Foundations of Quantum Gravity

Daniele Oriti

Depto. de Física Teórica and I.P.A.R.C.O.S.,
Universidad Complutense de Madrid, Spain, EU

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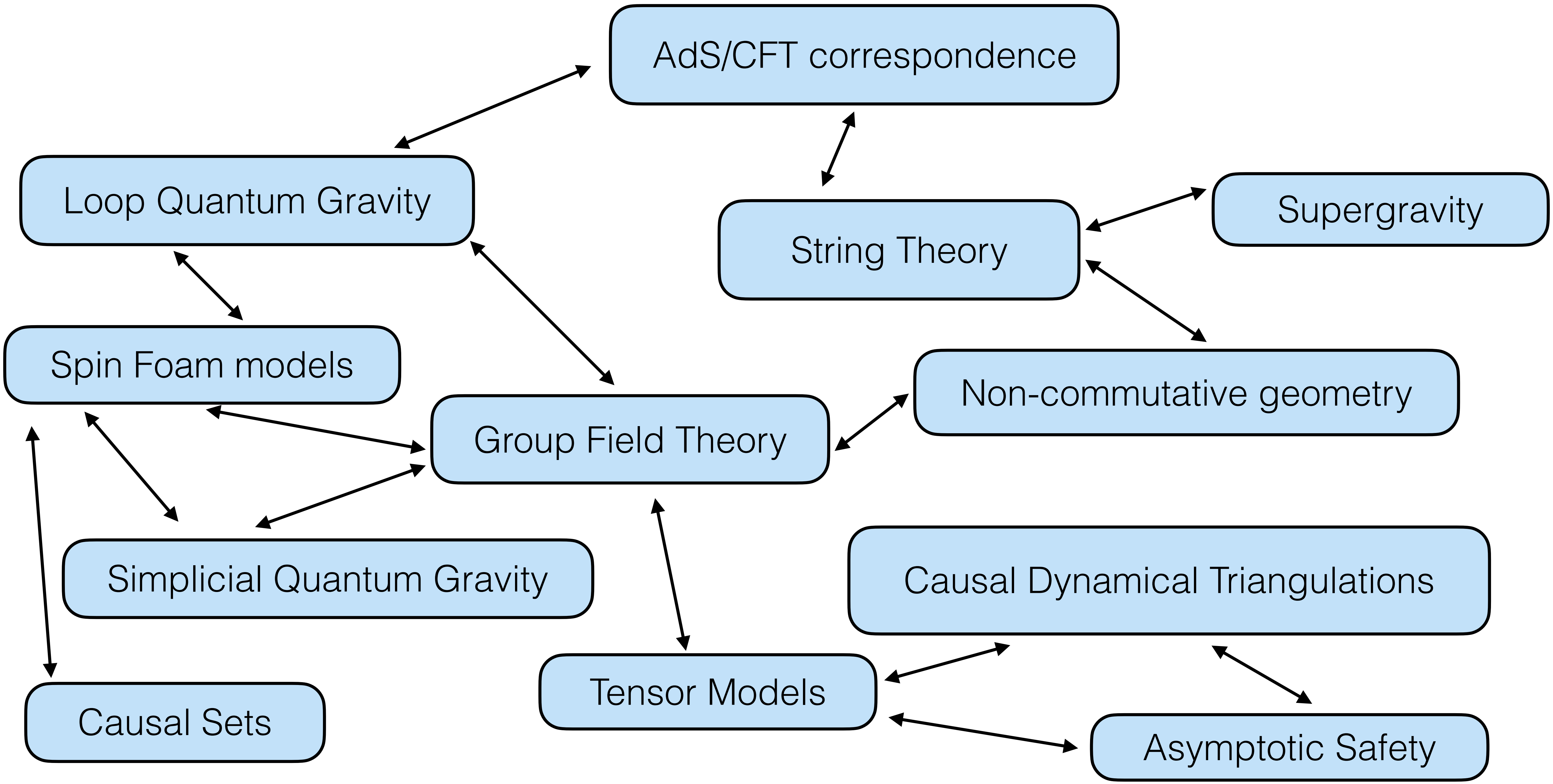
Quantum Gravity landscape: contemporary approaches

great variety; many mutual relations; many shared issues; mostly same goals



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but QG issues and goals are indeed shared, and often technical tools are also related, and ideas of more general use .....

this lecture

other lectures at
this summer school

this lecture

other lecturers at
this summer school

goals

outline general issues in QG, choices to be made, alternative perspectives and possibilities, more than specific results

suggest what to look for in various QG formalisms, going beyond technical issues, mostly focusing on conceptual aspects

hint at overlap with quantum foundations

show how QG is "as foundational as it gets"

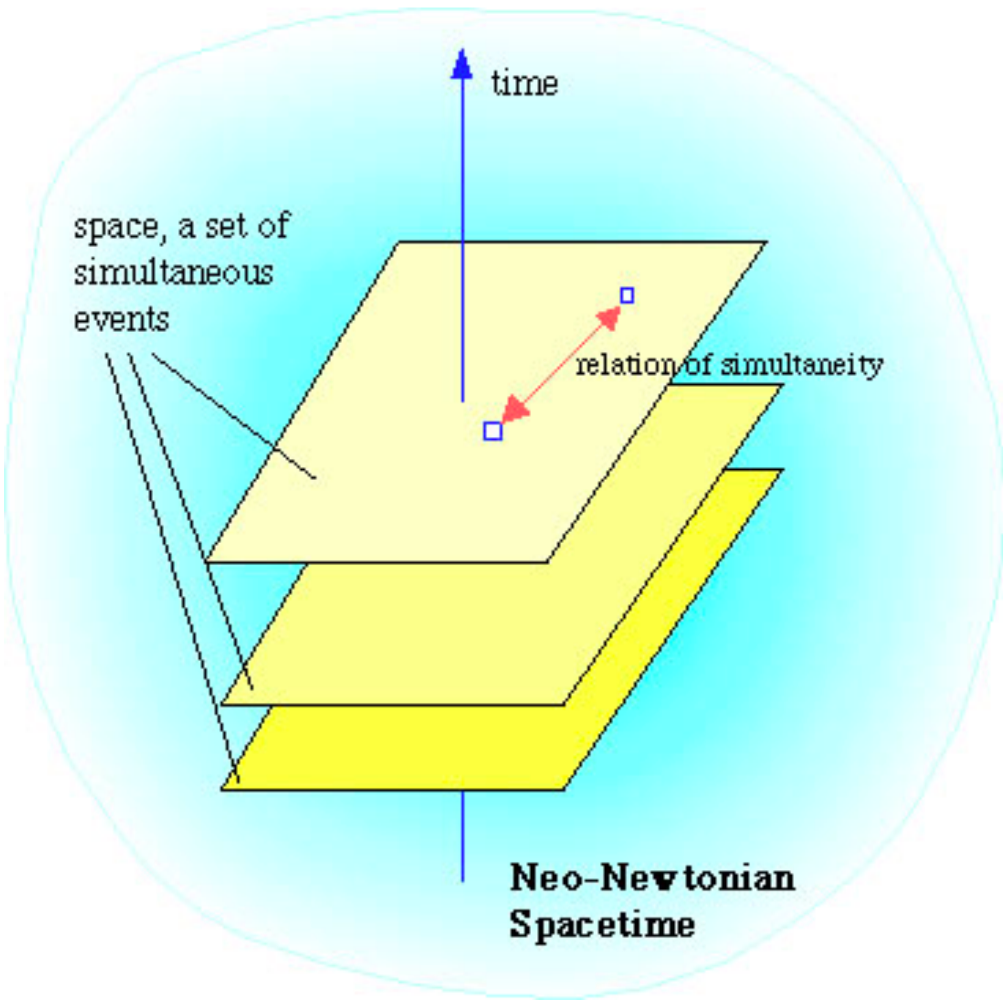
First things first:
what we know about
(physical) space and time

Newtonian physics   absolute space, absolute time

with preferred (temporal) coordinate/direction

physical, but not dynamical nor subject to influence of other entities

- continuum nature
- preferred foliation of spacetime manifold
- Galilean invariance (no preferred spatial direction, relativity of inertial frames))



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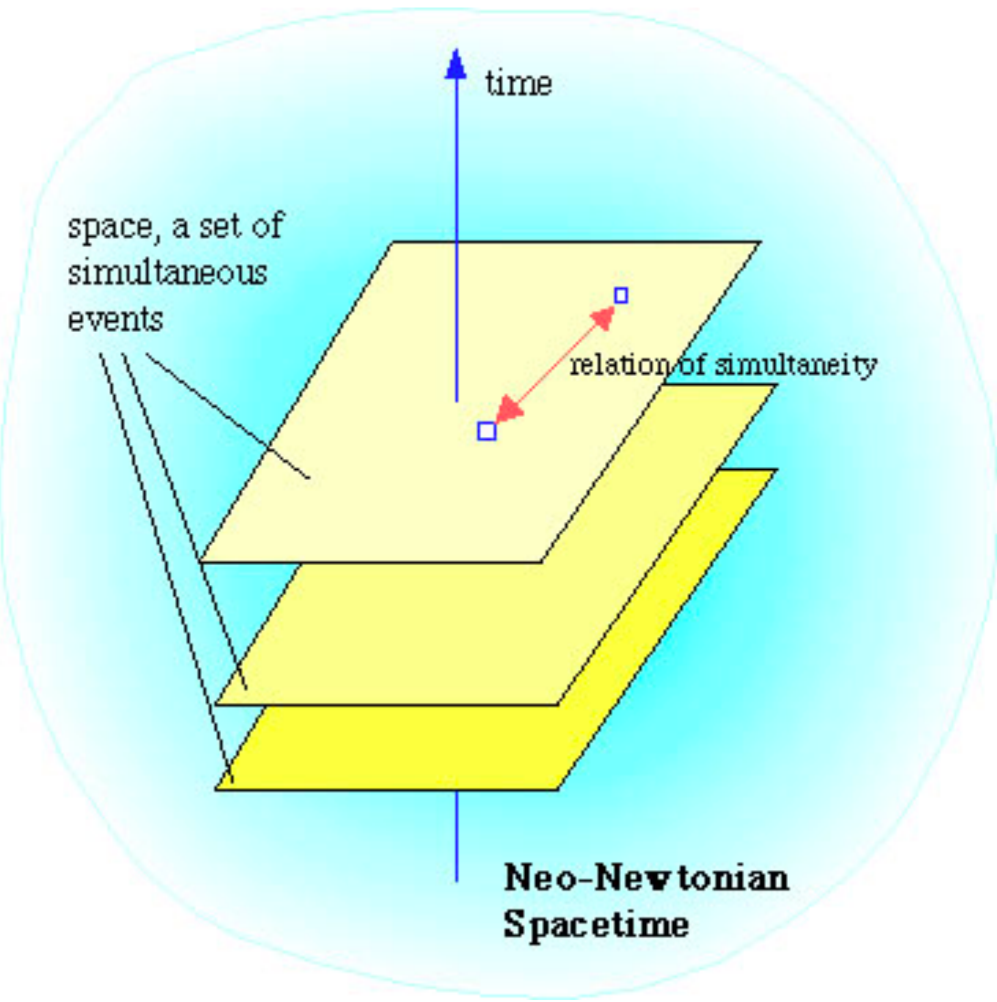


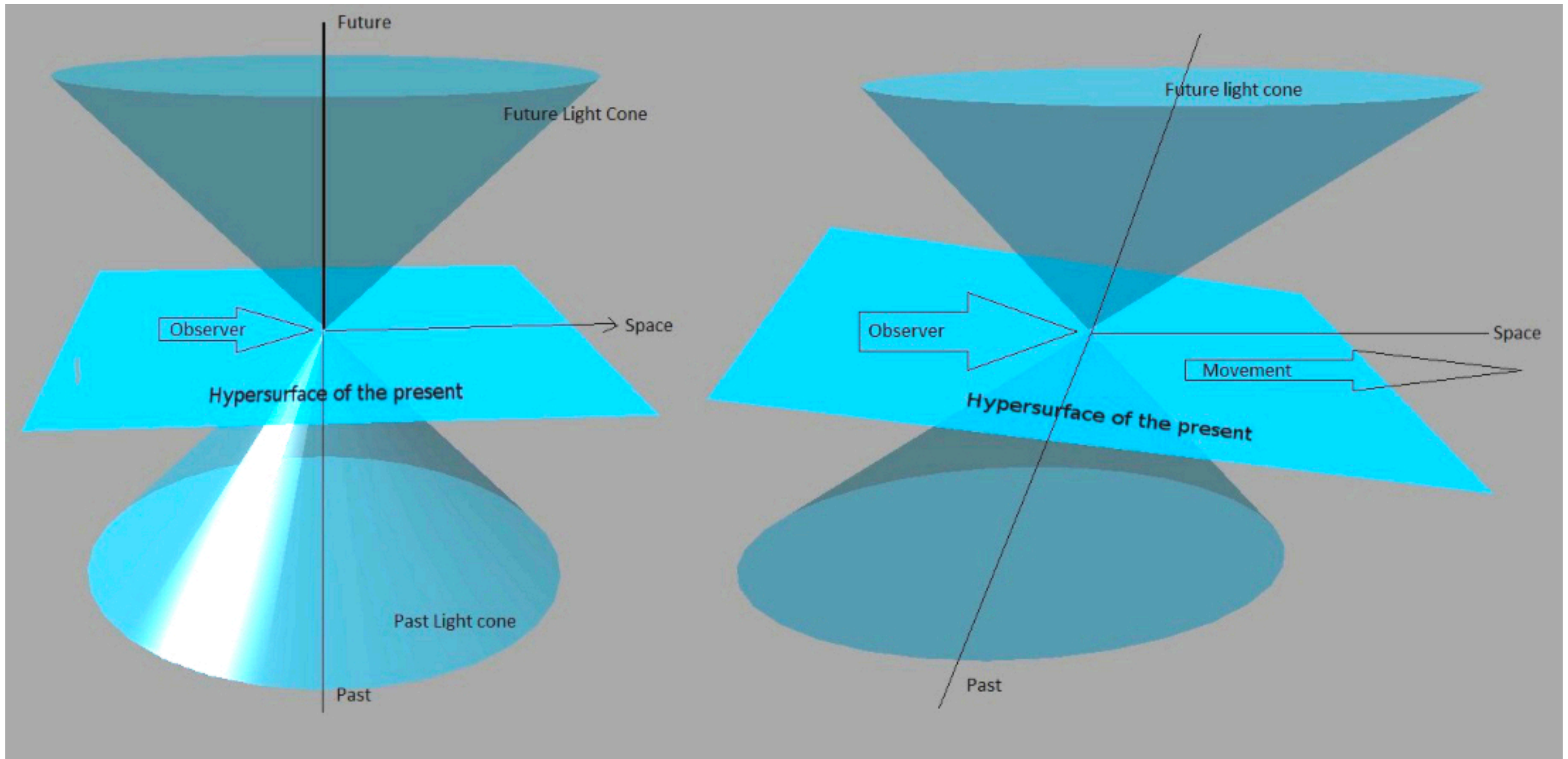
Special Relativity absolute spacetime

with preferred class of (spatio-temporal) coordinates/directions

physical but not dynamical nor subject to influence of other entities

key point: finite (and absolute and maximal) propagation speed of light

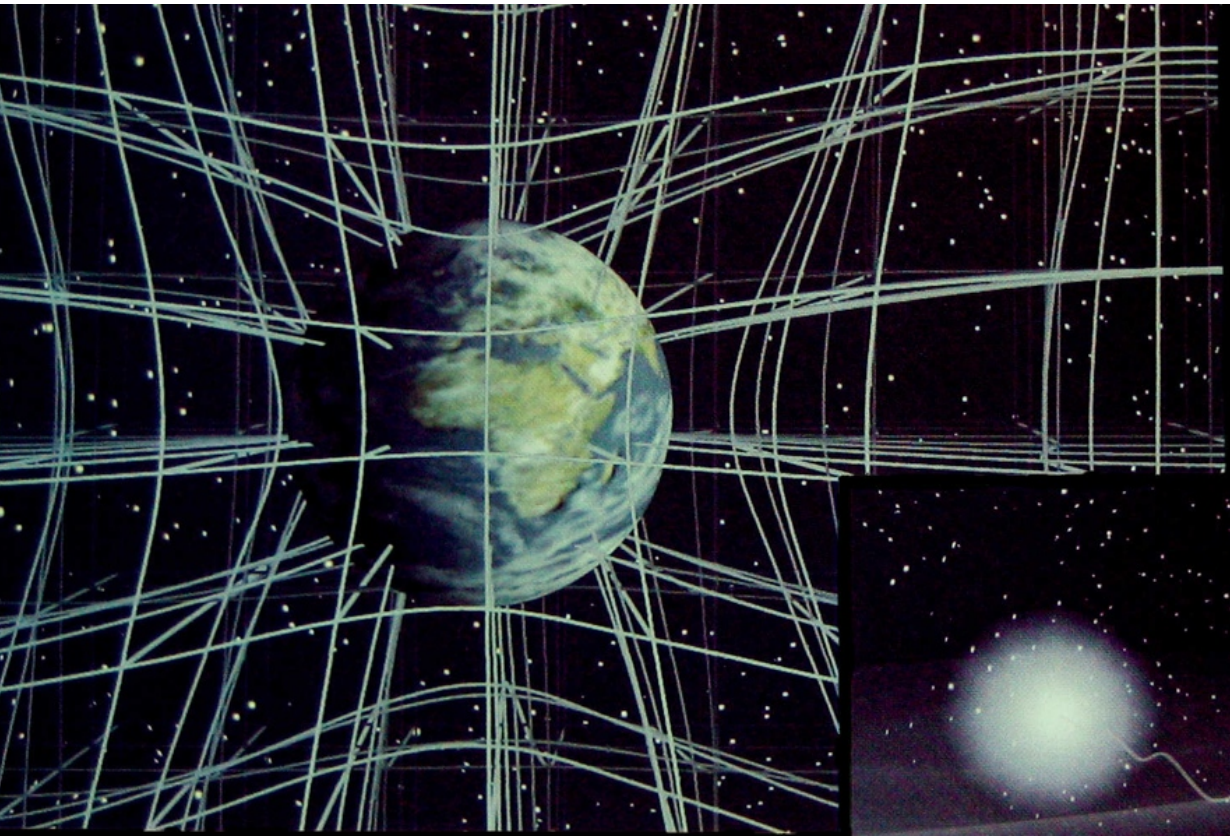
- continuum nature, foliability
- Lorentz invariance (relativity of inertial frames)

