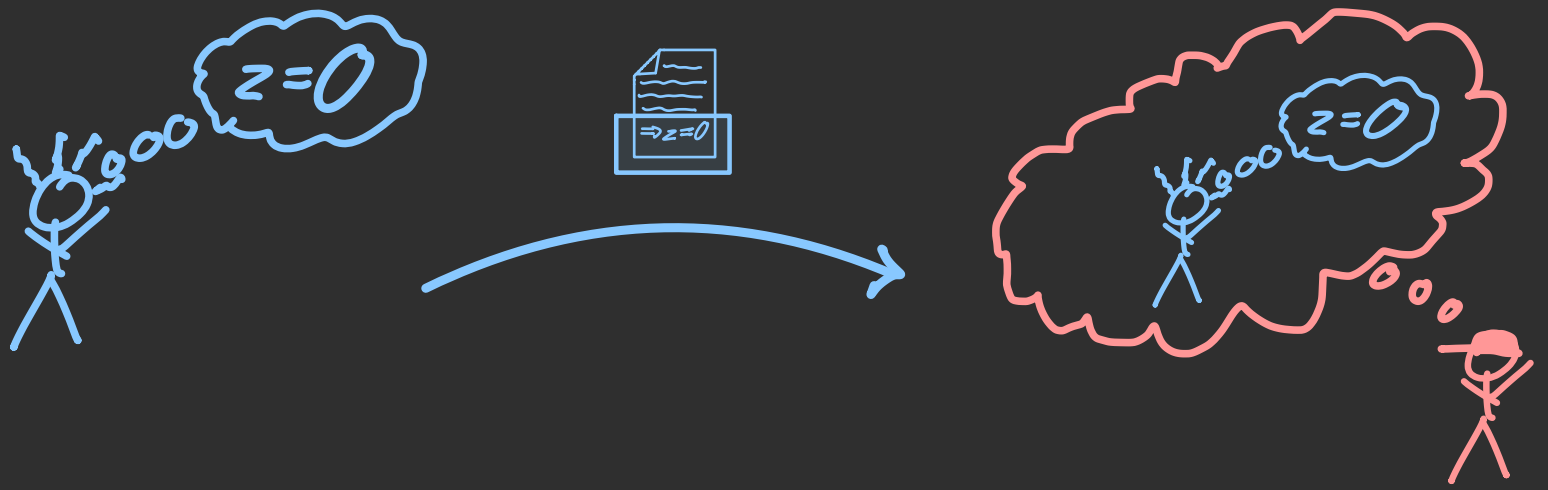
What is a paradox?