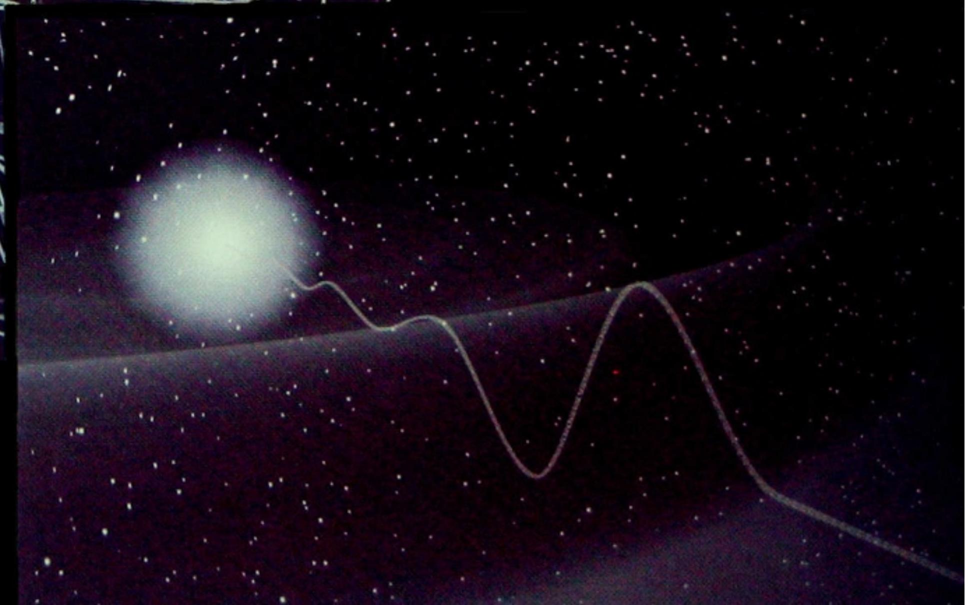


What we know about gravity and spacetime

- gravitational physics well described by General Relativity
  - basis for our description of astrophysics and cosmology
  - predicts amazing new phenomena (deflection of light, gravitational distortion of space and time measurements, gravitational waves, black holes, expansion of universe, …….)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$





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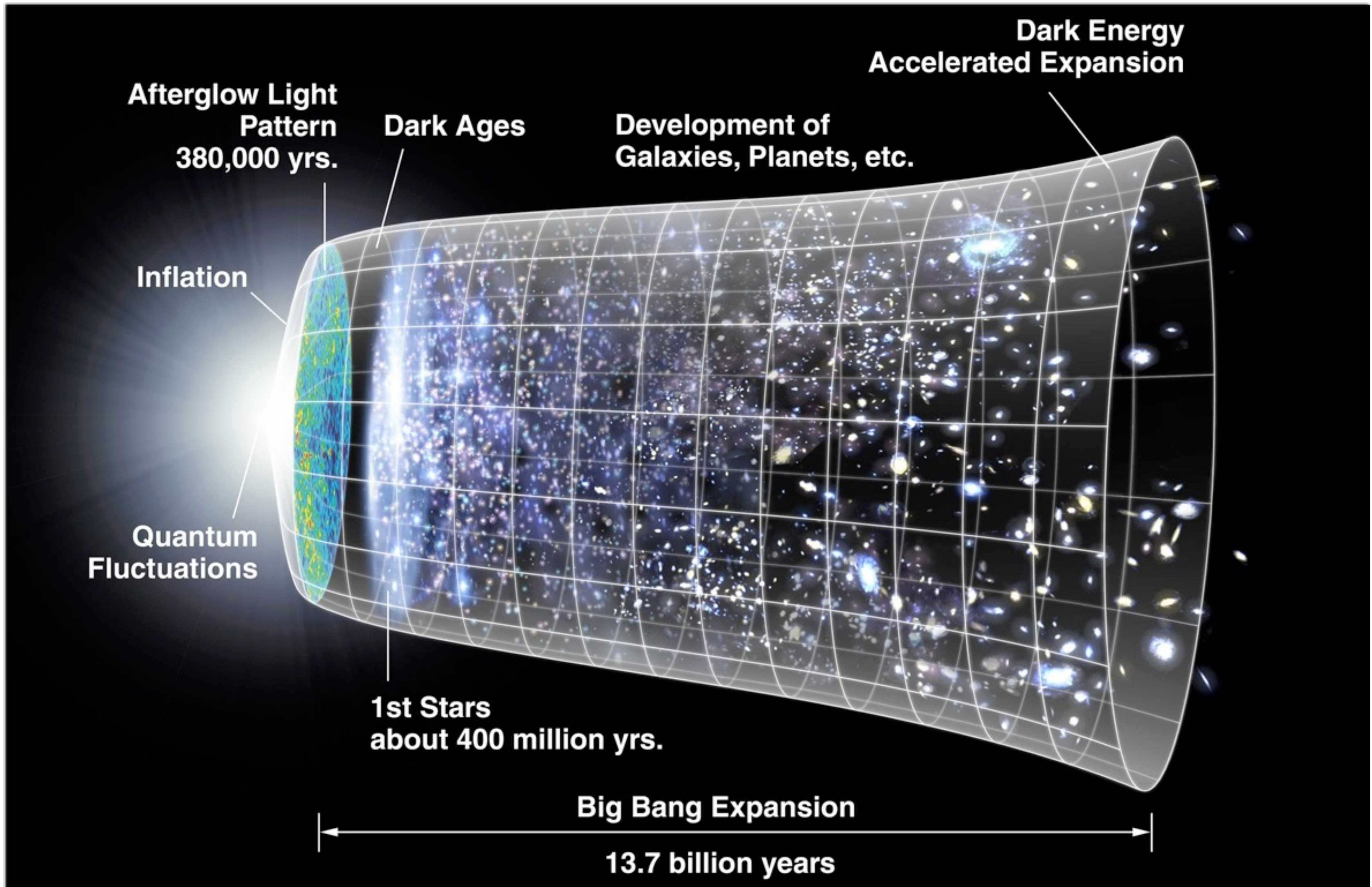

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- what do we learn from GR?
  - gravitational interaction described (macroscopically) by geometry of spacetime
  - continuum, local picture of spacetime adequate
  - dynamics and (local) interaction with matter described by Einstein's equations: "matter tells spacetime how to curve, spacetime tells matter how to move"
  - spacetime itself is physical system
  - there is no fixed background over which things happen, if not as approximation
    - deeper understanding of gravity is deeper understanding of space and time

Nature of spacetime: lessons from GR

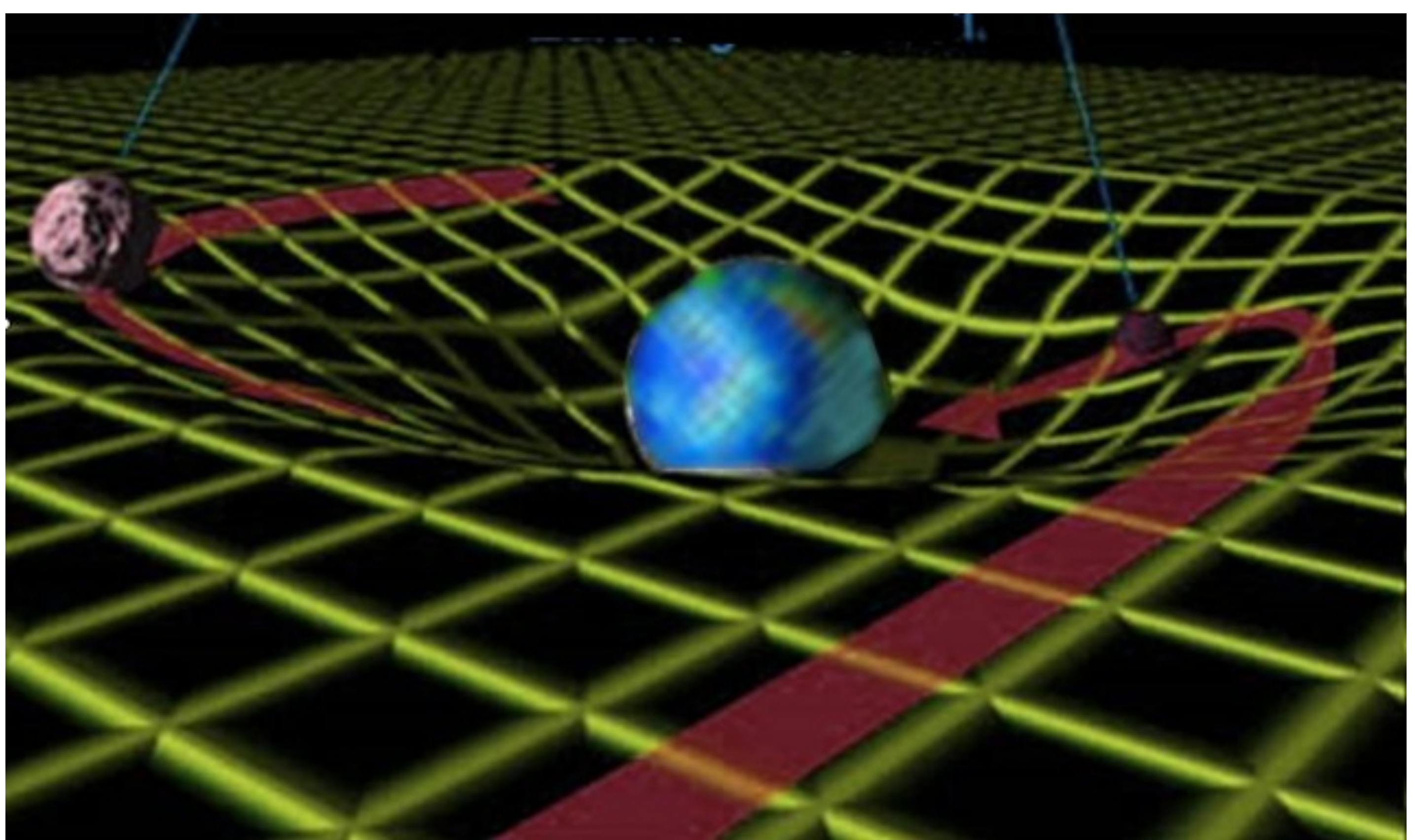
main lesson: spacetime is a physical system

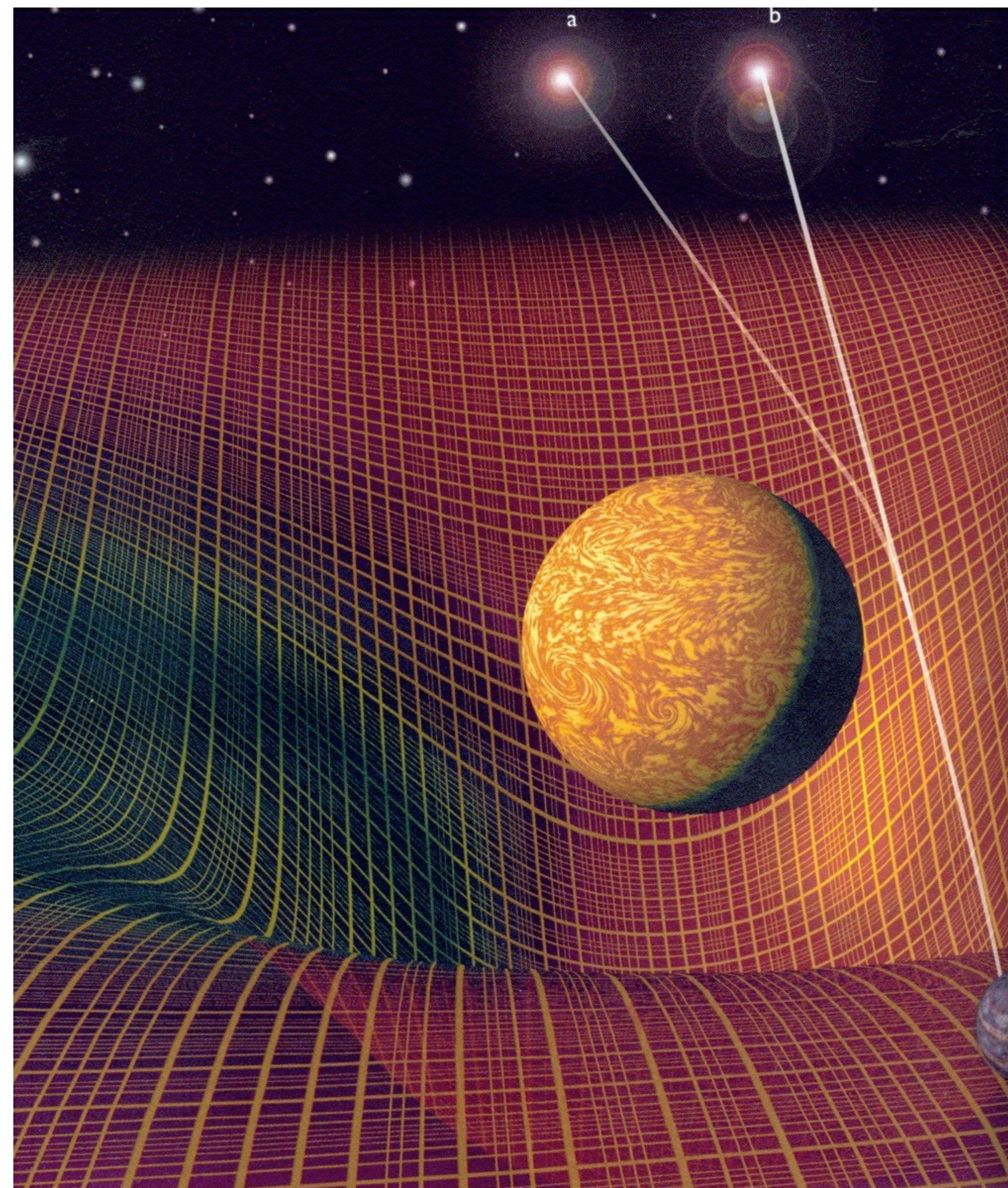
Einstein's equations (constraint for allowed configurations of spacetime geometry and matter fields)

$$g_{\mu\nu}(t,x) \qquad\qquad ds^2 = g_{tt}dt^2 + g_{12}\,dx_1 dx_2$$

$$R_{\mu\nu}[g(x)] - \frac{1}{2}R[g(x)] + \Lambda g_{\mu\nu}(x) = 8\pi G_N T_{\mu\nu}[\phi(x),...]$$

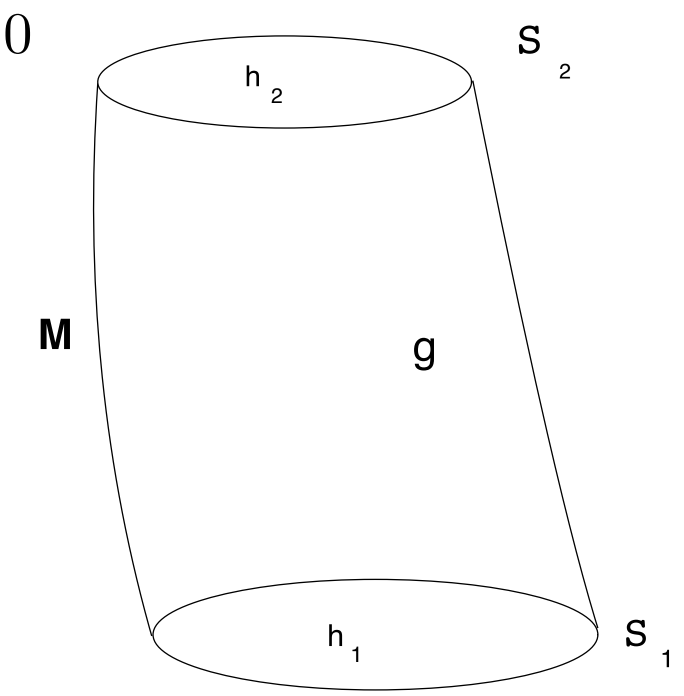
- gravity = spacetime geometry (spatial distances, time intervals, curvature of space, volumes, …..)
- mass-energy of material bodies "deformes" spacetime, this deformation affects motion of other material bodies
- deformation of spacetime is what we call "gravity"
- spacetime deformation itself has own dynamics