⇒z=0
⇒z=1
z
z


⇒z=0
?
⇒z=1
Alice
Z
z
Bob


⇒z=0
⇒z=0
⇒z=1
?
Z
z

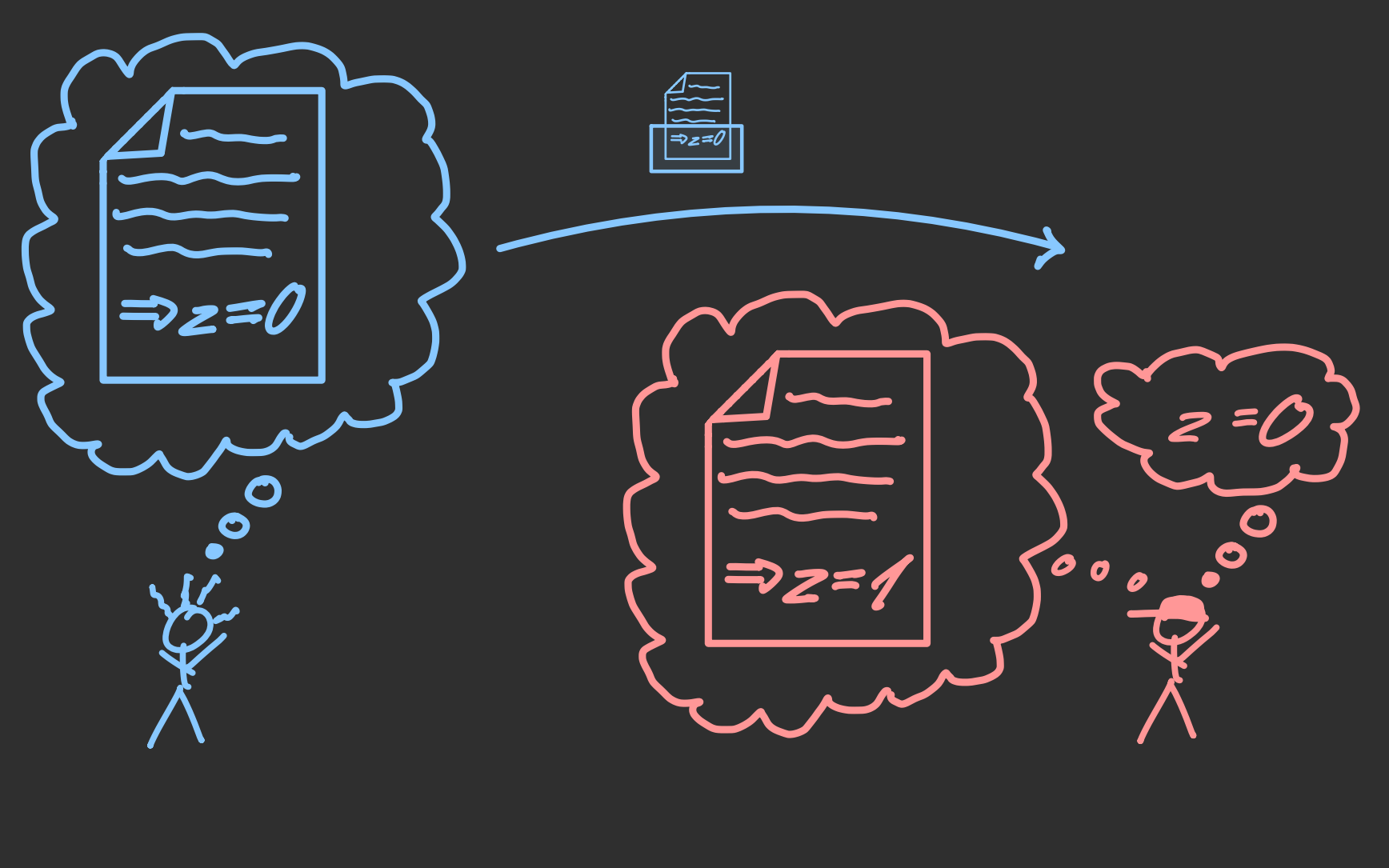
Communication

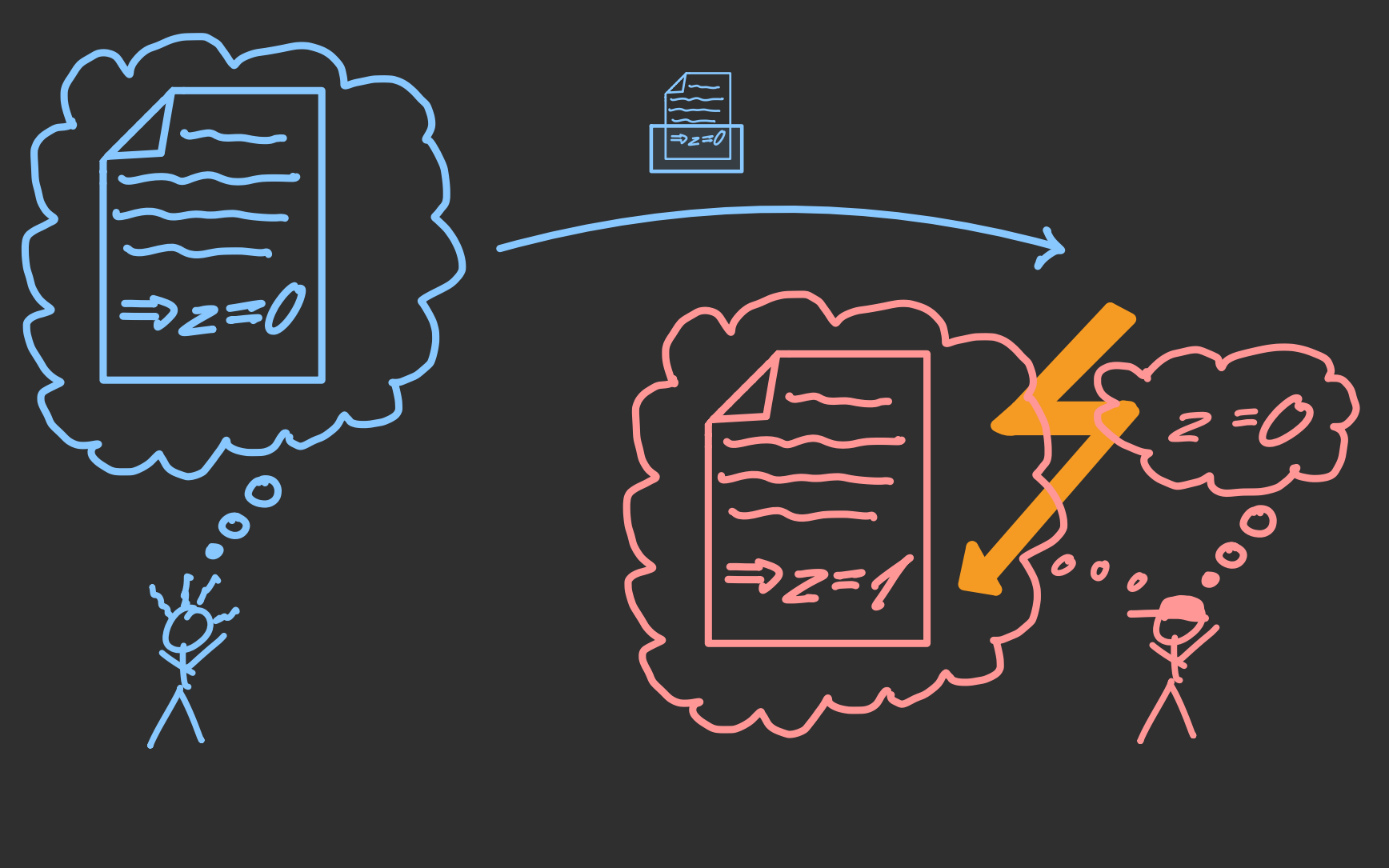

z=0
⇒z=0
z=0


z=0
⇒z=0
z=0
z=0

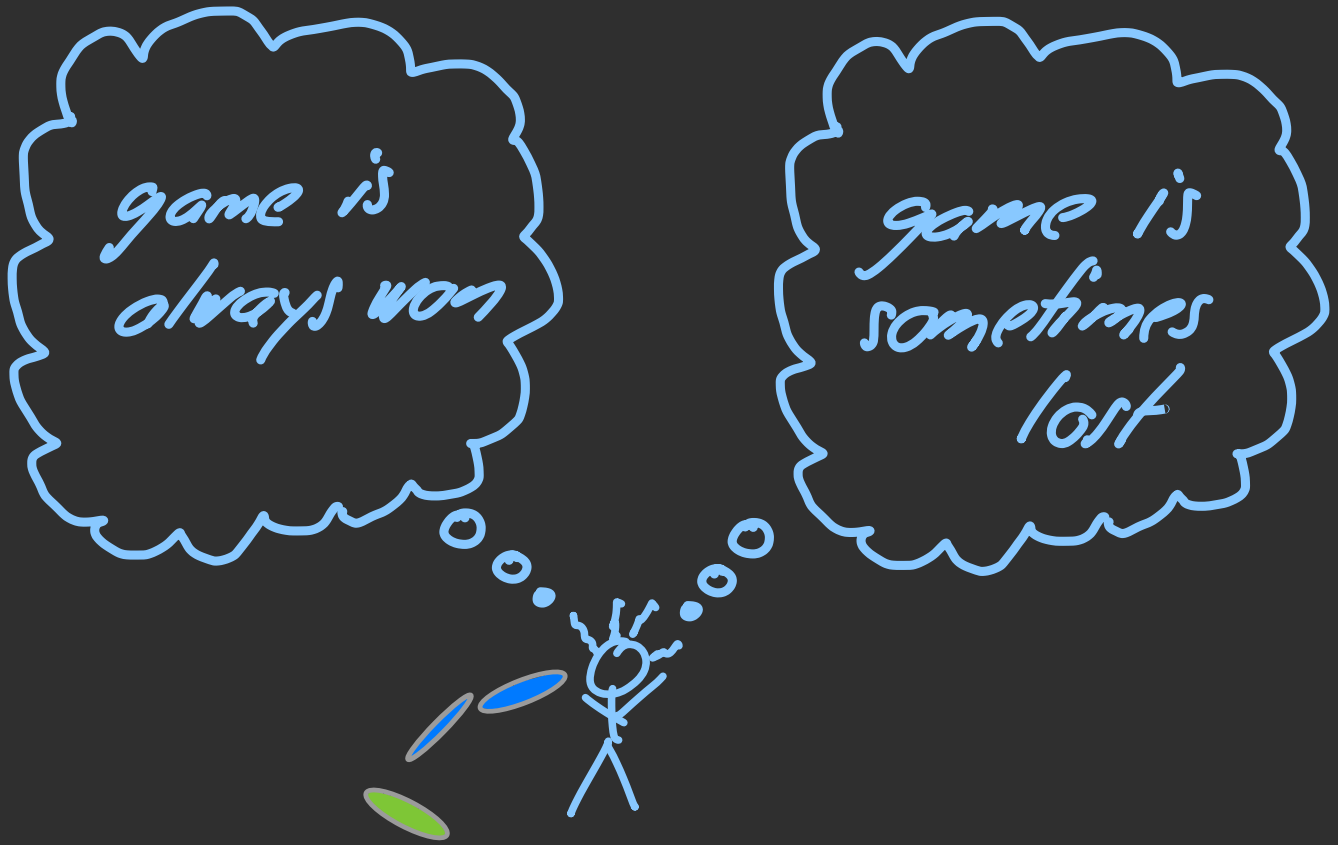
Consistency (C)

Suppose Alice has identical initial knowledge about the experiment and uses the same reasoning rules as me.