Space and Time in General Relativity

classical theory: $(\mathcal{M}\,,g)$ $\quad S_{\mathcal{M}}(g)$ $\quad R_{\mu\nu}(g) - \frac{1}{2}R(g)g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 0$



spacetime structures:

topological manifold

differentiable structure

continuum metric field

matter/gauge fields

GR key ingredients: only dynamical fields + diffeomorphism invariance

- no preferred time/space direction - infinity of equally valid local notions of time/space
- manifold points, paths on manifolds, values of fields at points or regions, are -not- physical per se
- they have to be made physical (given some operational meaning) by defining them via dynamical fields

these structures are basis for local, spatiotemporal description of physical universe in terms of field theory

all these spacetime structures have to be questioned

none of these spacetime structures should be simply assumed as fundamental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The problem of Quantum Gravity

starting point: conceptual, physical, mathematical clash

framework and ingredients of GR are incompatible with what we learned from Quantum Mechanics

→ two incompatible conceptual (and mathematical) frameworks for space, time, geometry and matter

GR	QFT
spacetime (geometry) is a dynamical entity itself	spacetime is fixed background for fields' dynamics
there are no preferred temporal (or spatial) directions	evolution is unitary (conserved probabilities) with respect to a given (preferred) temporal direction
physical systems are local and locally interacting	nothing can be perfectly localised
everything (incl. spacetime) evolves deterministically	everything evolves probabilistically
all dynamical fields are continuous entities	interaction and matter fields are made of "quanta"
every property of physical systems (incl. spacetime) and of their interactions can be precisely determined, in principle	every property of physical systems and their interactions is intrinsically uncertain, in general

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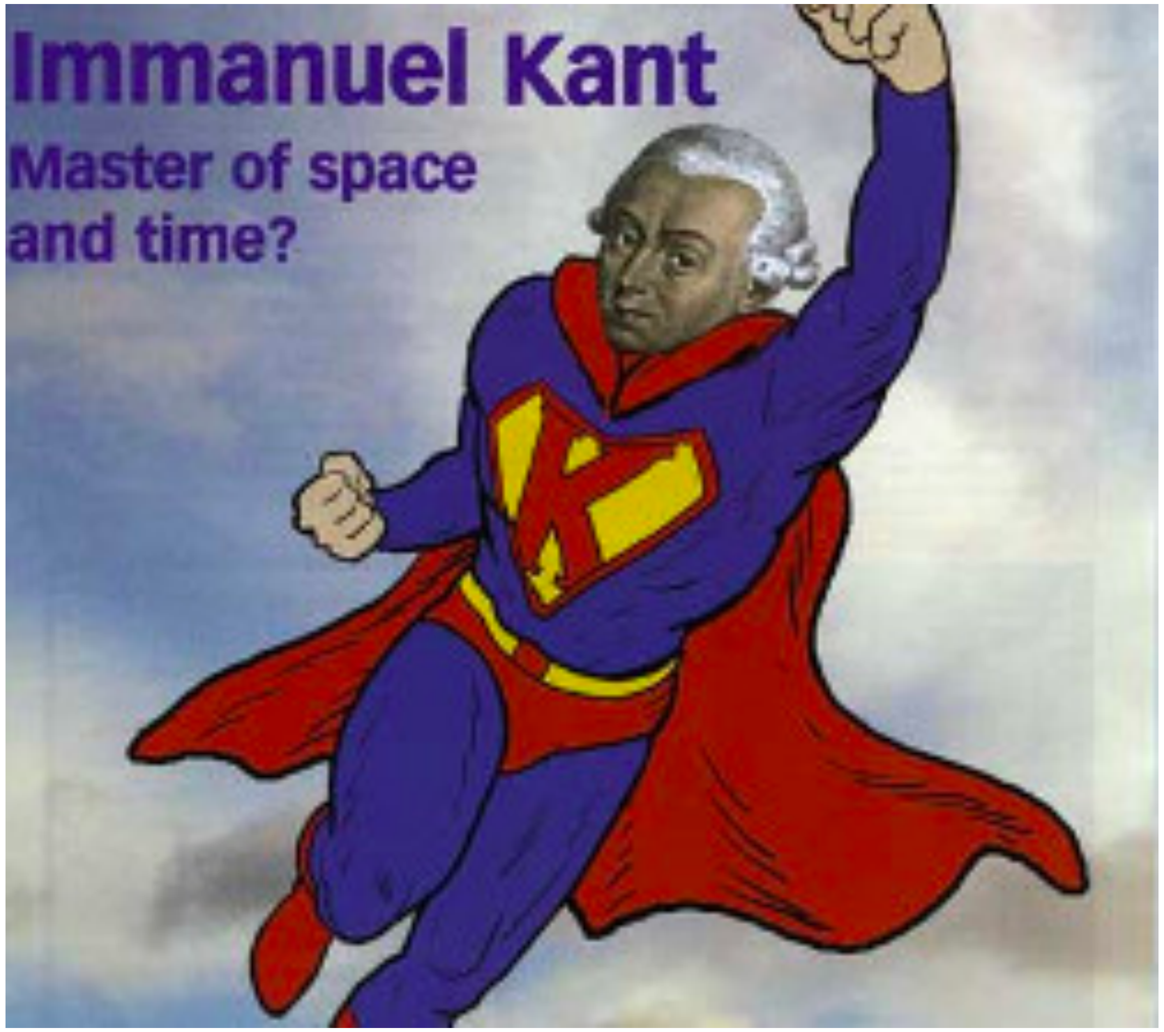
two frameworks come with different associated mathematical language and tools

conceptual + mathematical clash is clear deeper understanding of gravity

is

deeper understanding of space and time

we have to learn to think deeper about the nature of space and time themselves, thus we have to learn to (re-)**think the world without (assuming) space and time**



but:

space and time are the basic infrastructure and condition *sine qua non* of our conceptualisation of the world .....

...... difficult

summary of physical issues

- no proper understanding of interaction of geometry with quantum matter, if gravity is not quantized

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \langle \Psi | \hat{T}_{\mu\nu} | \Psi \rangle$$ not a consistent fundamental theory

- QFT framework problematic if background spacetime is dynamical (and spacetime metric has no isometries)
- eqns need to be solved by self-consistent iteration:
    - metric --> e-m tensor --> expectation value --> for new metric --> new em tensor --> ...
        - the process does not converge; equations too non-linear
- expectation value of em tensor at a point diverges; regularization is tricky
- renormalization of quantum effects produce modifications of Einsteins' equations (e.g. R^2 terms) ....
- UV regime of QFT on given background is problematic (e.g. black hole production? then, BH evaporation? unitarity?)
- expectation values not enough: quantum fluctuations of matter should induce fluctuations of geometry
- which vacuum state? inequivalent quantum theories ....

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three old arguments:

- classical gravitational measurements can lead to (possibly large) violations of basic QFT principles (momentum conservation, bound on signalling speed) - Eppley-Hannah (1977)
- no localization of sources of gravitational field, thus non-local gravity - Page-Geilker (1981)
- classical gravitational modifications of quantum mechanics (becoming non-linear) - Carlip (2008)

exemplary of a number of similar ones

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general expectation, various arguments + formal reasons, but maybe also experimental window

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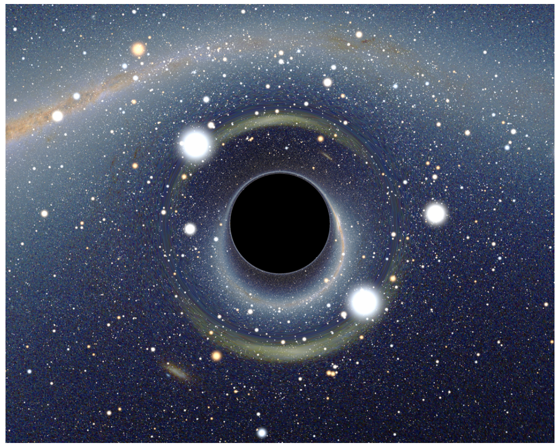
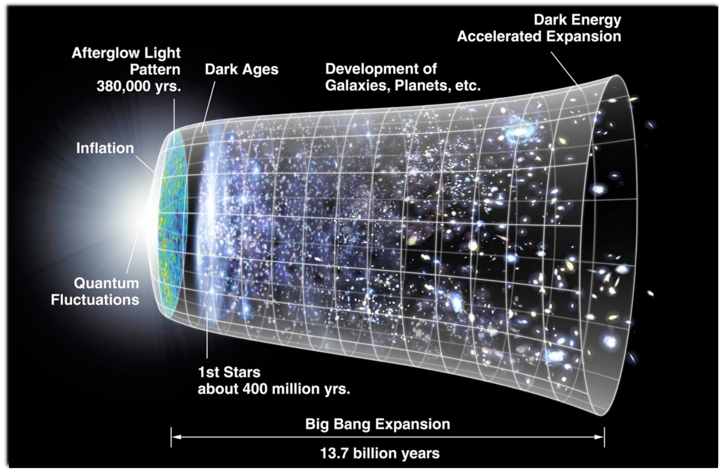
- spacetime singularities: breakdown of GR for strong gravitational fields/large energy densities - inevitable in classical GR center of black holes, big bang - quantum effects expected to be important





- cosmological scenarios for the early universe need QG completion

R. Brandenberger, ’10, ’11, ’14

why a close to homogeneous and isotropic universe?

why an approximately scale invariant power spectrum?

what happens "at" the Big Bang?

Inflation
- what produces inflation?
- physics of trans-Planckian modes (for long inflation)?
- inflation too close to Planck regime?
- inflationary spacetime still contains singularity

Bouncing cosmology
- new physics needed to describe/justify cosmological bounce

Emergent universe
(pre-big bang static phase)
- static phase and phase transition require new physics

summary of physical issues

new QG dofs? primordial (quantum) black holes?

new type of matter? new QG dof?

cosmological constant? why doesn't it gravitate?

modified gravity?

why holographic entropy?

spacetime microstructure? all require QG

violation of unitarity? locality? .....

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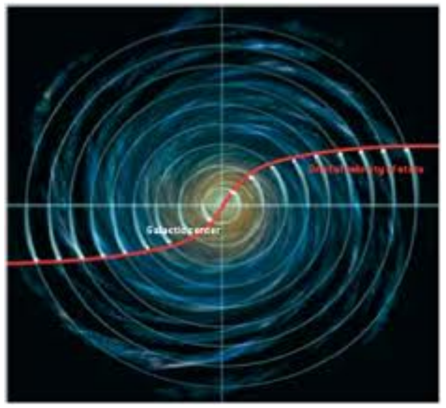

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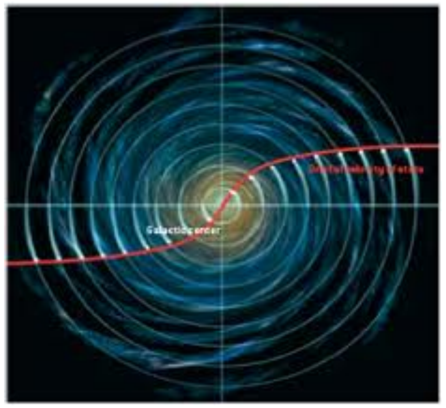

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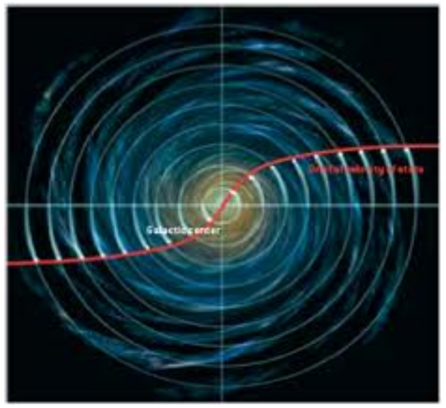

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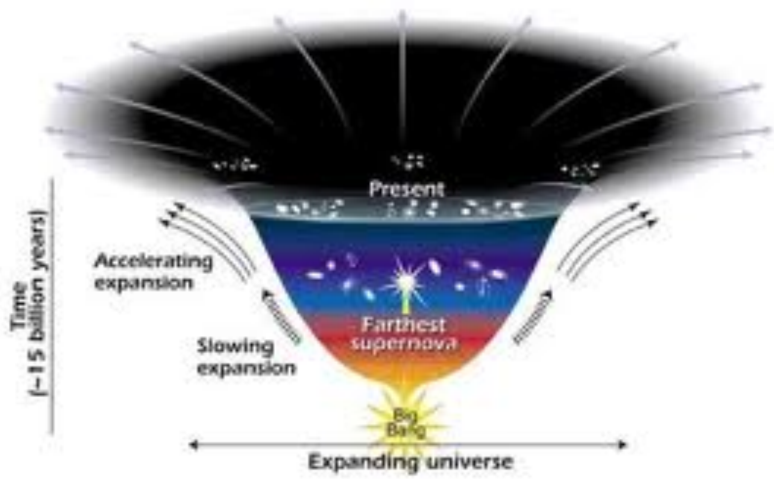

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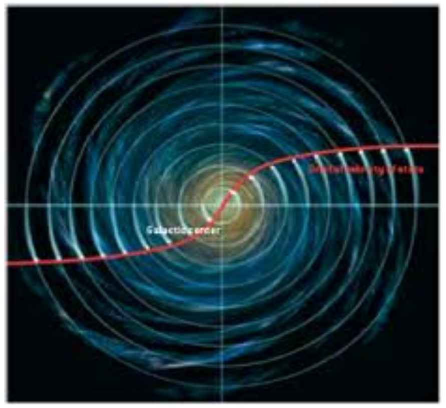

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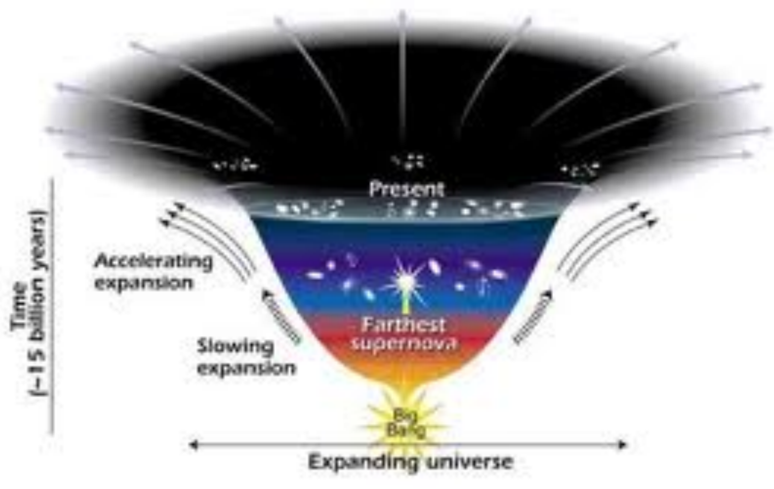

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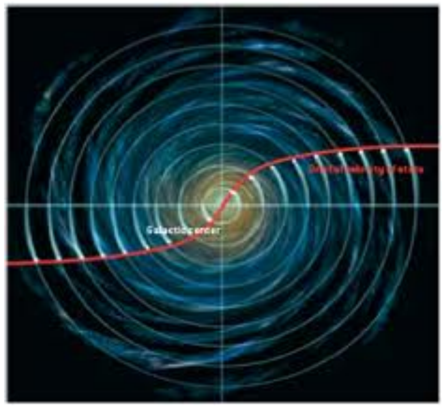

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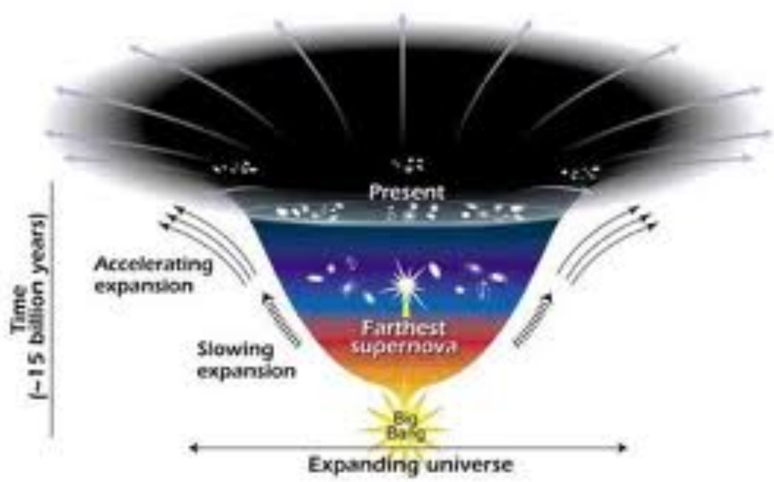

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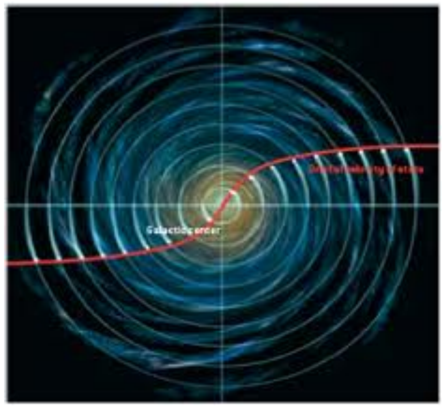

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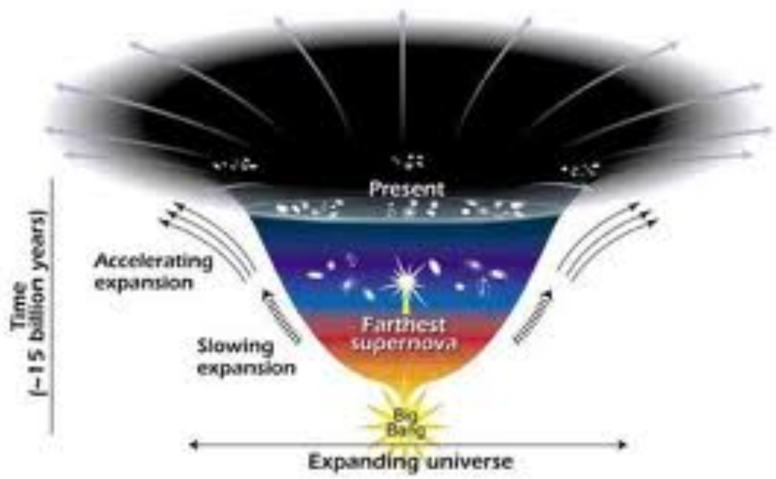

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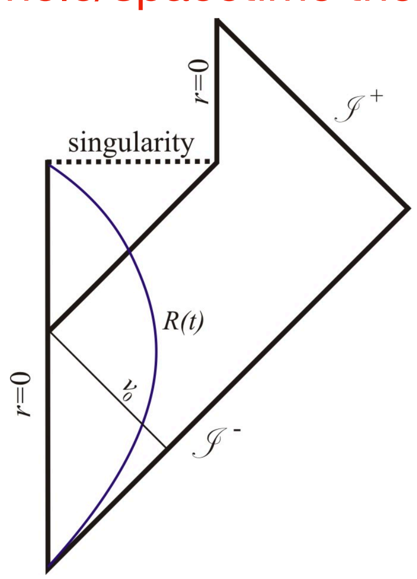

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how one approaches these physical issues depends on perspective on QG and spacetime, and of course on specific QG formalism

can we avoid or at least postpone conceptual/foundations issues?

most conservative strategy: perturbative QG

maintain usual QFT perspective: gravity as standard interaction

postpone conceptual issues (space, time, etc) - forget key lesson of GR

- start from classical GR action $$S = \int \mathcal{L}(x)\, \mathrm{d}^4x \; ; \qquad \mathcal{L}(x) = \sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}^{\mathrm{matter}}\right)$$
- to define a perturbative, linear theory, we expand the generic metric as: $$\boxed{g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu}}$$

    i.e. we only deal with small perturbations around flat spacetime, as the dynamical field

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- the gravitational coupling is dimensionful with dimension of length square $[G] = L^2$

  and it also plays the role of expansion parameter $\varepsilon = \sqrt{16\pi G}$

- quantize using standard QFT methods around free theory, relying on background structures provided by flat metric

most conservative strategy: perturbative QG

maintain usual QFT perspective: gravity as standard interaction

postpone conceptual issues (space, time, etc) - forget key lesson of GR

- start from classical GR action $S = \int \mathcal{L}(x)\, \mathrm{d}^4 x\ ; \qquad \mathcal{L}(x) = \sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}^{\mathrm{matter}}\right)$

- to define a perturbative, linear theory, we expand the generic metric as: $\boxed{g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu}}$

  i.e. we only deal with small perturbations around flat spacetime, as the dynamical field

- key idea/motivation: GR as the unique non-perturbative completion of a linear theory of massless spin-2 fields

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$\longrightarrow$ perturbative QG is non-renormalizable

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lessons

- QG "vacuum" is not Fock vacuum, UV theory not free; fundamental QG dofs are not gravitons;
- perturbative framework to be obtained as EFT, though, from any fundamental QG formalism

one key goal of any QG

perturbative framework (QFT + GR) to be obtained as EFT from any fundamental QG formalism

background spacetime/geometry, including flat Minkowski spacetime

also to be recovered from more fundamental description

+

QFT (including gravitons) on top

Quantum Gravity:

main routes

then, two main options:

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1. quantize GR non-perturbatively

    apply quantization methods to spacetime/geometry as a whole

    maintain fields, spacetime and geometry as basic dofs/entities

    UV theory (of metric + matter fields) is non-perturbative

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   UV theory (of metric + matter fields) is non-perturbative

2. extend QFT framework, also in perturbation theory, with new dofs and/or symmetries

   extend perturbative QFT formalism with new dofs and symmetries

   quantize perturbatively ensuring UV consistency

   new dofs/symmetries and perturbative theory guide toward full non-perturbative formulation

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- unification? sure, but which unification?

unified framework — single mathematical/conceptual framework (thus same underlying principles) to treat all fields/interactions

QG necessarily unified theory in this sense

unified substance — single entities give rise (look like) different particles/fields in different regimes

stronger condition; welcome if realized; not (logically) necessary; it implies first

early attempts

General strategy being followed at first (up to '90s):

quantise GR, adapting and employing standard quantization techniques

different research directions, corresponding to different quantization techniques:

perturbative quantization, canonical quantization, covariant (path integral) quantization

all achieved key insights

all got stuck and died of starvation (or are maintained alive in a vegetative state)

Quantum Gravity: canonical quantization

Bergmann, Dirac (1950-1959): canonical quantization of (constrained) gauge systems

Arnowit, Deser, Misner (1961): Hamiltonian formulation of General Relativity, diffeomorphism constraints

Bergmann-Komar, Peres, DeWitt, Wheeler (1962-1967): canonical quantum gravity in ADM (metric) variables

choose foliation + identify phase space variables + impose "dynamics"

$$\{h_{ij}(x), K^{kl}(x')\} \propto \delta_{ik}\delta_{jl}\delta(x-x')$$

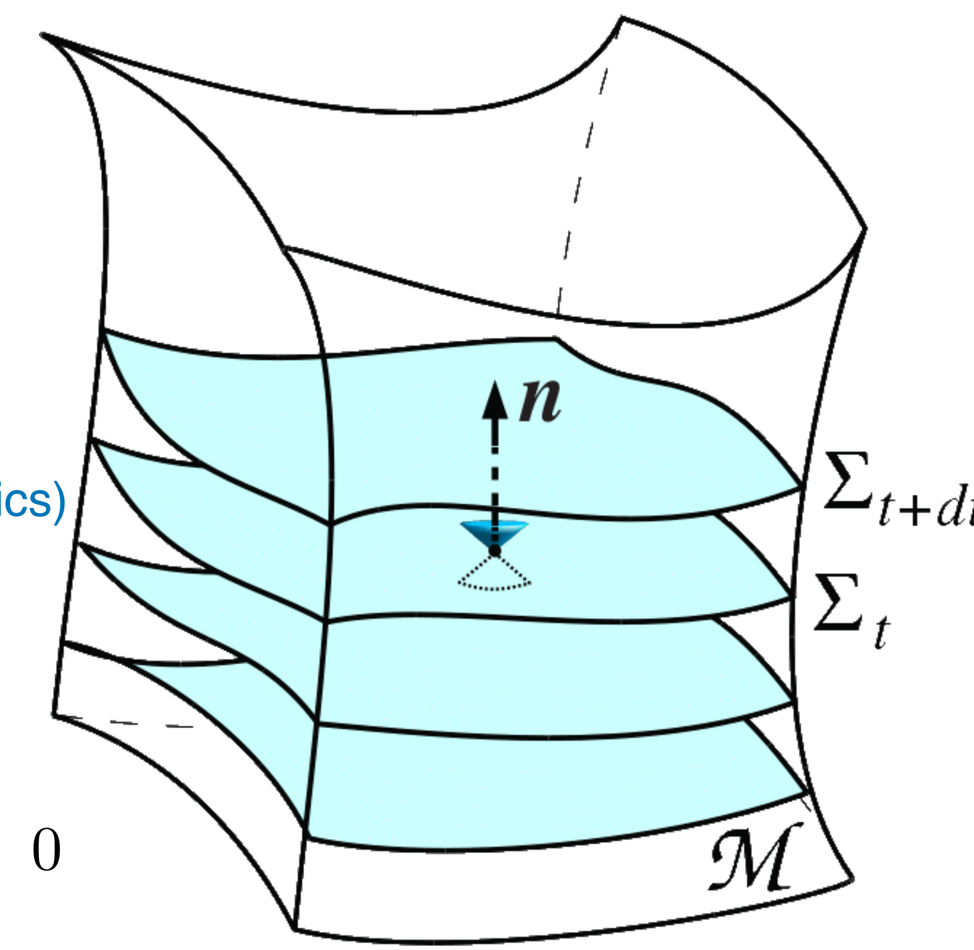
spatial 3-metric   extrinsic curvature

$$H_i(h_{ij}, K_{kl}) = 0$$

$$H(h_{ij}, K_{kl}) = 0$$

invariance under spatial and temporal diffeomorphisms ~
invariance under change of foliation

(totally constrained theory: constraints encode whole dynamics)



quantum level:

$$\Psi(h_{ij}) \in \mathcal{H}$$

$$\widehat{H}_i\left(h_{ij}, \frac{\delta}{\delta h_{kl}}\right)\Psi(h_{ij}) = 0 \qquad \widehat{H}\left(h_{ij}, \frac{\delta}{\delta h_{kl}}\right)\Psi(h_{ij}) = 0$$

Wheeler, DeWitt, Teitelboim, Kuchar, Isham…. (1967-1987, …): properties of “superspace of 3-geometries”, problem of time, scalar product on quantum states, quantum cosmology, lots of semiclassical analyses, ….
too ill-defined at mathematical level to be solid approach to QG (beyond semi-classical or formal analyses)

Quantum Gravity: covariant path integral quantization

Misner, Wheeler,… (1957-): sum-over-histories formulation of QG, non-perturbative transition amplitudes (and scalar product) between QG states via sum over spacetime geometries

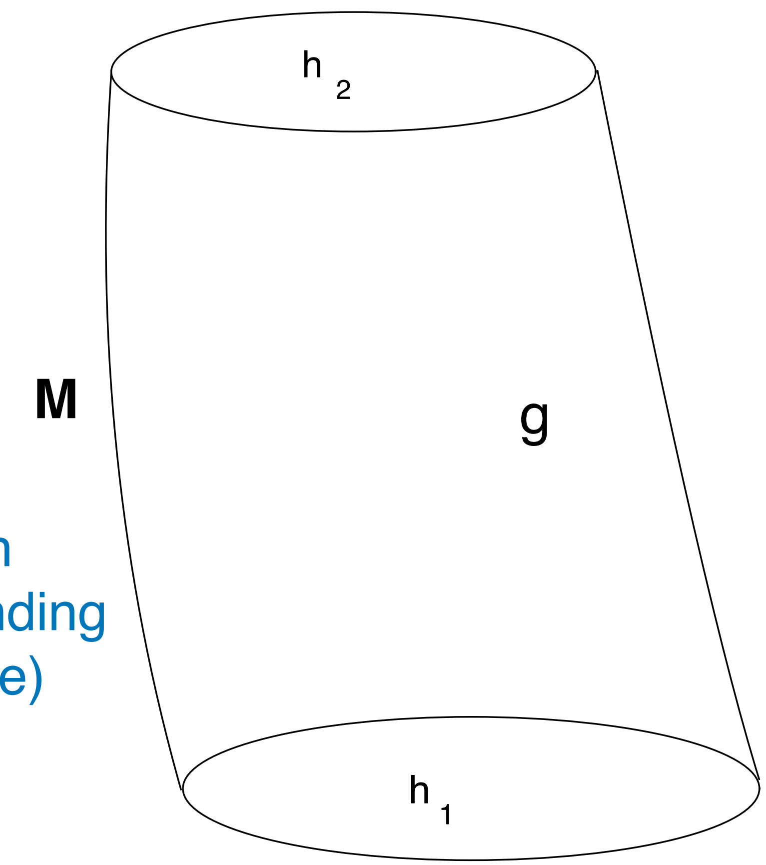
Wheeler (1963): define it via discrete lattice (Regge) regularization —-> quantum Regge calculus

transition amplitude (or scalar product) from one 3-geometry to another

$$\langle h_1 | h_2 \rangle = \int_{h_1, h_2} \mathcal{D}g \; e^{i\, S_{\mathcal{M}}(g)}$$

sum over spacetime 4-geometries

probability amplitude for each "history" (4-geometry), depending on GR action (or modified one)

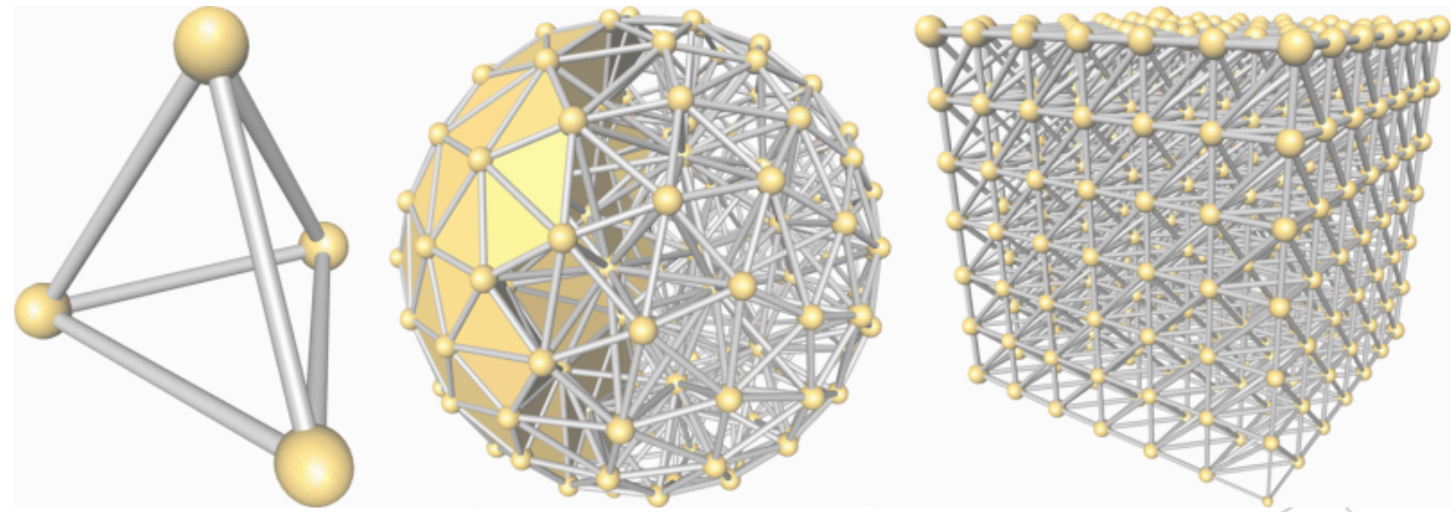


Hawking, Hartle, Teitelboim, Halliwell,… (1978-1991, …): Euclidean continuation, covariant (no-boundary) definition of "wave function of the universe", relation to canonical theory, implementation of diffeomorphism symmetry, covariant quantum cosmology, lots of semi-classical applications, ……..

too ill-defined at mathematical level to be solid approach to QG (beyond semi-classical or formal analyses)

modern versions

Lattice Quantum Gravity

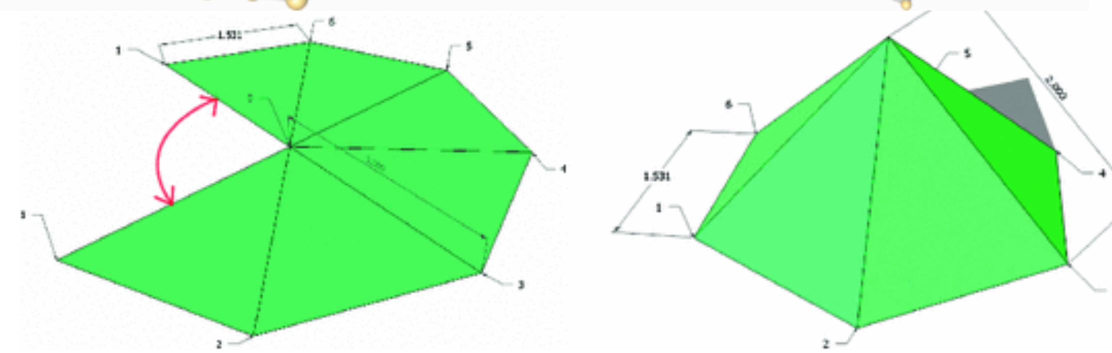


Basic idea: covariant quantisation of gravity as sum over “discrete geometries”

Continuum spacetime manifold replaced by simplicial lattice; metric data encoded in edge lengths

Gravitational action is discretised version of Einstein-Hilbert action (Regge action)

$$S_R = \frac{1}{8\pi}\sum_t A_t \varepsilon_t$$



Quantum Regge calculus

T. Regge, R. Williams, H. Hamber, B. Dittrich, B. Bahr, ….

Path integral of discrete geometries:

fixed simplicial lattice, sum over edge length variables

continuum limit via lattice refinement

$$Z = lim_{\Delta\to\infty} \int d\mu(\{L_e\})\, e^{-S_R^{\Delta}(\{L_e\})}$$

(Causal) Dynamical Triangulations

J. Ambjorn, J. Jurkiewicz, R. Loll, D. Benedetti, A. Goerlich, T. Budd, …

Path integral of discrete geometries:

sum over all possible (causal) simplicial lattices (fixed topology), fixed edge lengths

continuum limit via sum over finer and finer lattices

$$Z = lim_{a\to 0} \sum_{\Delta} \mu(a,\Delta)\, e^{-S_R^{\Delta}(\{L_e=a\})}$$

Loop Quantum Gravity (and spin foam models)