⇒z=0
⇒z=0
⇒z=1
z=0


⇒z=0
⇒z=0
⇒z=1
z=0

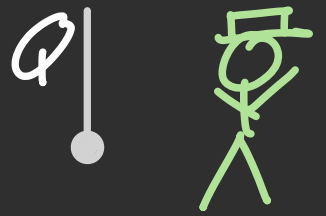
A relerant example


game is always won
game is sometimes lost

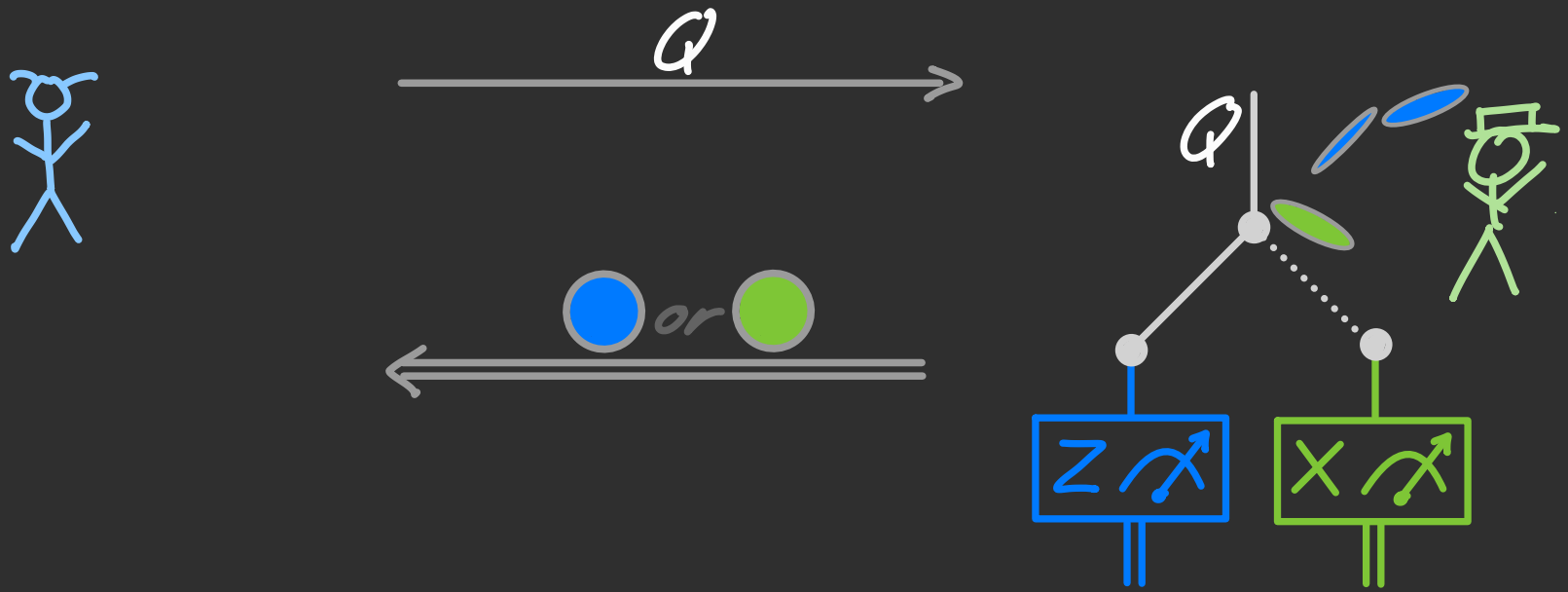
Game 1

Player

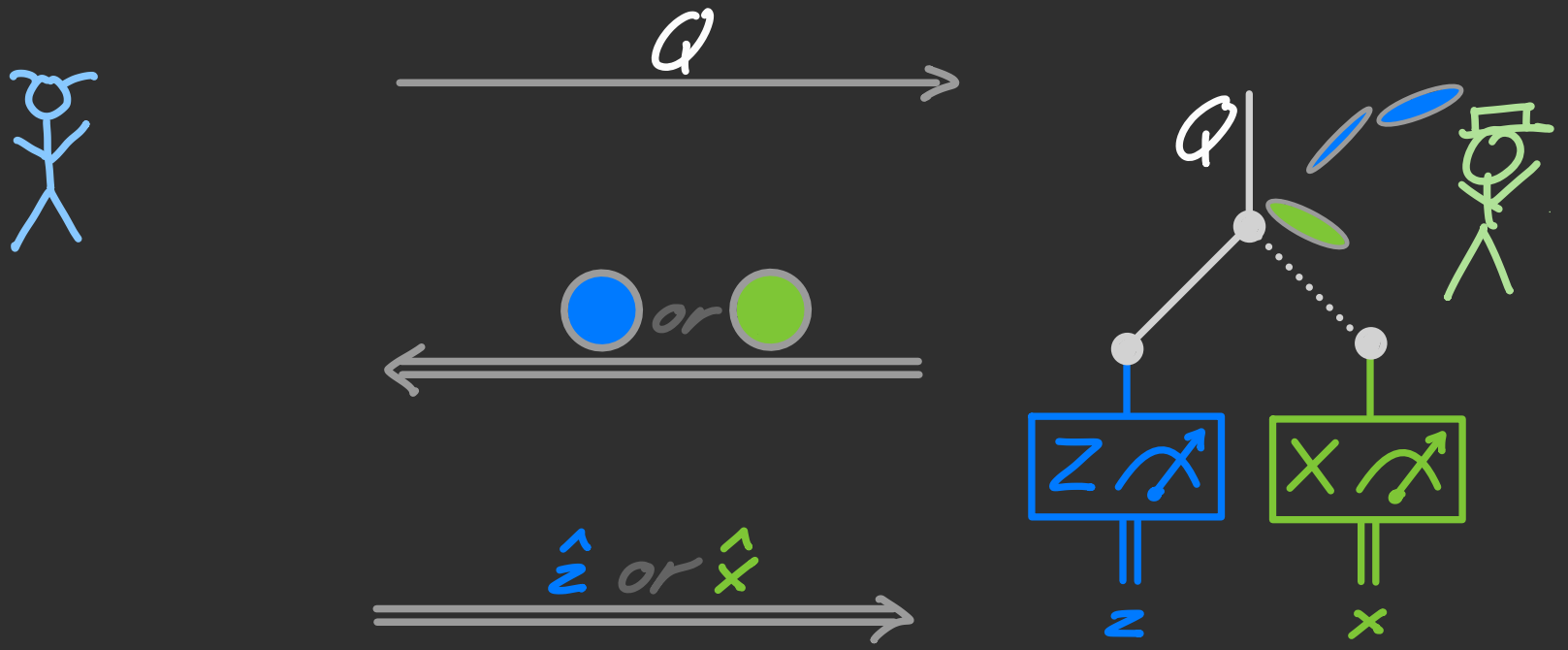
Referee



Game 1

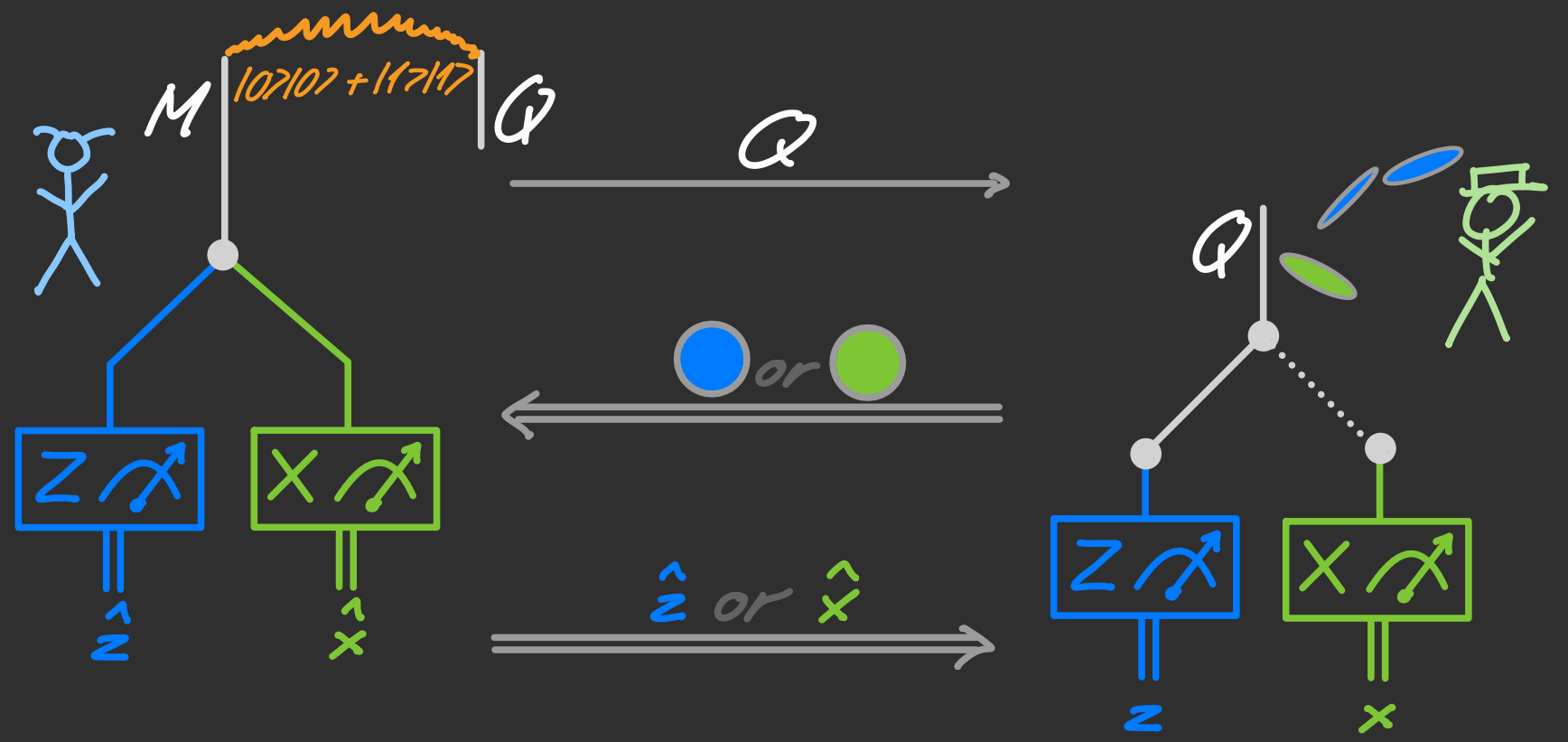


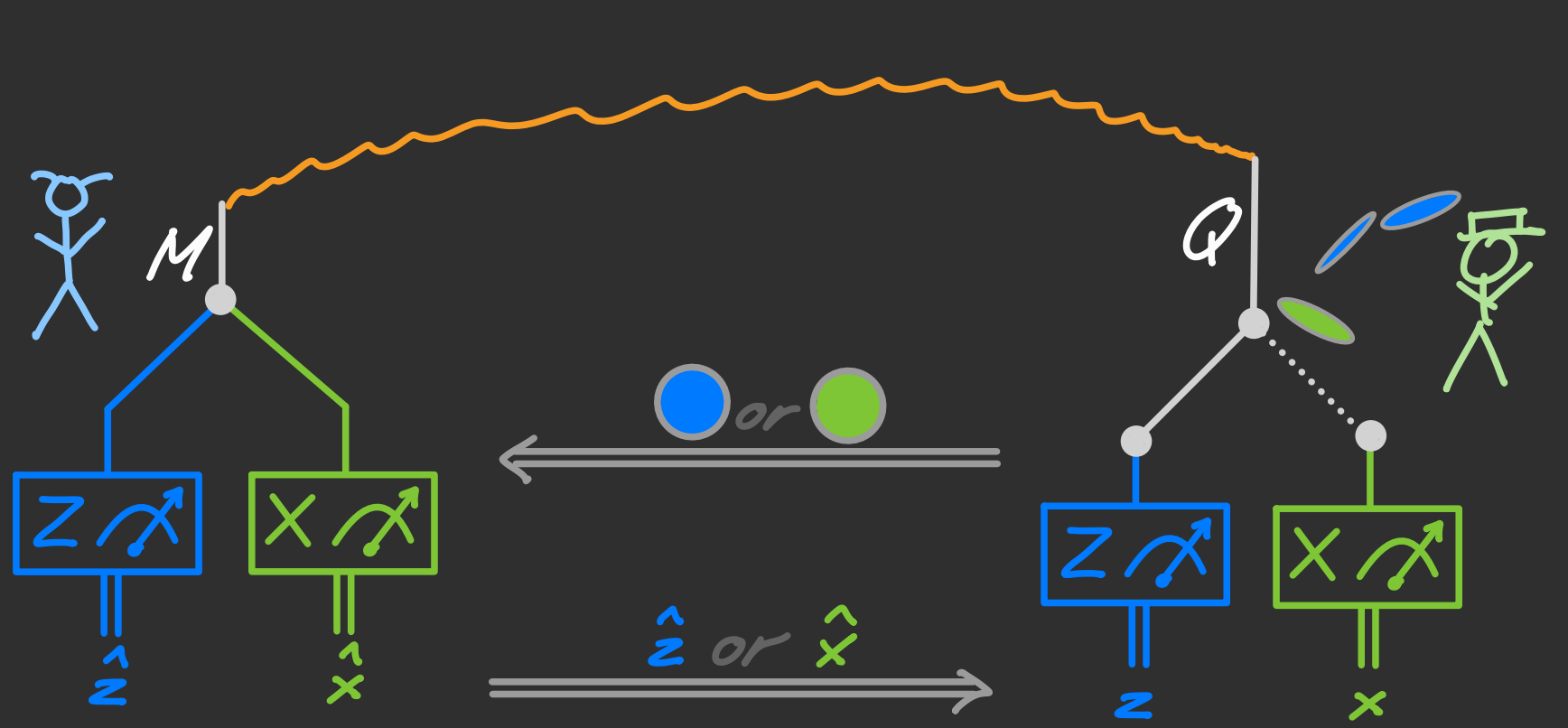
Game 1