A. Ashtekar, C. Rovelli, L. Smolin, T. Thiemann, J. Lewandowski, J. Pullin, H. Sahlmann, B. Dittrich, ...

Canonical quantization of GR as gauge theory (connection variables): $(A_a^i \ , \quad E_i^b = \frac{1}{\gamma}\sqrt{e}\, e_i^b)$

choose foliation, identify phase space variables, impose dynamics + quantize

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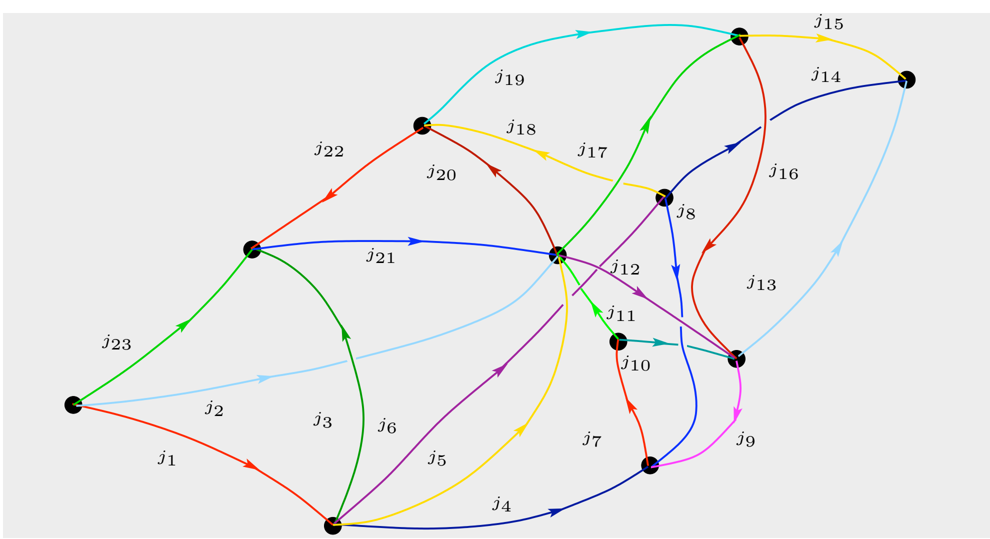
quantum states of “space” are graphs labeled by algebraic (group-theoretic) data: spin networks

kinematical Hilbert space of quantum states: $\mathcal{H} = \lim_{\gamma} \frac{\bigcup_\gamma \mathcal{H}_\gamma}{\approx} = L^2\left(\bar{\mathcal{A}}\right)$

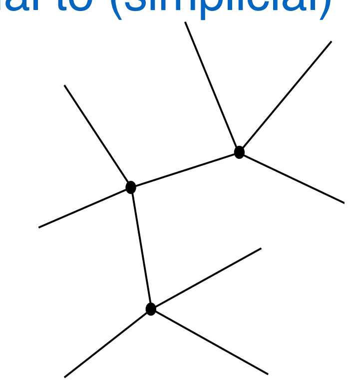
$$\mathcal{H}_\gamma = L^2\left(G^E/G^V, d\mu = \prod_{e=1}^{E} d\mu_e^{Haar}\right)$$

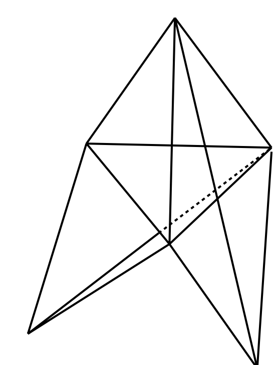
G= SU(2)



spin networks can be understood as (generalised) piecewise-flat discrete geometries

underlying graphs are dual to (simplicial) lattices





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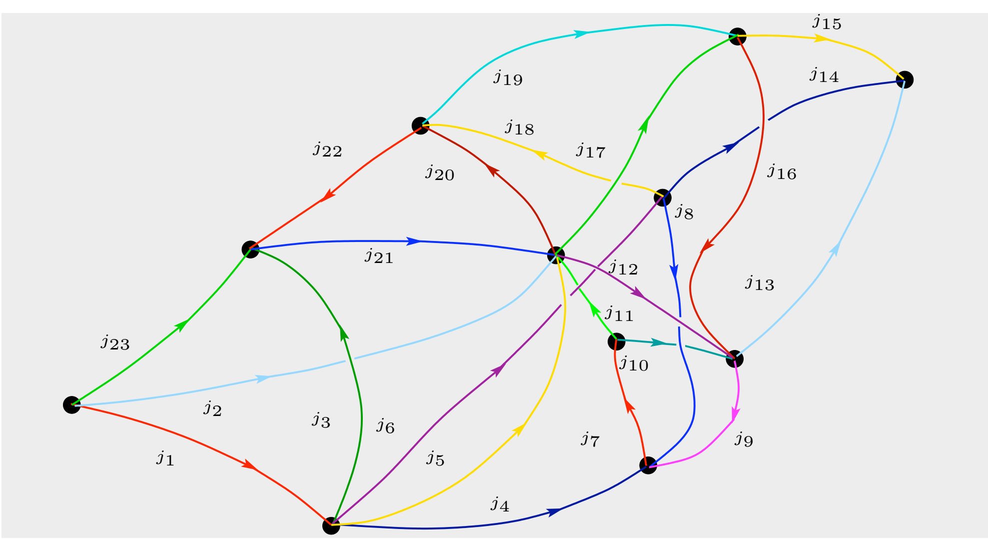

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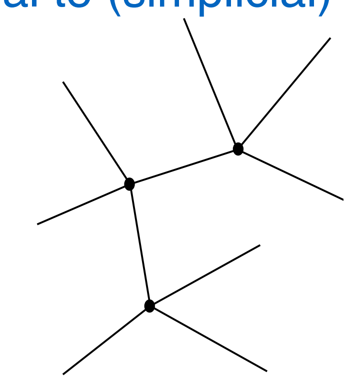

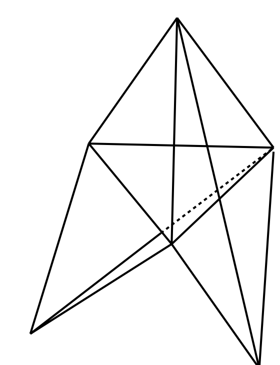

Geometric observables correspond to operators; some of them have discrete spectrum: discretization of quantum geometry! (Rovelli, Smolin, Ashtekar, Lewandowski, 1995-1997)

$$\widehat{A}\, |\bigcirc_j\rangle = 8\pi\beta l_p^2 \sqrt{j(j+1)}\, |\bigcirc_j\rangle$$

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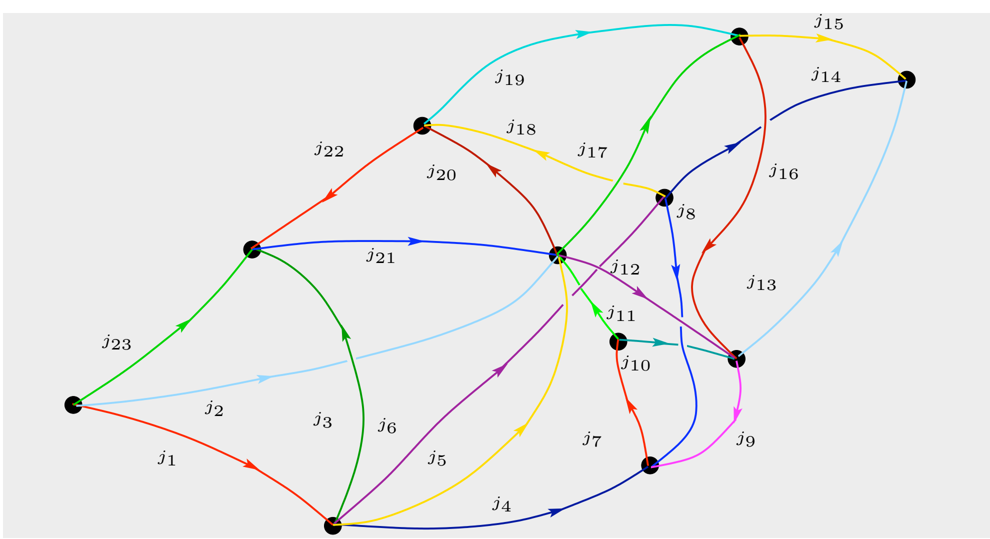

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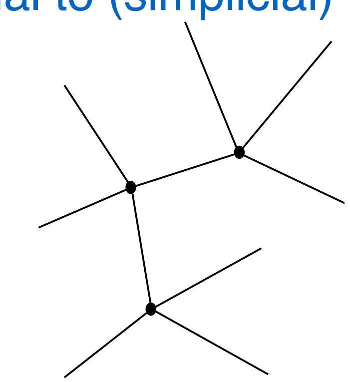

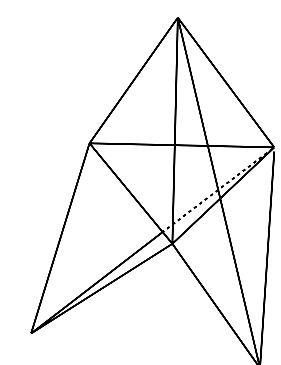

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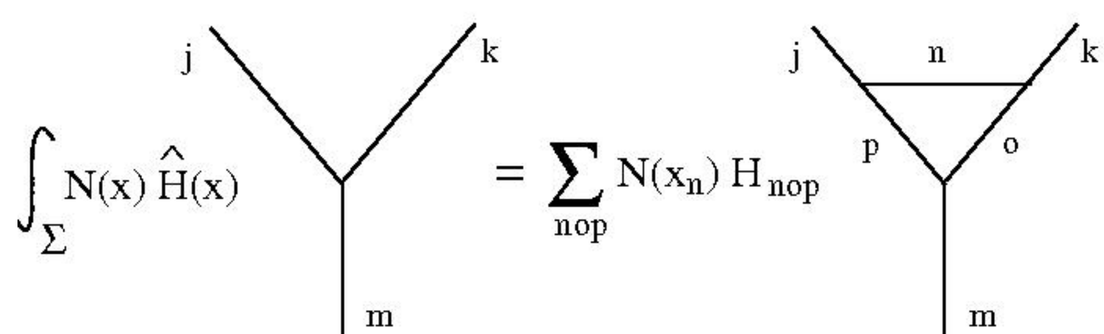
complicated combinatorial+algebraic "dynamics"
(action of Hamiltonian constraint):

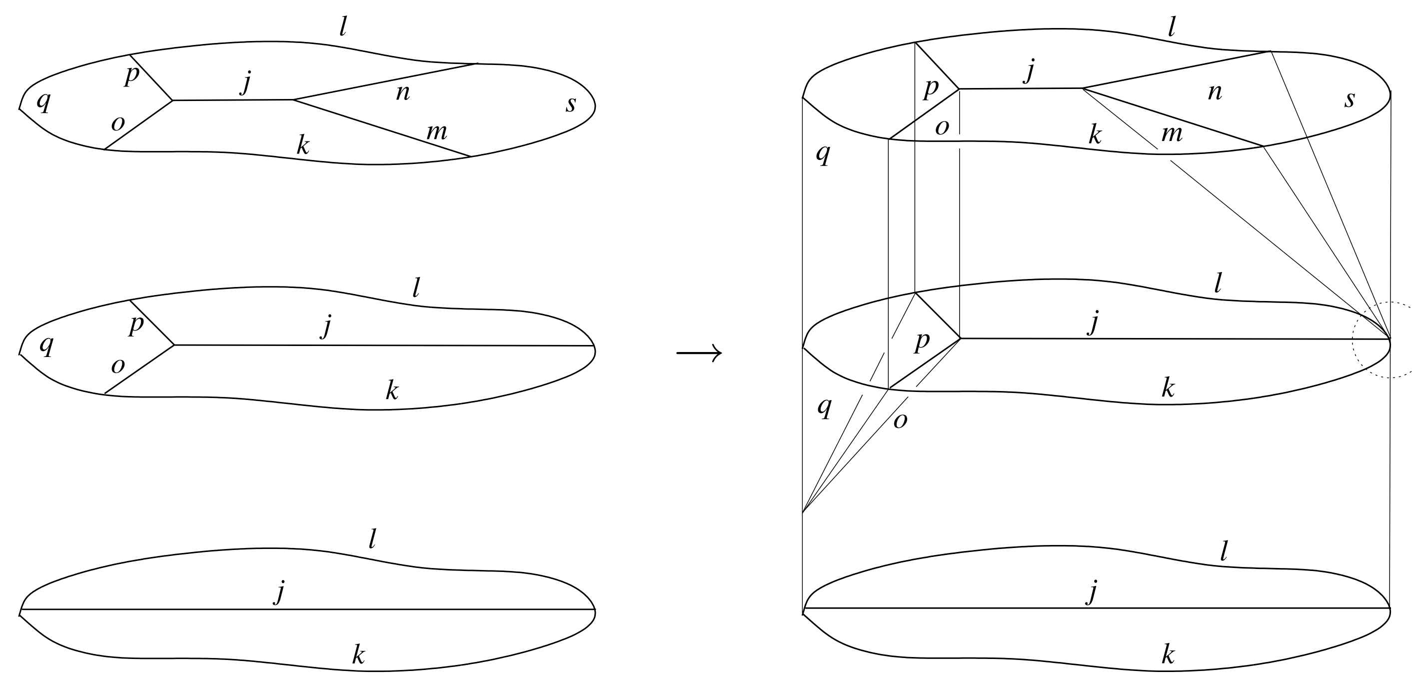


Loop Quantum Gravity (and spin foam models)

M. Reisenberger, C. Rovelli, J. Baez, J. Barrett, L. Crane, A. Perez, E. Livine, DO, S. Speziale, ……

evolution of spin networks involves changes in combinatorial structure and in algebraic labels

“histories” (dynamical interaction processes) are also purely algebraic and combinatorial: spin foams



purely algebraic and combinatorial
“path integral for quantum gravity”

$$\langle \Psi_\gamma(j,i)\,|\,\Psi_{\gamma'}(j',i')\rangle \;=\; \sum_{\Gamma|\gamma,\gamma'} w(\Gamma) \sum_{\{J\},\{I\}|j,j',i,i'} \mathcal{A}_\Gamma\,(J,I) \qquad \approx \;\text{“}\int \mathcal{D}g\, e^{i\,S(g)}\,\text{”}$$

spin networks/spin foams can be understood as (generalised) piecewise-flat discrete geometries

the underlying graphs and 2-complexes are dual to (simplicial) lattices

Lots of results on quantum geometry and mathematics of quantum gravitational field; inspiring models of quantum black holes and quantum cosmology

Group field theories

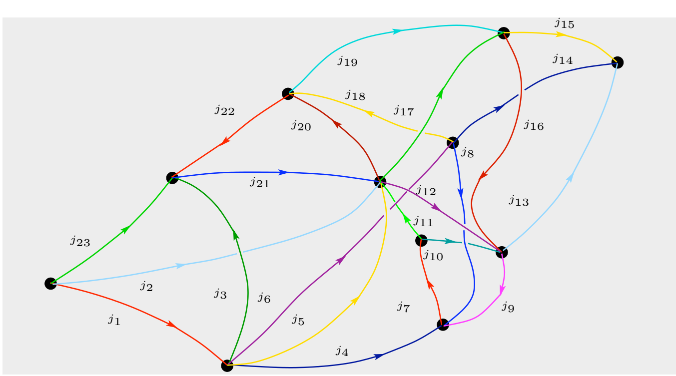
(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

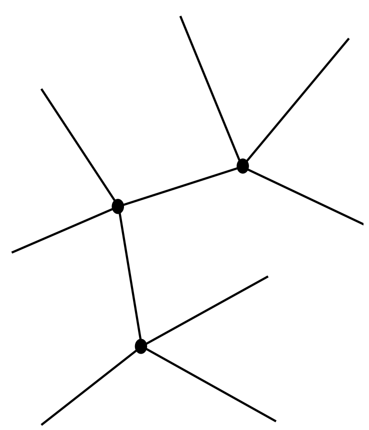
Quantum field theories over group G (enriching tensor models with group data) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

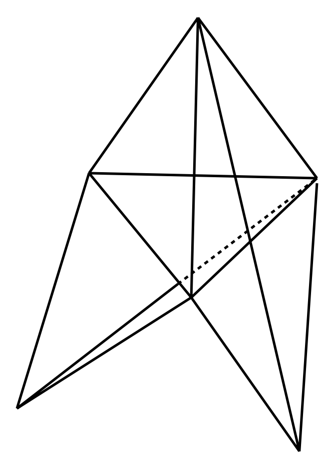
for gravity models, G = local gauge group of gravity (e.g. Lorentz group)

$$S(\varphi, \overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia}, \bar{g}_{iD}) \quad + \quad c.c.$$

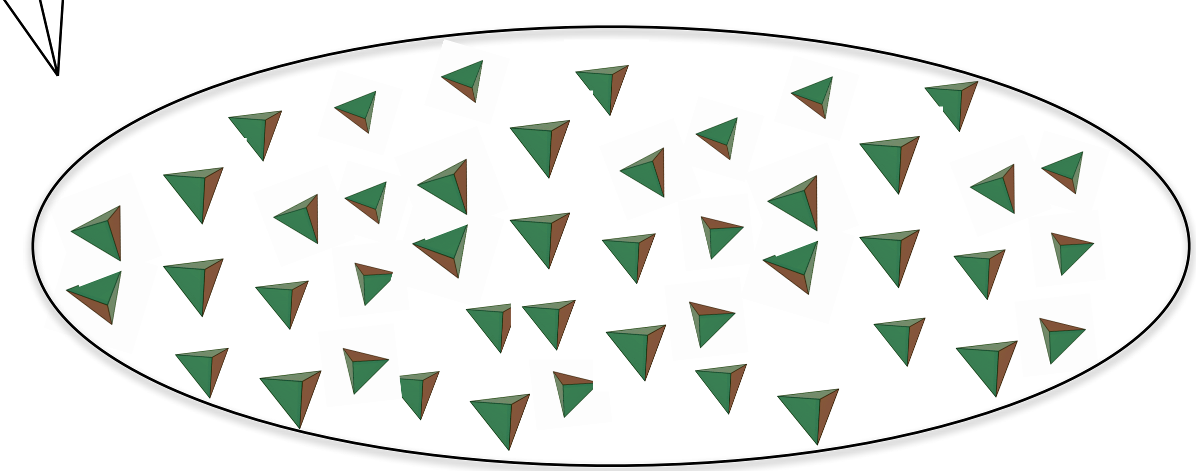
single field "quantum": spin network vertex
or tetrahedron







generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)



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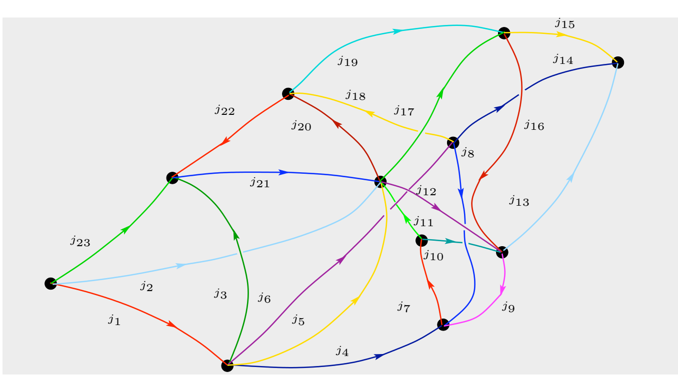
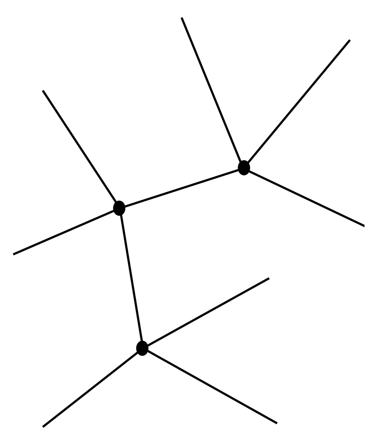
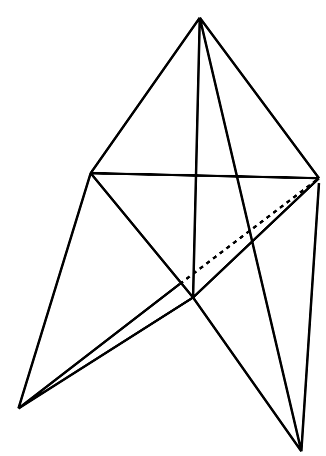
quantum states are 2nd quantised spin networks/simplices







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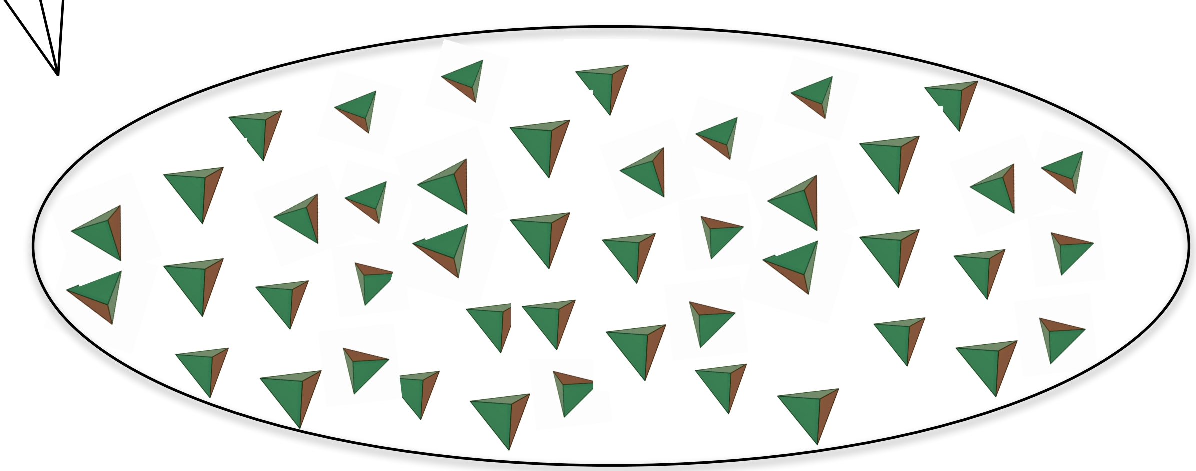

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Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

(simplicial case: simplicial complexes obtained by gluing d-simplices)

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Feynman amplitudes (model-dependent):

equivalently:

- spin foam models (sum-over-histories of spin networks ~ covariant LQG)

  Reisenberger,Rovelli, '00
- lattice path integrals (with group+Lie algebra variables)

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Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

Feynman perturbative expansion around trivial vacuum

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

(simplicial case: simplicial complexes obtained by gluing d-simplices)

Feynman amplitudes (model-dependent):

equivalently:

- spin foam models (sum-over-histories of spin networks ~ covariant LQG)

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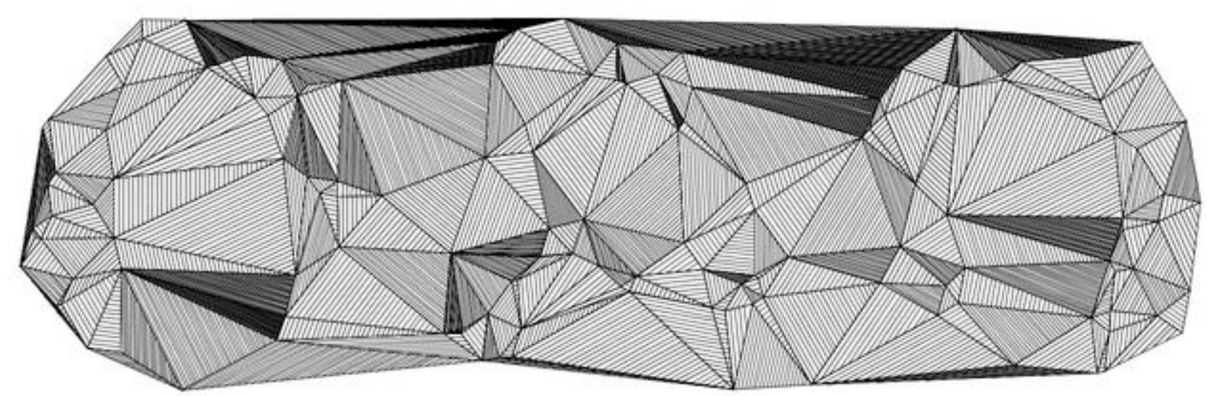

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GFT as lattice quantum gravity:

$\boxed{\text{dynamical triangulations}}$ + $\boxed{\text{quantum Regge calculus}}$

....and more……

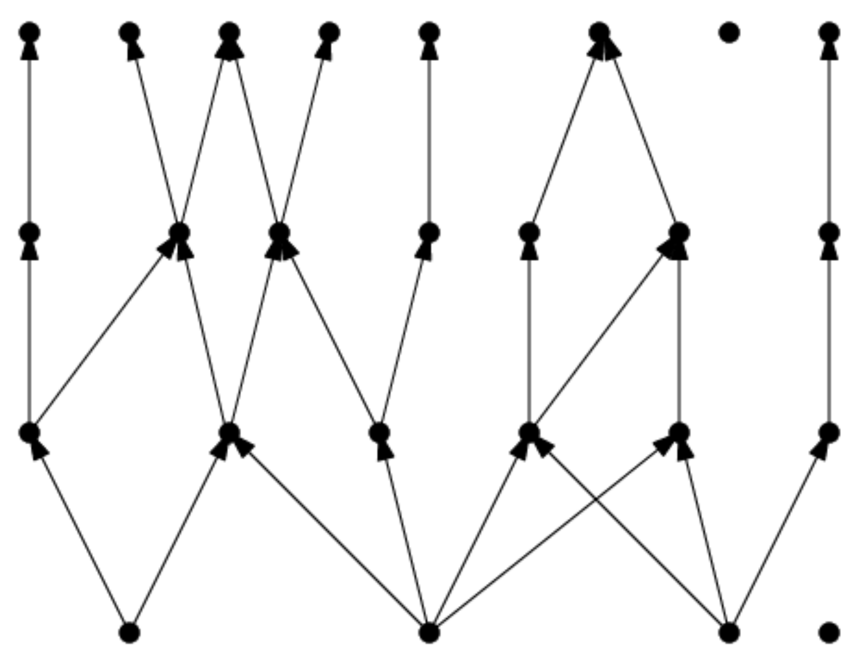
there are quite a few other quantum gravity approaches, with different goals
and different levels of development

non-commutative geometry

algebras of functions (incl. coordinate functions) on spacetime are central object; they are turned into non-commutative algebras, thus “non-commutative spacetime and geometry”; 2 subdirections: Connes’ spectral triple (based on Dirac operator; possible route to unification) and “quantum spacetimes” (based on Hopf algebra symmetries, basis of much phenomenology); difficult to turn on dynamics of geometry and spacetime itself

causal set theory

intrinsically discrete sub-structure for spacetime, given by fundamental causal relations between finite set of “events”, giving a “partially ordered, locally finite set”. quantum dynamics defined ideally by “sum-over-causets” weighted by quantum amplitude; continuum spacetime should emerge from this sum, as approximation



asymptotic safety, quantum graphity, twistor theory, ….

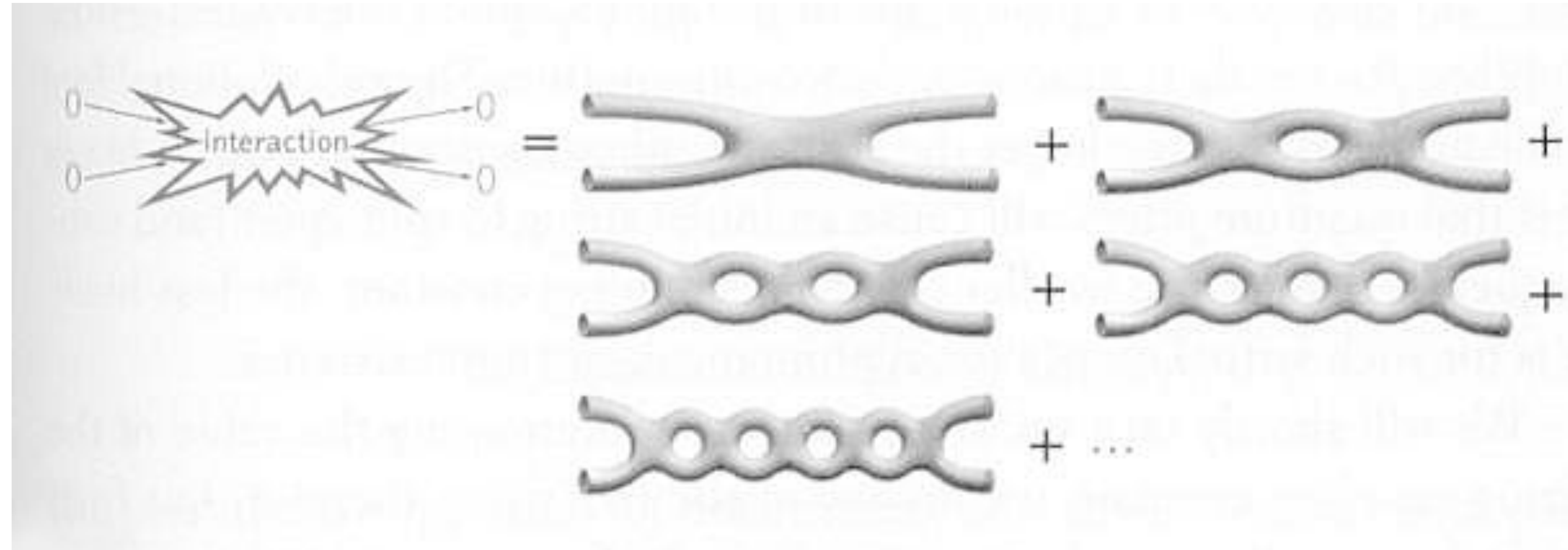
not going to discuss them here…..

new dofs/symm

--> string theory

extend fields/particles to strings/string vibration modes

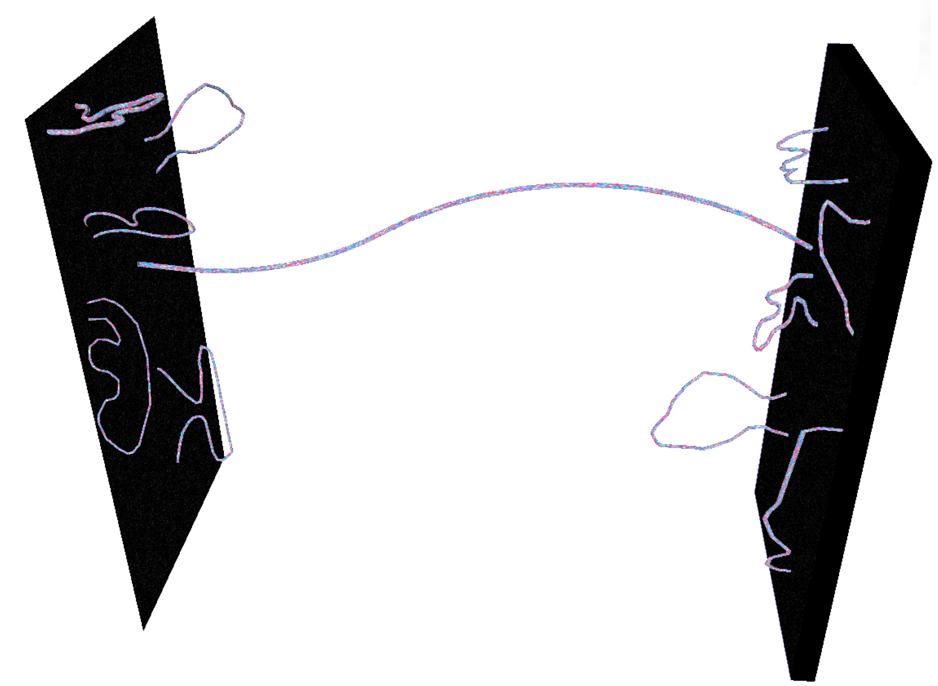
need supersymmetry



string excitations: particles of any spin/mass;
incl. graviton = quantum of gravitational field

consistent (around flat space) and finite perturbation theory in 10d

background spacetime satisfies GR equations



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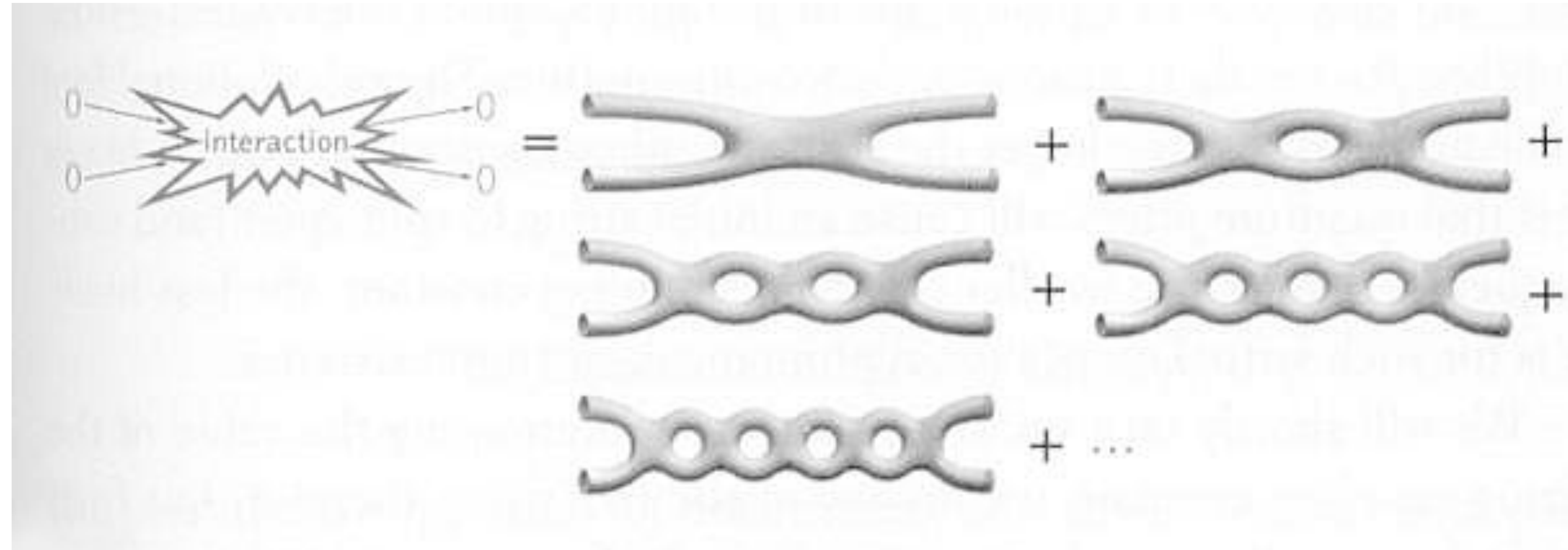