check for $\hat{z}=z$ or $\hat{x}=x$

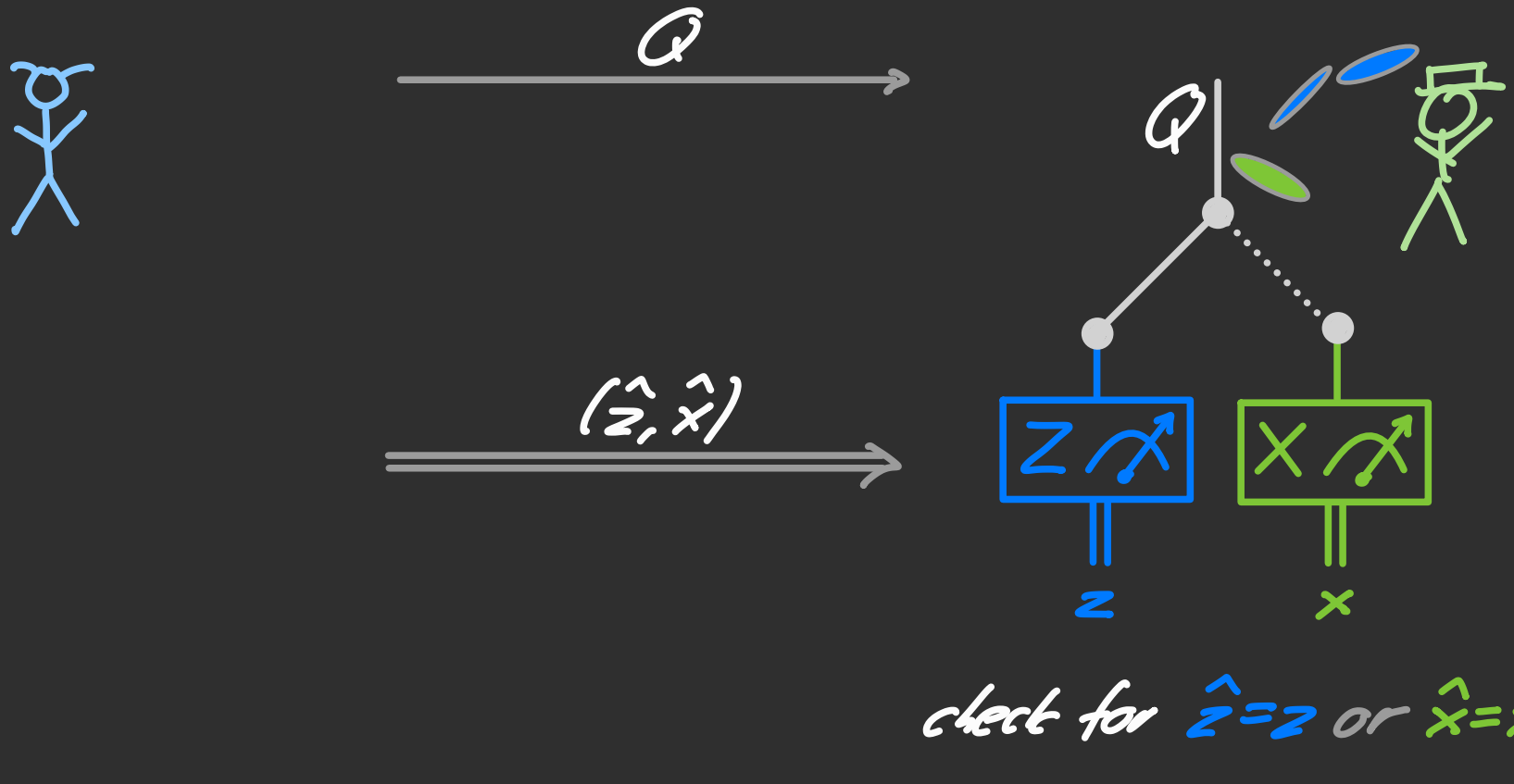
Game 1: A winning strategy

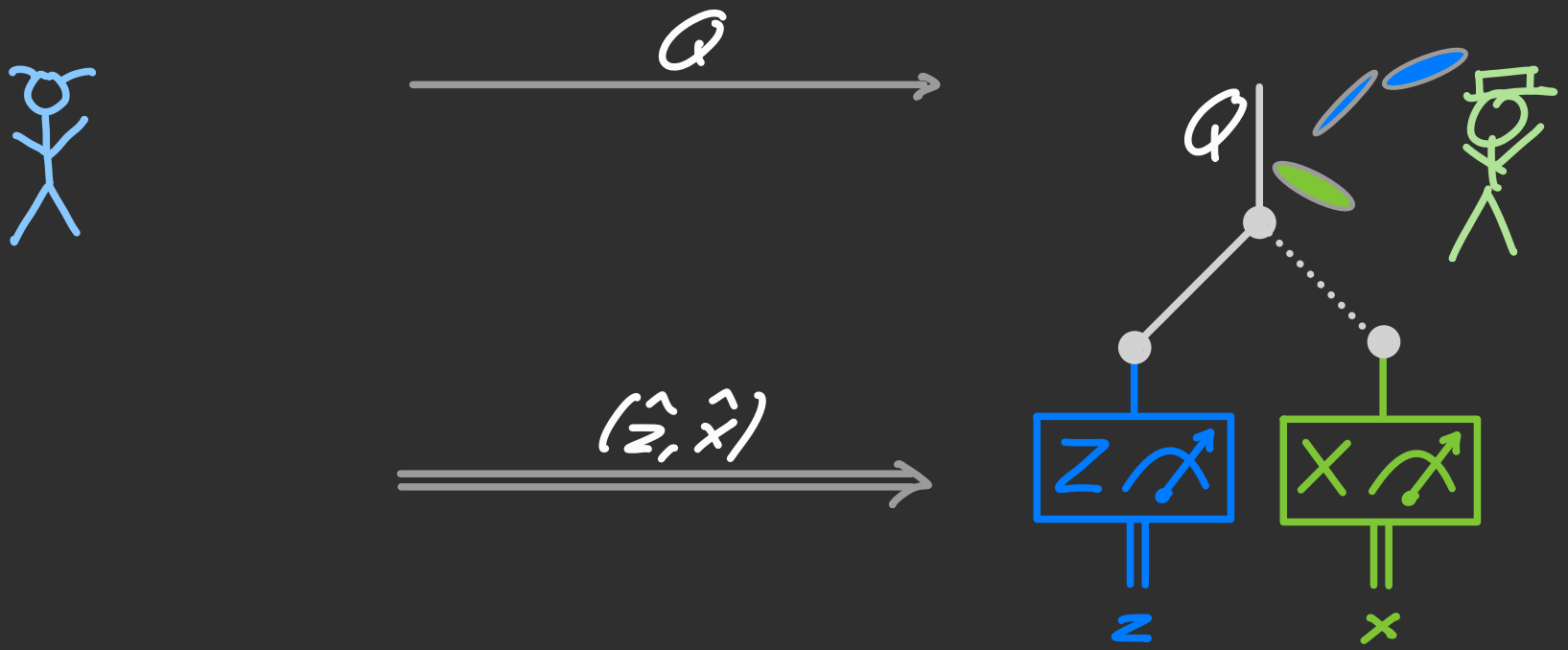




$$Pr[win] = Pr[\hat{z}=z \text{ or } \hat{x}=x] = 1$$

Game 2: Complementarity Game

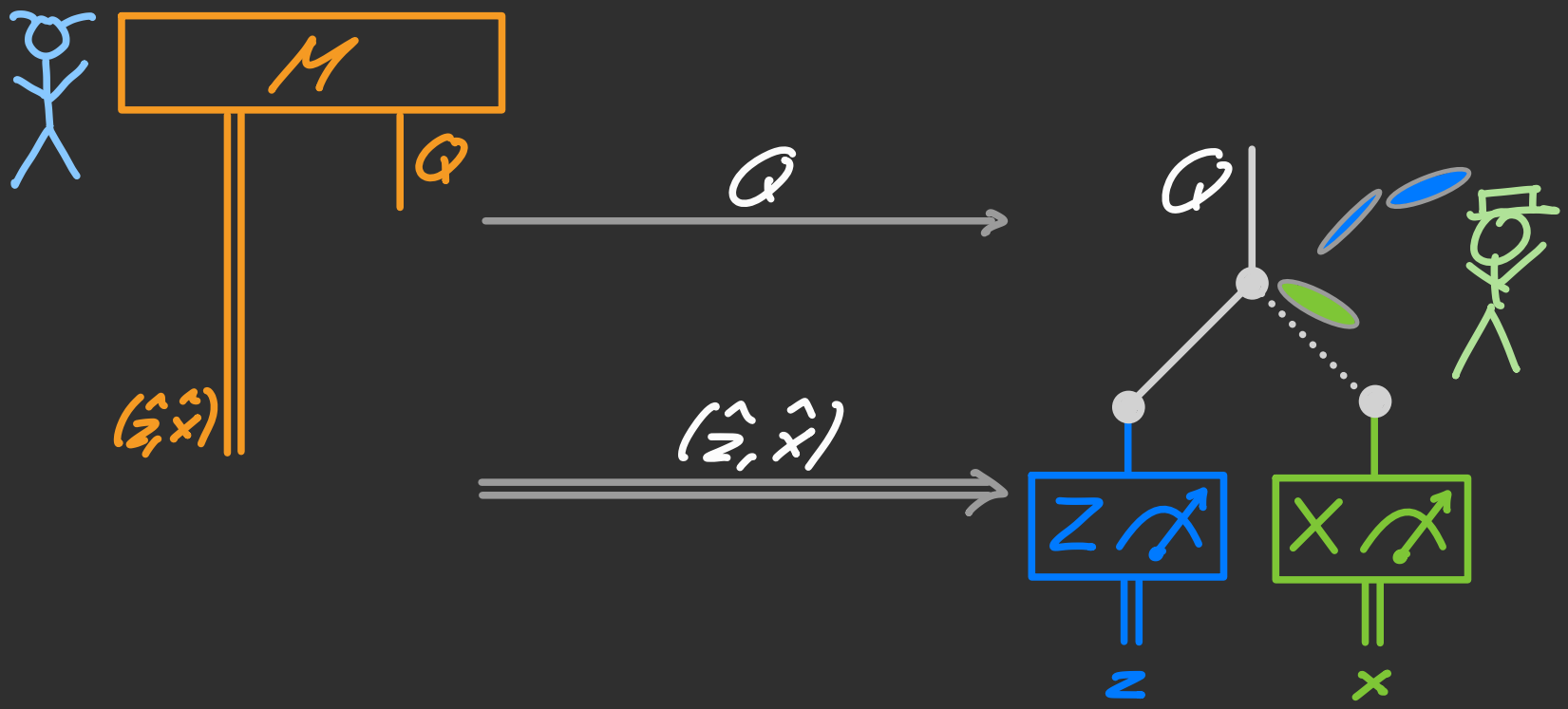




$P_r[win] \leq \mu < 1$

$\mu = \frac{1}{2} + \sqrt{\frac{1}{8}}$
[Reichardt, 2017]