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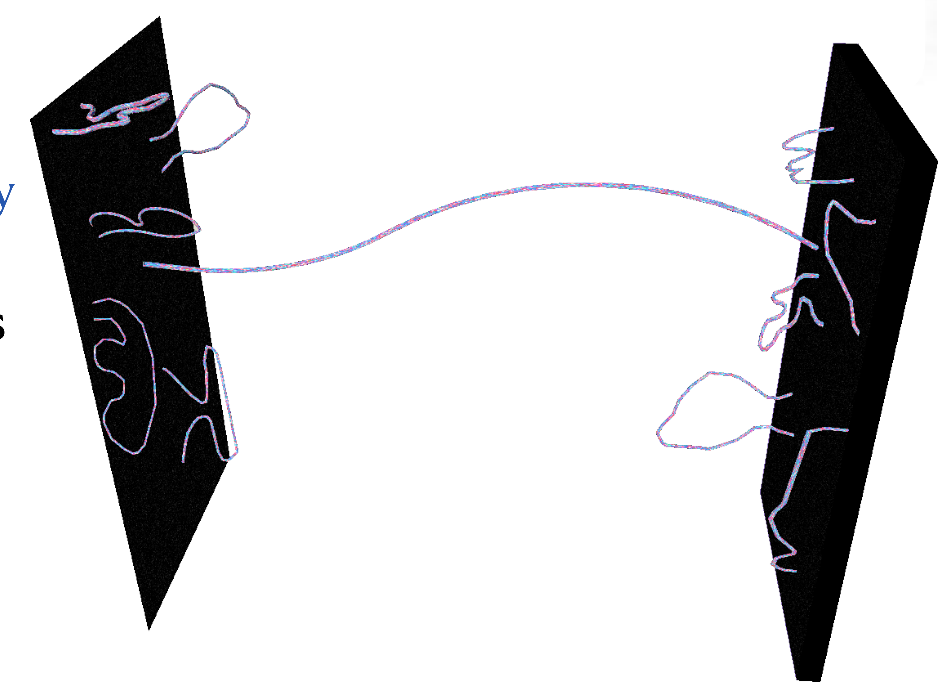
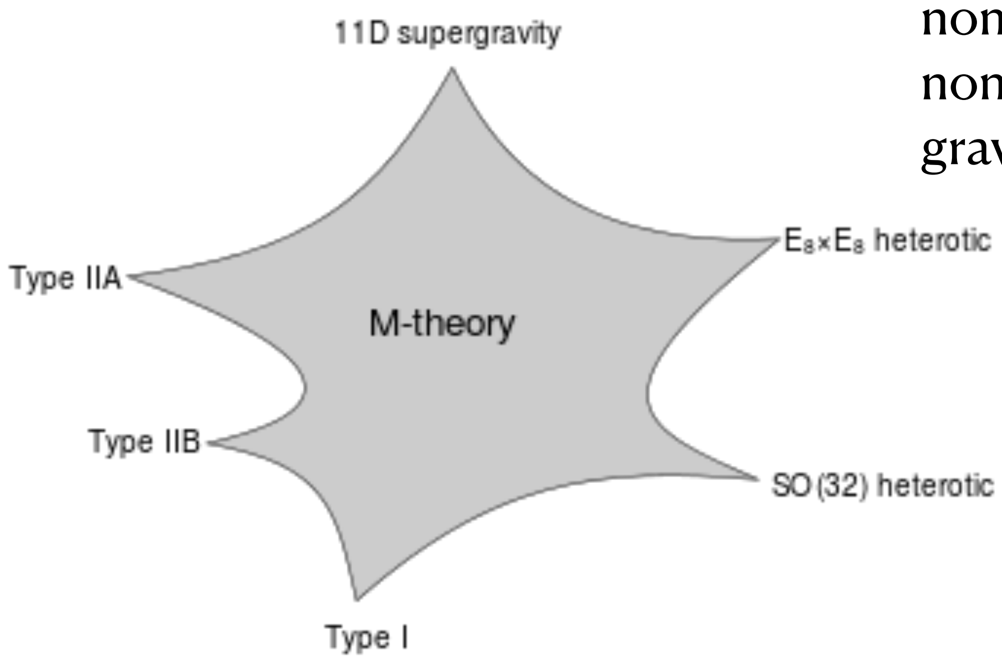
- need non-perturbative extension, including new dofs and symmetries, anyway





non-perturbative extension includes non-perturbative quantization of gravity/spacetime/geometry

need to make sense of it like in "quantum GR" approaches

new dofs/symmetries may help

third option:

"Quantum Gravity is NOT quantum GR"

-

emergent spacetime, emergent gravity

Other things we learned

spacetime thermodynamics

BH thermodynamics generalised to cosmological horizons, similar for surfaces in flat space (Unruh effect)

is any (region of) spacetime a thermodynamic. system?

Einstein's equations as equation of state

T. Jacobson (1995), ….., T. Padmanabhan, ……

$$\delta Q = T dS$$

GR dynamics is effective equation of state for any microscopic dofs collectively described by a spacetime, a metric and some matter fields

IDEA **geometric entropy functional** + **local matter-energy perturbations** => **Einstein eq. as equation of state** $G(g) \propto T(\varphi, g)$

crucial: "holographic" behaviour

$$\delta S = \alpha\, \delta A$$

analogue gravity in condensed matter systems

C. Barcelo, S. Liberati, M. Visser, '05

effective curved metric (from background fluid) and quantum matter fields (excitations over fluid) from non-geometric atomic theory (quantum liquids, optical systems, ordinary fluids, …)

Unruh, Parentani, Visser, Weinfurtner, Jacobson, … (1981-…)

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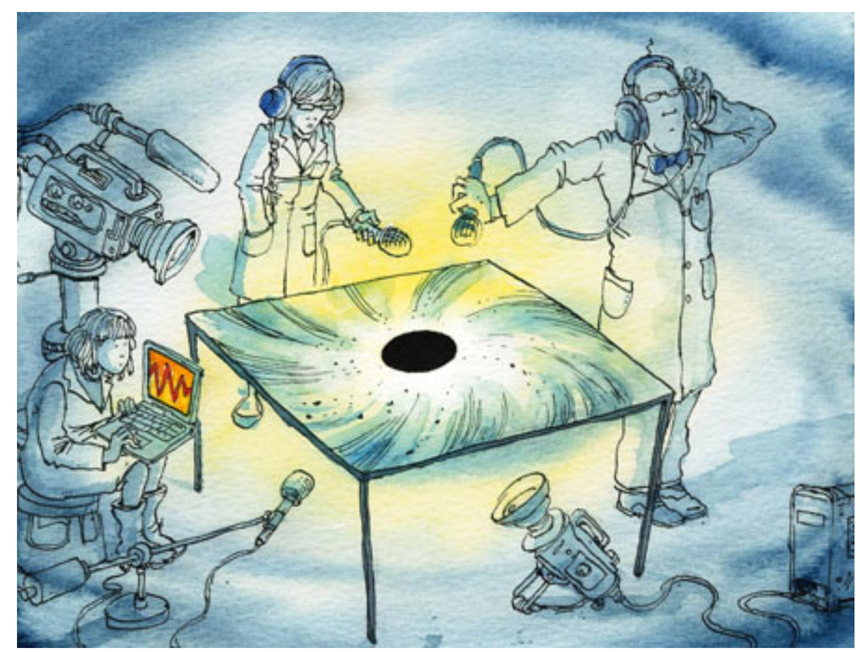

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→ **Is gravity an emergent phenomenon?**
Are spacetime and fields just collective emergent entities?

Beyond Relativistic SpaceTime - hints of more radical disappearance of spacetime itself

- challenges to “localization” in semi-classical GR (& minimal length scenarios)
- spacetime singularities in GR
- black hole thermodynamics
- Einstein’s equations as equation of state

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C. Wuetrich, 2017; DO, 2013; DO, 2018

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C. Wuetrich, 2017; DO, 2013; DO, 2018

idea further supported by results from contemporary QG formalisms, which have identified candidate non-spatiotemporal building blocks and/or further hints that spatiotemporal structures are not fundamental

DO, 2017, 2018

third option: replace QFT and GR structures by "something else" - emergent spacetime and geometry

is spacetime itself “emergent” from non-spatiotemporal, non-geometric, quantum building blocks (“atoms of space”)?

- new dofs ("atoms of space", no spacetime, not "geometry + other") (and new "beyond quantum" framework)
- not just emergent gravity; flat spacetime itself would be emergent, highly excited, collective state of "QG atoms"

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e.g.

discrete gravity, spin foam models

tensor models, group field theory

causal sets

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geometry and matter (probably together) are both collective description of "something else"

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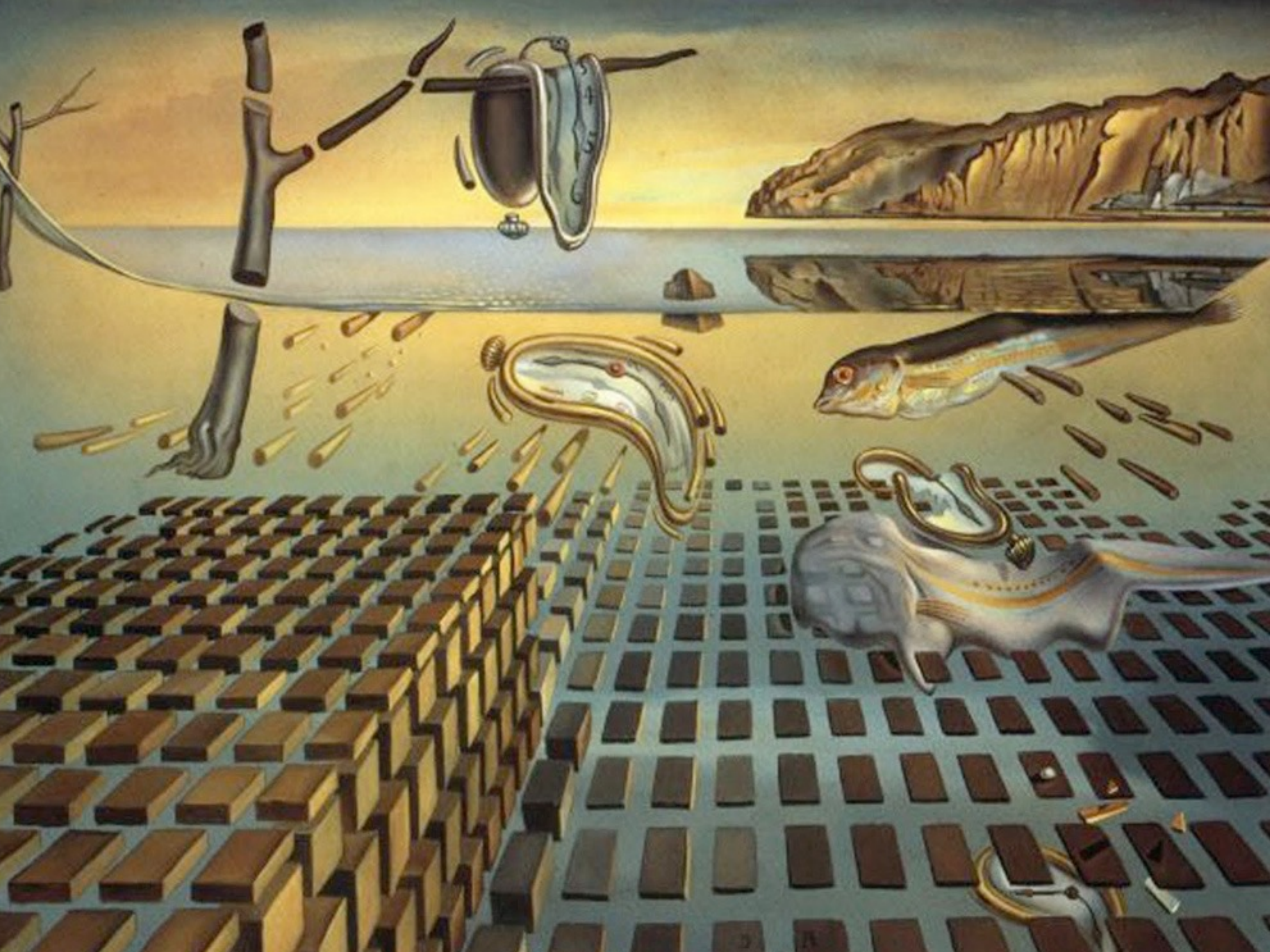
- it means abandoning (one or more) basic principles of GR and QFT: locality, unitarity, Lorentz, ...

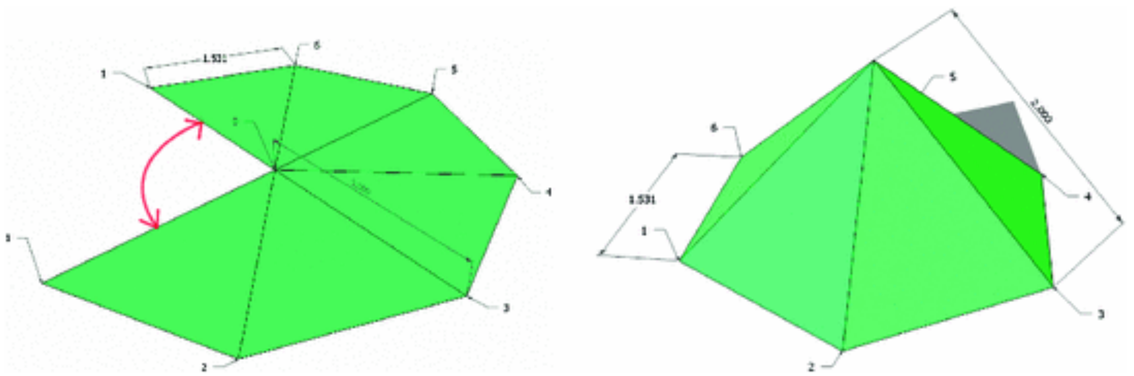


in fact, also other strategies suggest emergent spacetime/geometry scenario

- canonical LQG --> piecewise-degenerate quantum geometries encoded in combinatorial/algebraic data

(embedded) spin network states encode dofs of continuum connection/tetrad fields which are degenerate (identically vanishing) everywhere (in manifold) except on links of spin network graph

- lattice QG --> piecewise-flat quantum geometries

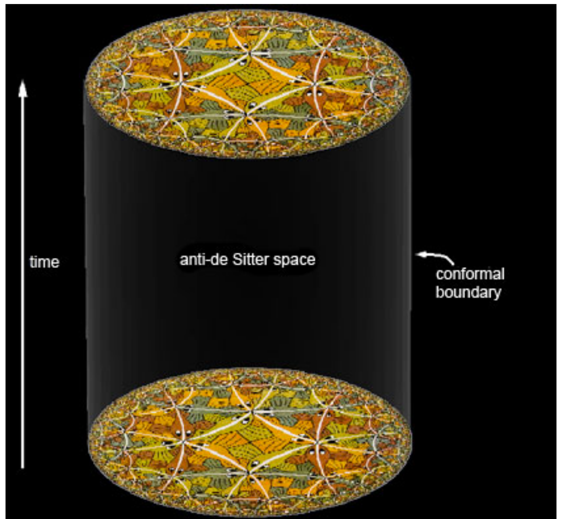


discrete gravity data on lattice are those of continuum but distributional (non-smooth) metrics which are identically flat inside simplices (curvature arise from gluing)

- string theory dualities --> non-spacetime based dofs for M-theory?

string dualities relate ST on spacetimes with different topology and dimension; suggest more fundamental description not based on spacetime at all

- AdS/CFT provides concrete example of "emergent gravity" + "emergent space"



AdS bulk: curved spacetime of dimension d

CFT: flat spacetime of dimension d-1

extra spatial direction "reconstructed" from quantum CFT dofs examples of (partial) realizations of emergent spacetime scenario

examples of (partial) realizations of emergent spacetime scenario

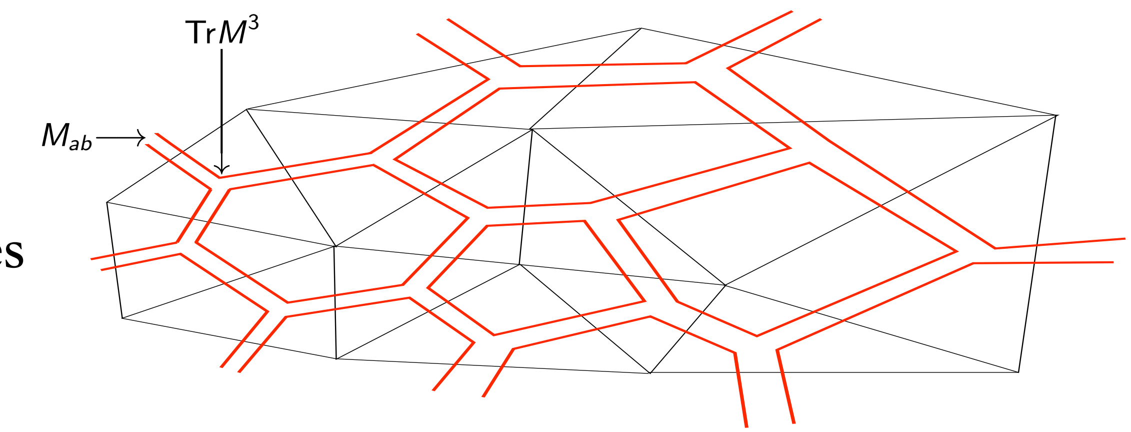
matrix models for 2d (Riemannian) QG

abstract theory of (random) NxN matrices

$$S(M) = \frac{1}{2} trM^2 - \frac{g}{\sqrt{N}} trM^3 = \frac{1}{2} M^i{}_j K^{jl}{}_{ki} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V^{jnl}{}_{mki}$$

- partition function

$$Z = \sum_\Gamma \left(\frac{g}{\sqrt{N}}\right)^{\frac{1}{2}} Z_\Gamma = \sum_\Gamma g^{V_\Gamma} N^{F_\Gamma - \frac{1}{2} V_\Gamma}$$