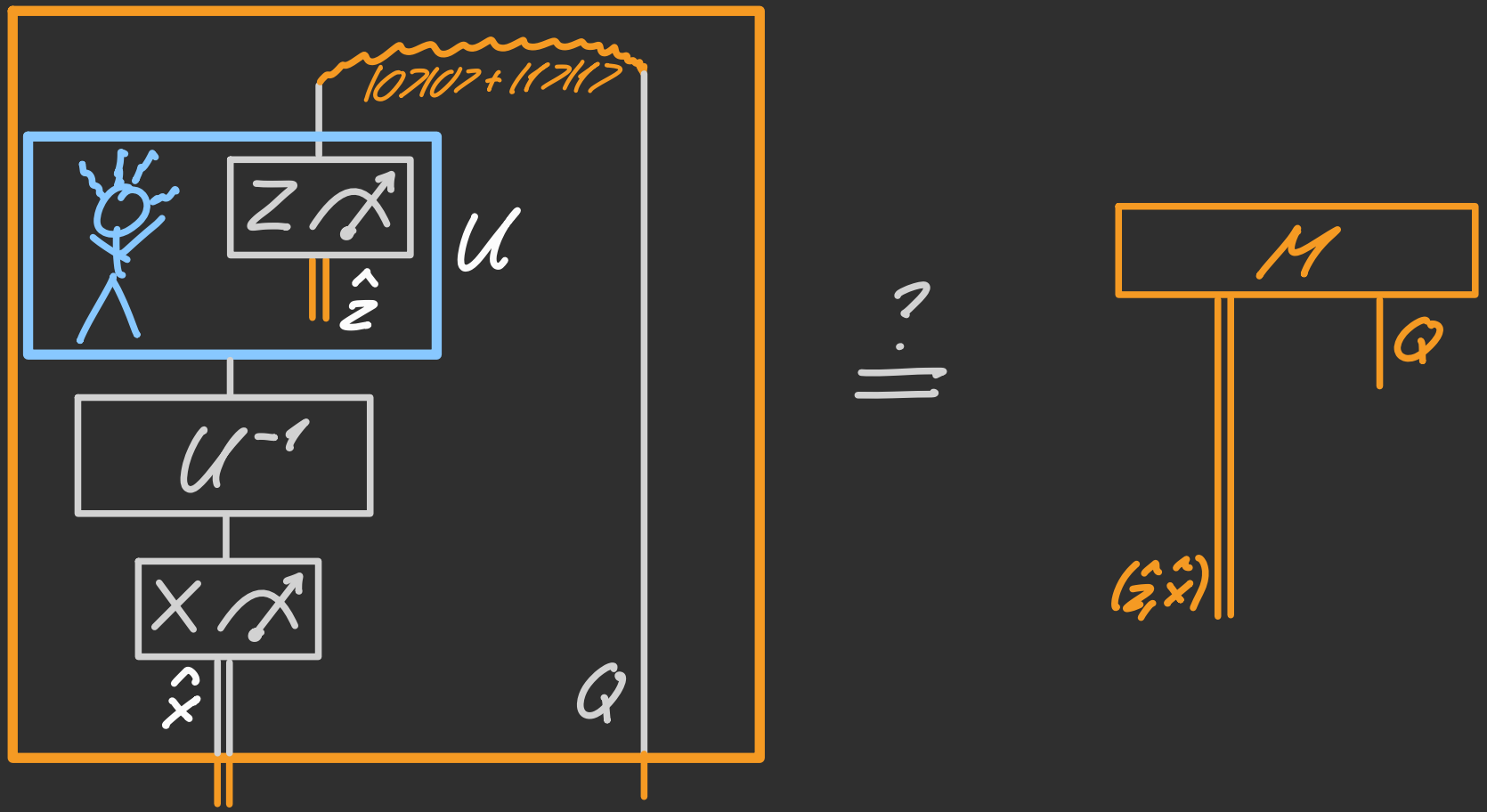
Magic Machine

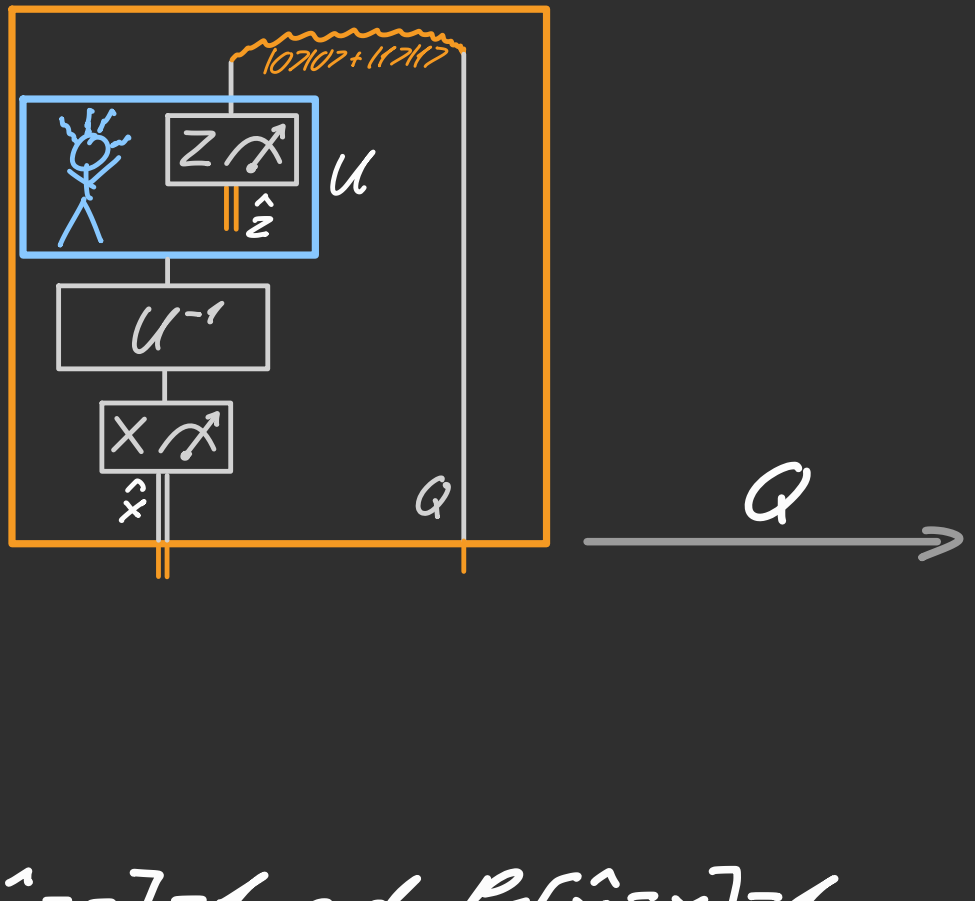


Defining condition: $Pr[\hat{z}=z \text{ or } \hat{x}=x] = 1$

Building a Magic Machine without gravity:

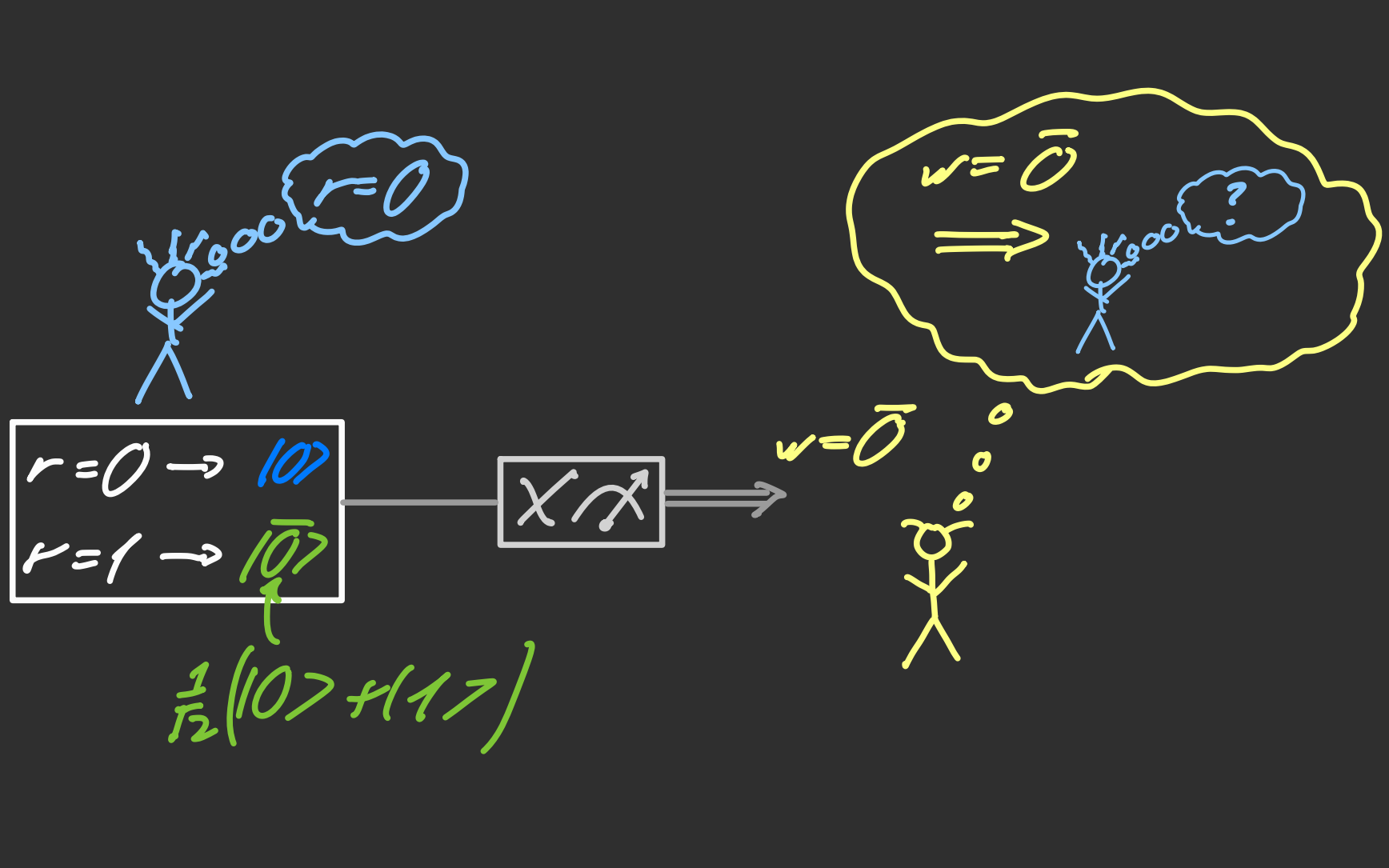
Attempt I

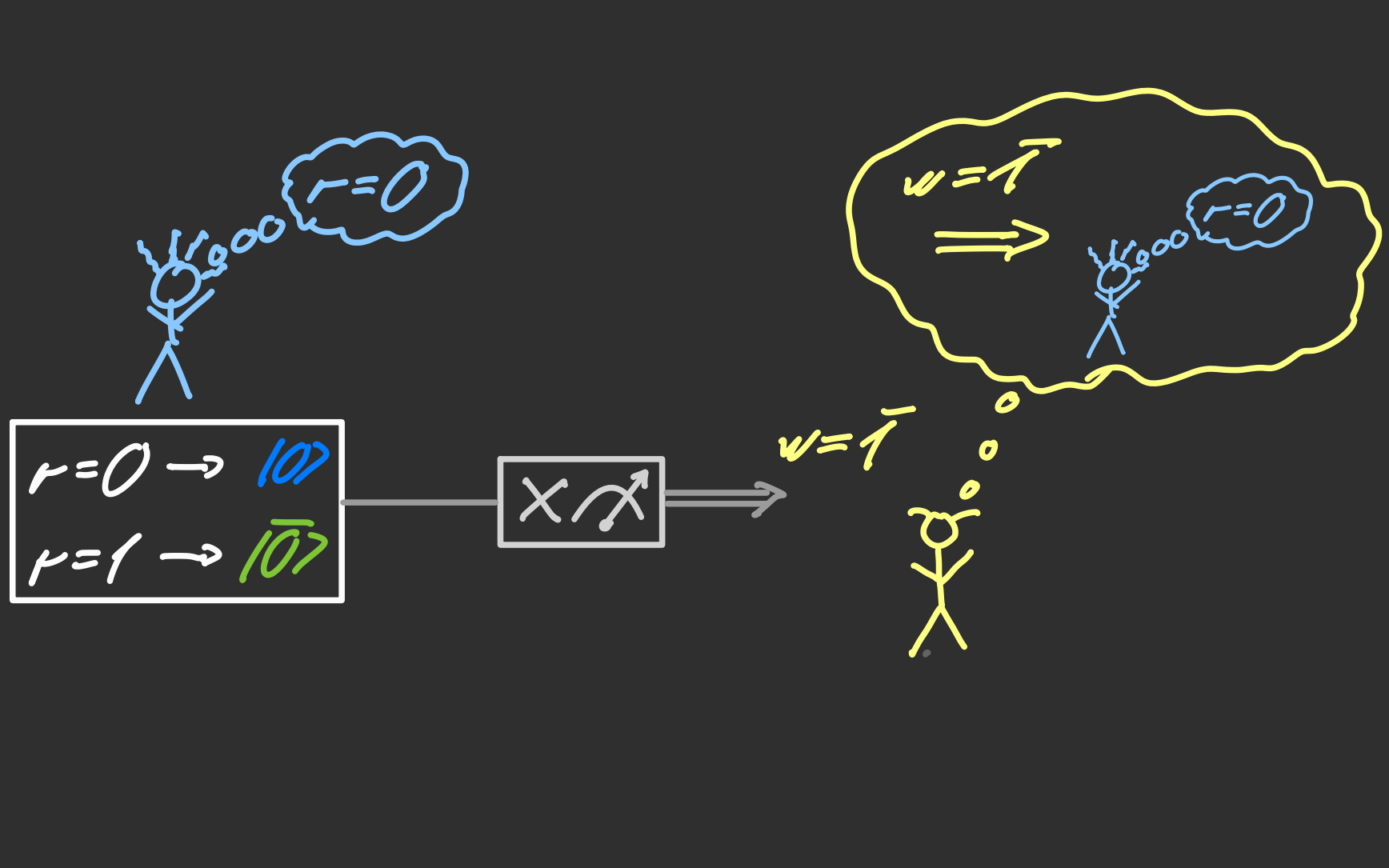

|0⟩⟨0| + |1⟩⟨1|
Z
U
$\hat{z}$
U^{-1}
X
$\hat{x}$
Q
?
=
M
Q
$(\hat{z}, \hat{x})$

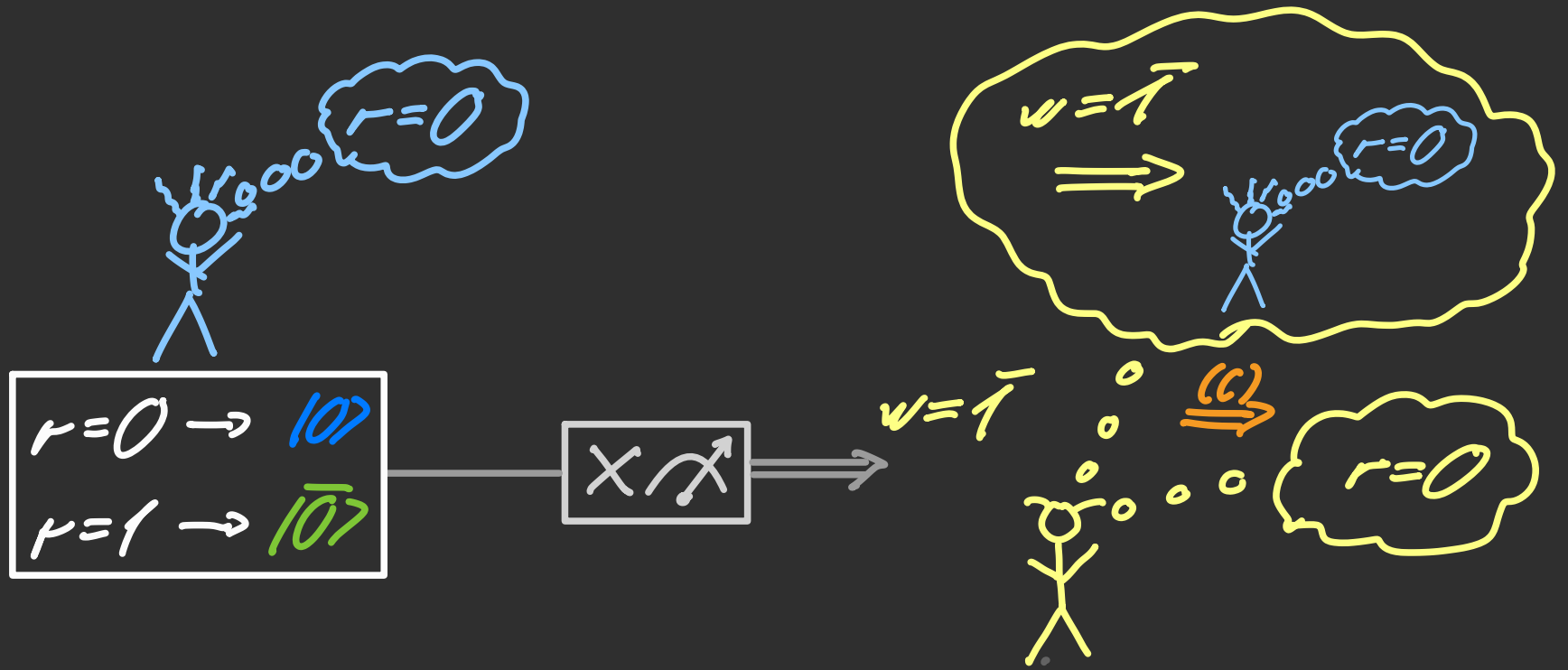


$Pr[\hat{z}=z]=1$ and $Pr[\hat{x}=x]=1$


Quantum whispering

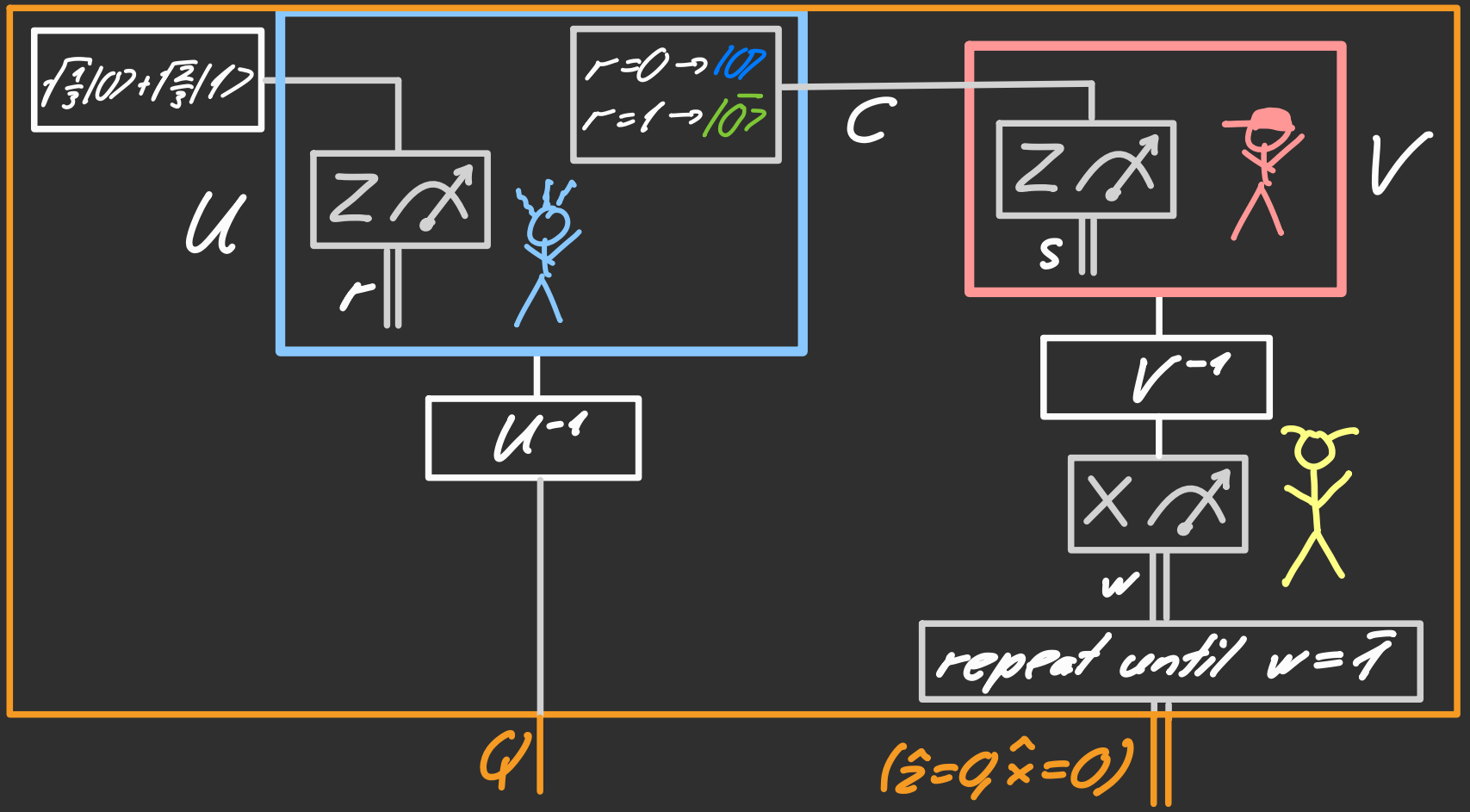

r=0
w=0
?
r=0 → |0⟩
r=1 → |1⟩
X
w=0
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$


r=0
r=0 → |0⟩
r=1 → |1⟩
X
w=1
w=1
r=0

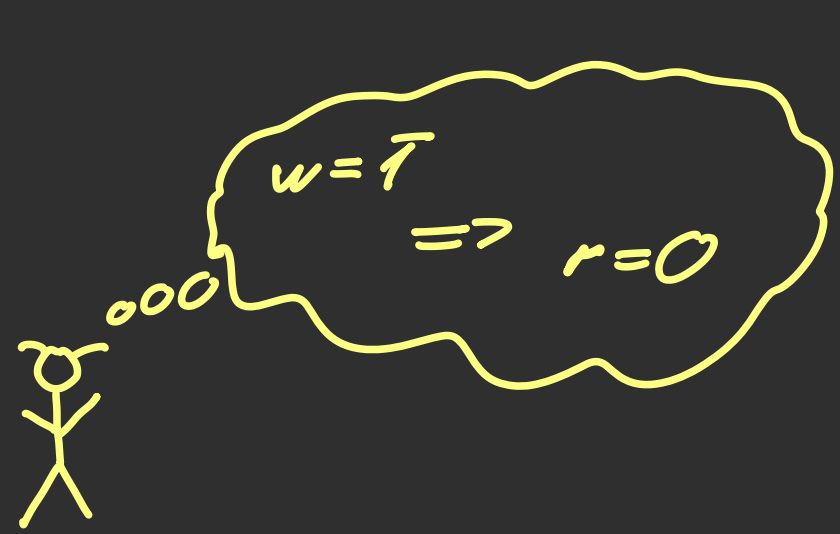

r=0
r=0 → |0⟩
r=1 → |1⟩
X
w=1
w=1
r=0
(c)
r=0

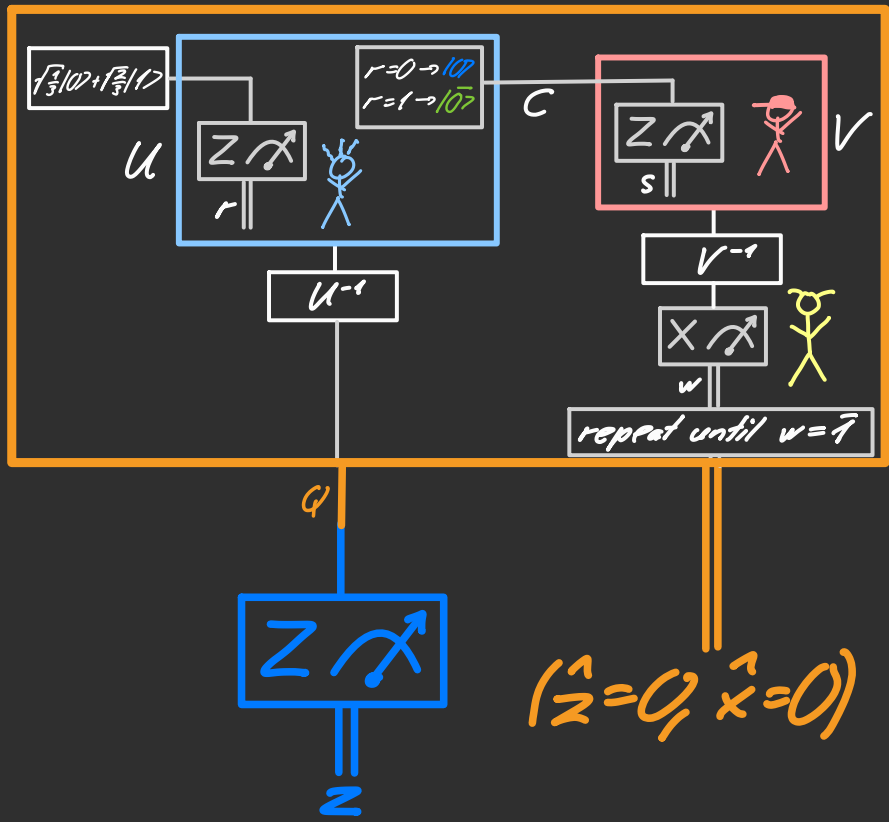
Building a Magic Machine without gravity:

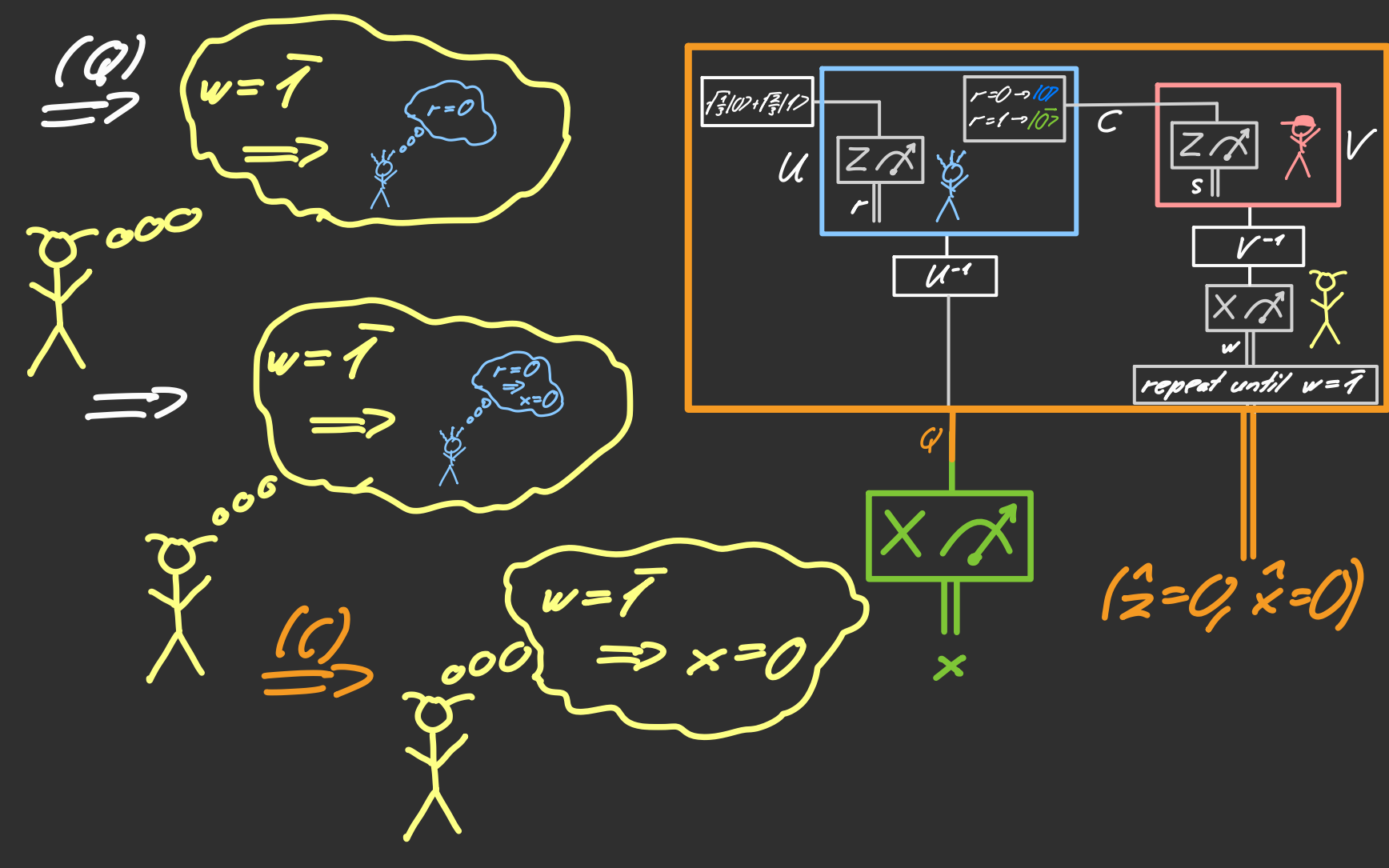
Attempt II



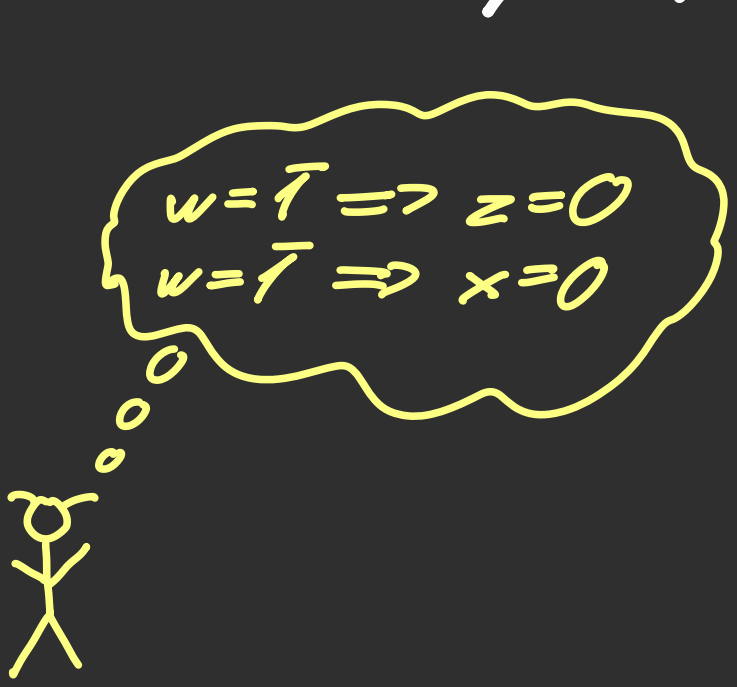
[Frauchiger, RR, 2018]

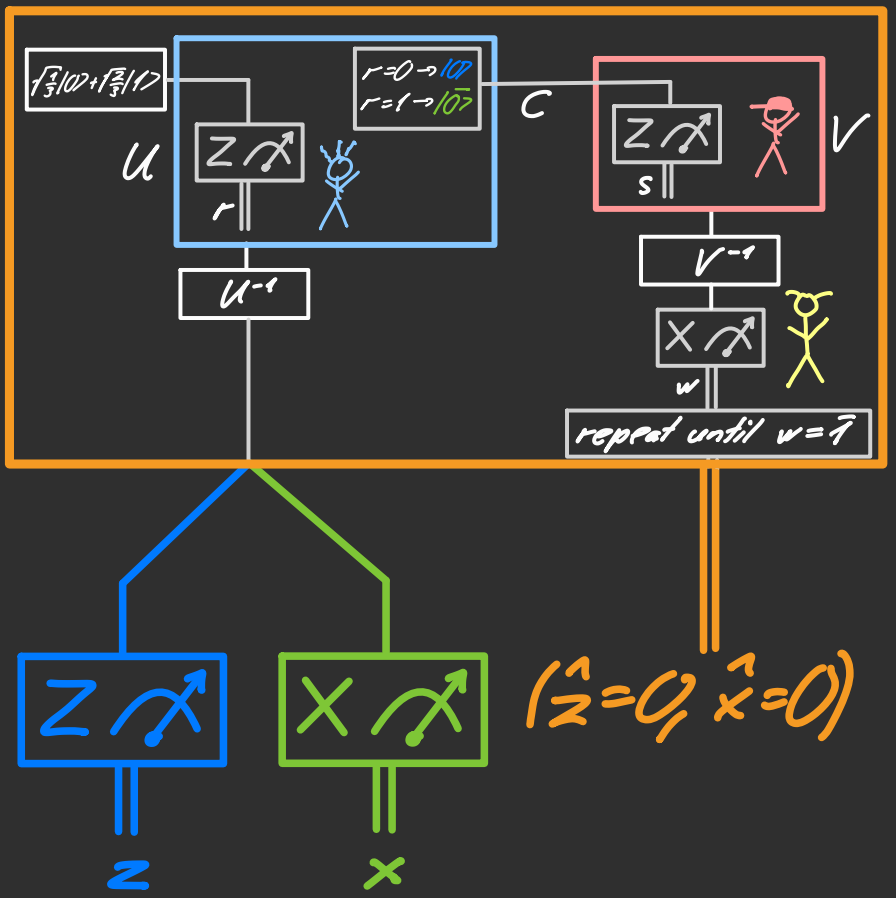

w=1
=>
r=0
ooo


√⅓|0⟩+√⅔|1⟩
U
Z
r
r=0→|0⟩
r=1→|1⟩
C
Z
s
V
V⁻¹
X
w
repeat until w=1
U⁻¹
Q
Z
z
(ẑ=0, x̂=0)


(Q)
w=1
r=0
w=1
r=0
x=0
(C)
w=1
x=0
U
C
V
Z
r
s
X
w
repeat until w=1
Q
x
(x=0, x=0)

Summary of Alice's inferences





$Pr[win] = 1$

Quantum collaboration paradox

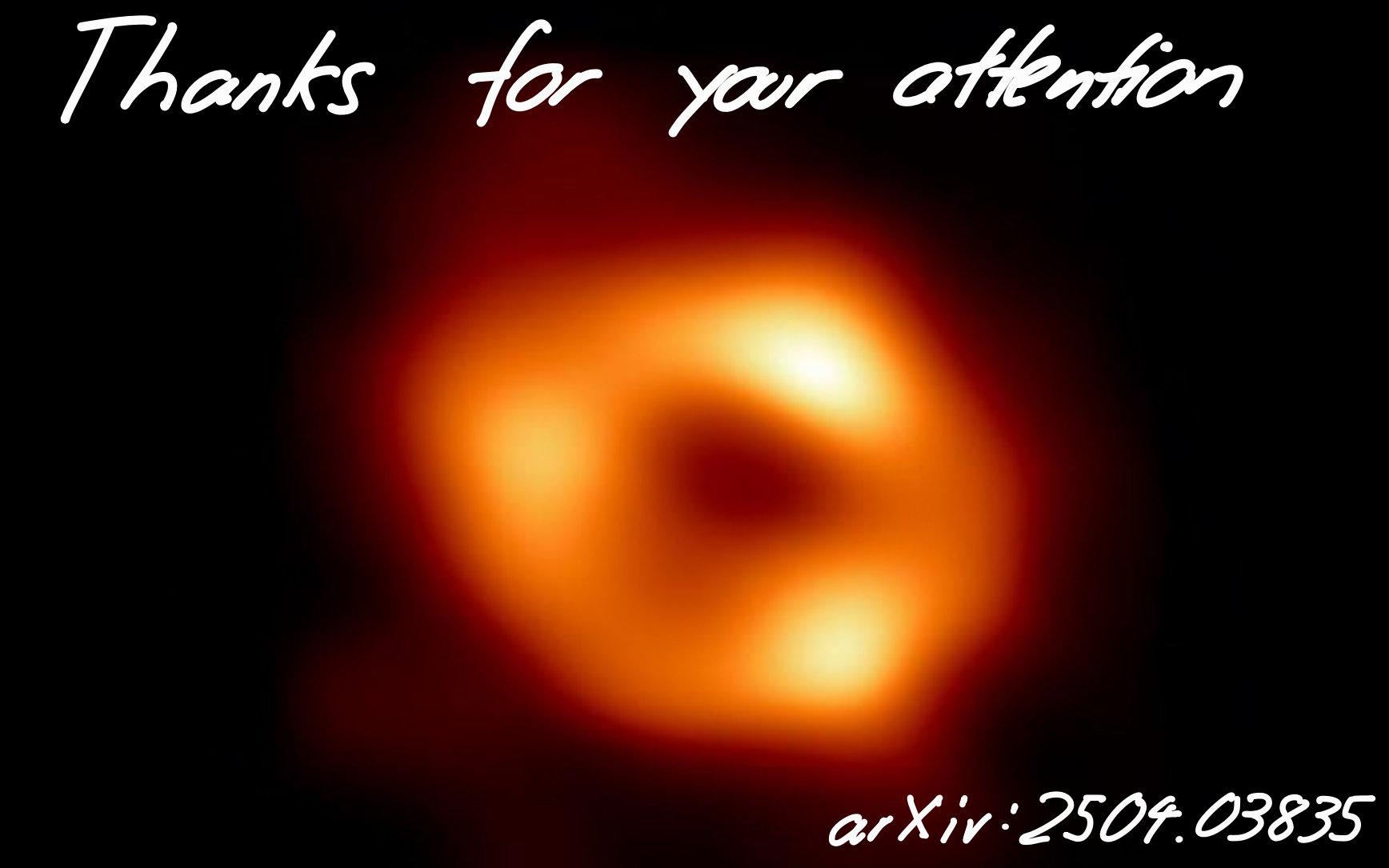
Theorem

These assumptions are incompatible:

(Q) Quantum theory

(C) Consistency

Thanks for your attention



arXiv:2504.03835