Feynman diagrams ~ 2d discrete surfaces

- purely combinatorial formulation of quantum dynamics

- continuum limit (large N, refined lattices, critical behaviour wrt coupling g) $\longrightarrow$ 2d Liouville QG

- SD equations for matrix n-point functions ~ Wheeler-deWitt equations (from canonical quantization of 2d gravity) examples of (partial) realizations of emergent spacetime scenario

examples of (partial) realizations of emergent spacetime scenario

group field theory

S. Gielen, DO, L. Sindoni, E. Wilson-Ewing, L. Marchetti, ...

- generalization of matrix models to d>2 + 2nd quantized formulation of LQG :
  abstract theory of tensor fields enriched with group-theoretic data

- QFT of quantum simplices - Feynman diagrams = d>2 lattices - Feyman amplitudes = lattice gravity path integrals

$$\varphi : G^{\times d} \to \mathbb{C} \qquad S(\varphi, \overline{\varphi}) = \frac{1}{2} \int [dg_i] \overline{\varphi(g_i)} \mathcal{K}(g_i) \varphi(g_i) + \frac{\lambda}{D!} \int [dg_{ia}] \varphi(g_{i1}) .... \varphi(\bar{g}_{iD}) \mathcal{V}(g_{ia}, \bar{g}_{iD}) \quad + \quad c.c.$$

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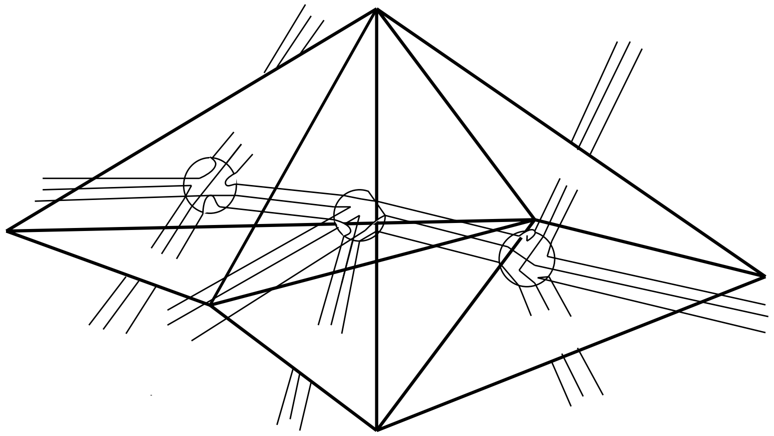

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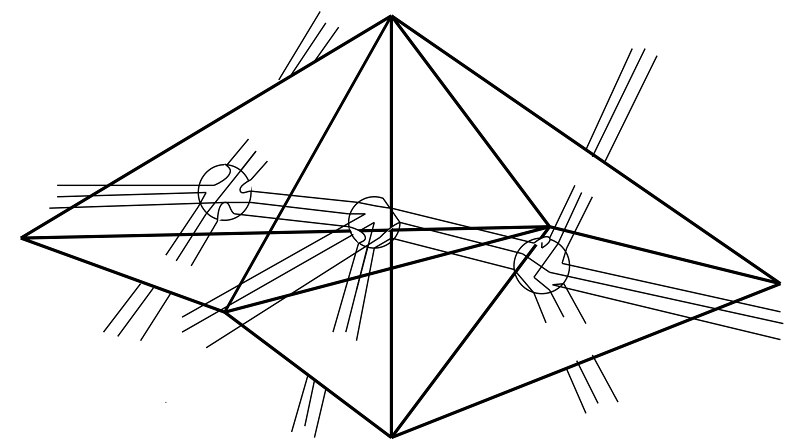
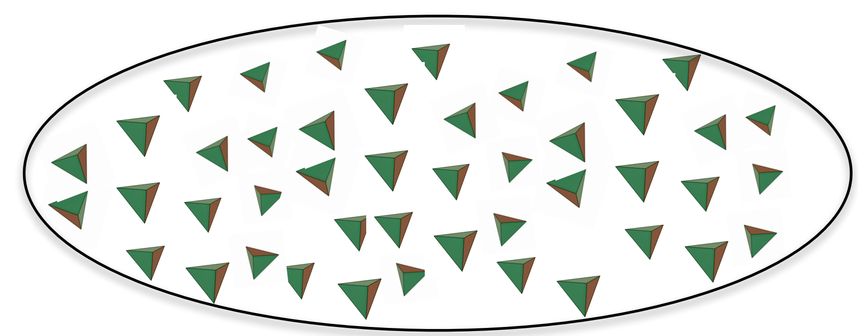


- for suitable "quantum geometric" models:
  - consider fluid of quantum tetrahedra ("universe as QG condensate")
  - mean field GFT condensate hydrodynamics ~ ~ non-linear eqn for "wavefunction" on minisuperspace (space of homogenous geom)
  - obtain effective dynamics for universe volume



$$\int [dg_i']\,\tilde{\mathcal{K}}(g_i,g_i')\sigma(g_i') + \lambda\frac{\delta\tilde{\mathcal{V}}}{\delta\varphi(g_i)}|_{\varphi\equiv\sigma} = 0$$

$$\boxed{\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j\,\rho_j\sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2}$$

$$\boxed{\frac{V''}{V} = \frac{2\sum_j V_j\left[E_j + 2m_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}}$$

- semiclassical Friedmann eqn at large volumes/late times

- quantum bounce replacing big bang singularity

Quantum Gravity:

general features

Quantum Gravity (~ quantized spacetime/geometry/gravitational field):

general structure what would it mean to have a quantum theory of geometry/spacetime?

also obtained as coarse-grained approximation in radical "emergent spacetime" QG

- a Hilbert space of quantum states $\mathcal{H} \ni |h_{ij}\rangle =$ | spatial geometry > =

= | spatial distances, curvature, volumes, ... >

- quantum geometric observables (operators acting on quantum states) $\widehat{\mathcal{O}(\hat{h},\hat{K})}\,|\Psi\rangle = |\tilde{\Psi}\rangle$ observable geometric quantities, e.g. spatial distances, volumes, curvature, ...

- a quantum dynamics

constraint operator acting on states and imposing that they correspond to admissible "spacetimes" or "spacetime geometries" (still not classical)

$$\widehat{\mathcal{D}}\,|\Psi\rangle = 0$$

counterpart of Schroedinger eqn

no time!

or

sum-over-histories allowing calculation of "transitions between geometries"

"histories of space" = "spacetimes"

$$\langle h_2|h_1\rangle = \sum_{g_{\mu\nu}|h_1,h_2} \mathcal{A}(g)$$

counterpart of Feynman's path integral

- a classical limit/approximation

to recover GR equations and classical gravitational field

"emergence of classical spacetime"

notice: classical limit is subtle even in standard QM

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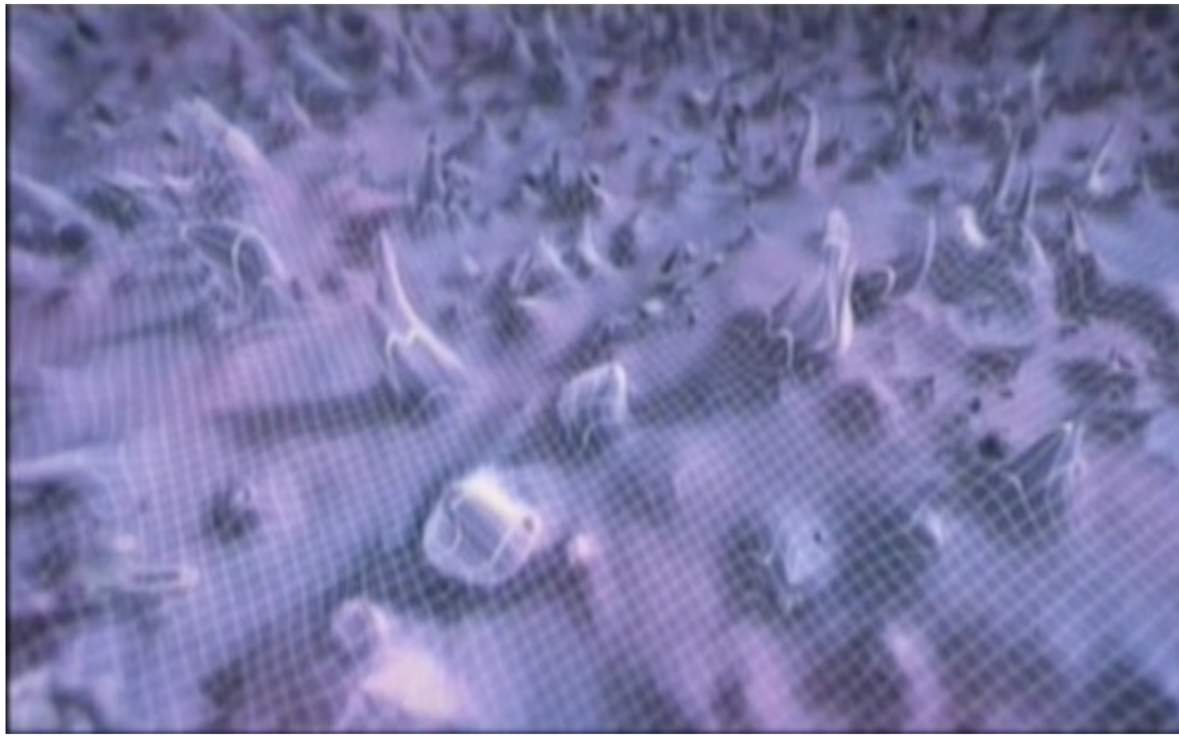
Space, Time, Geometry “fluctuate” and evolve probabilistically

quantum fluctuations of all geometric quantities (lengths, areas, time intervals, volumes, ...)

quantum fluctuations (no sharp meaning) for locality, events, ...



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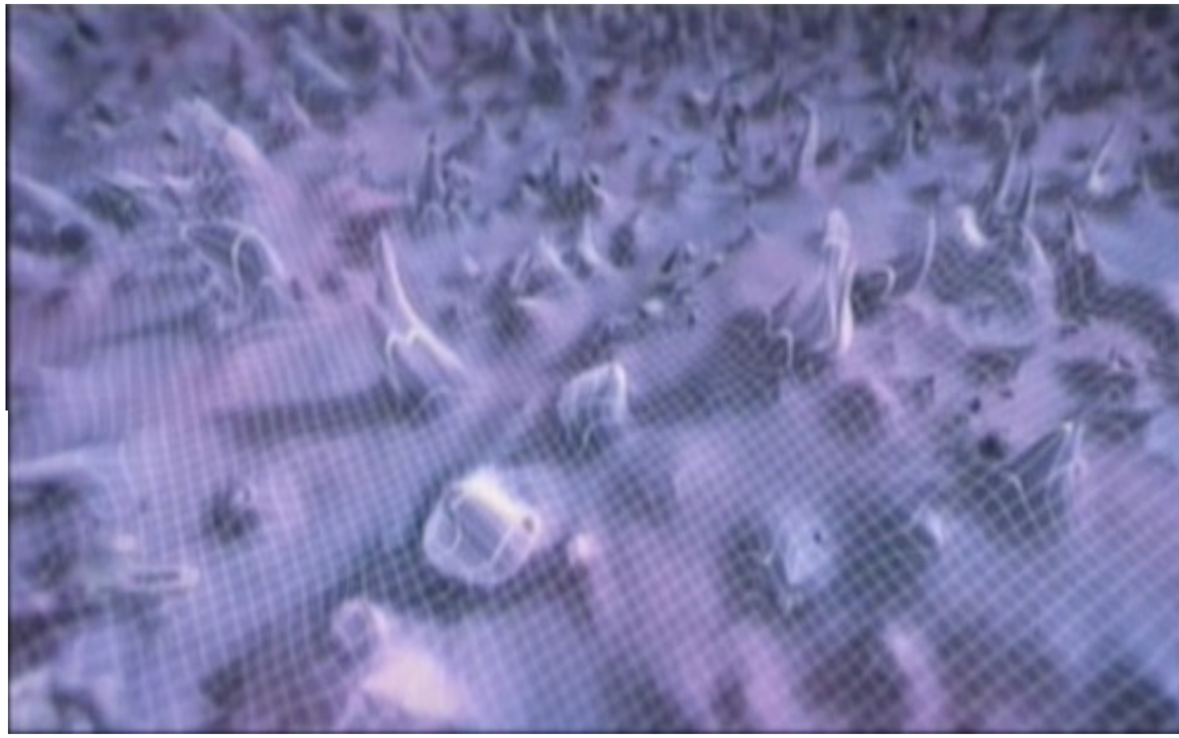

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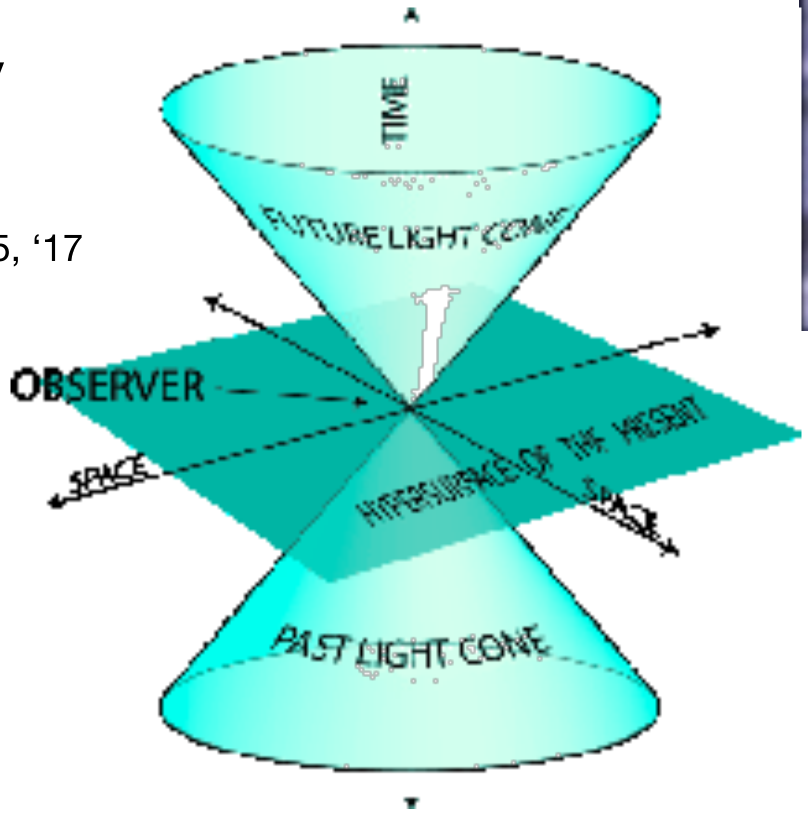
E. Castro-Ruiz, F. Giacomini, C. Brukner, '15, '17



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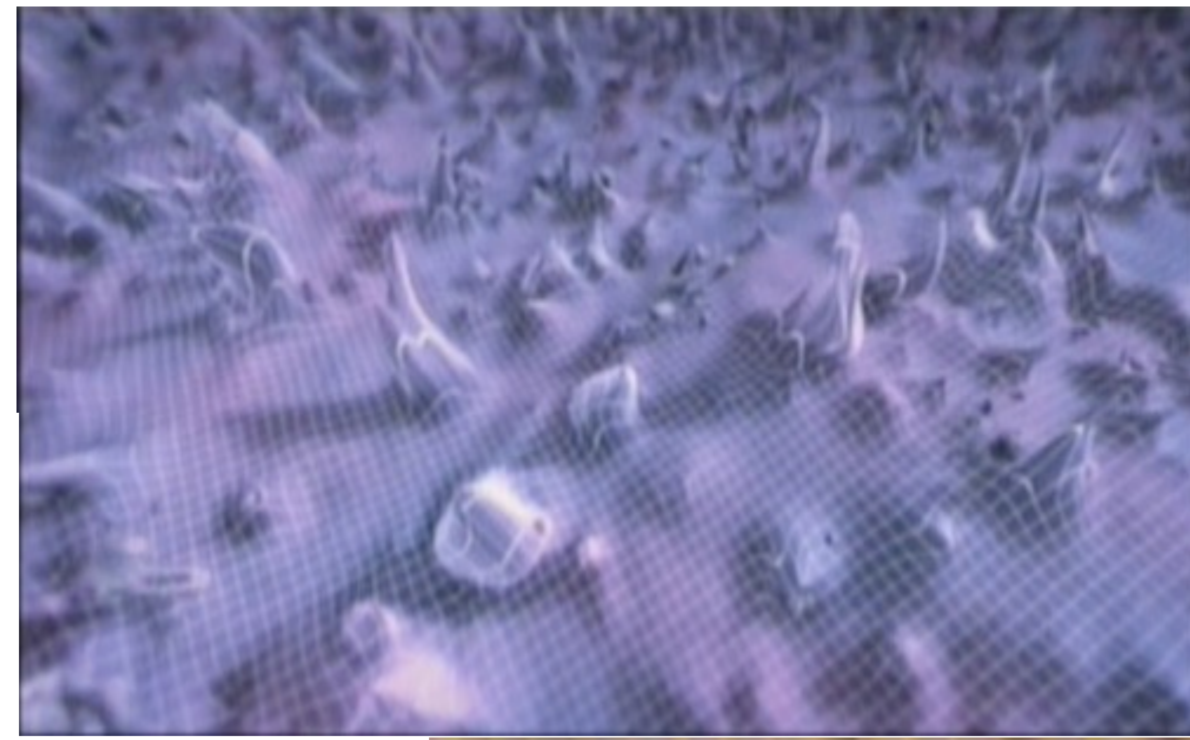

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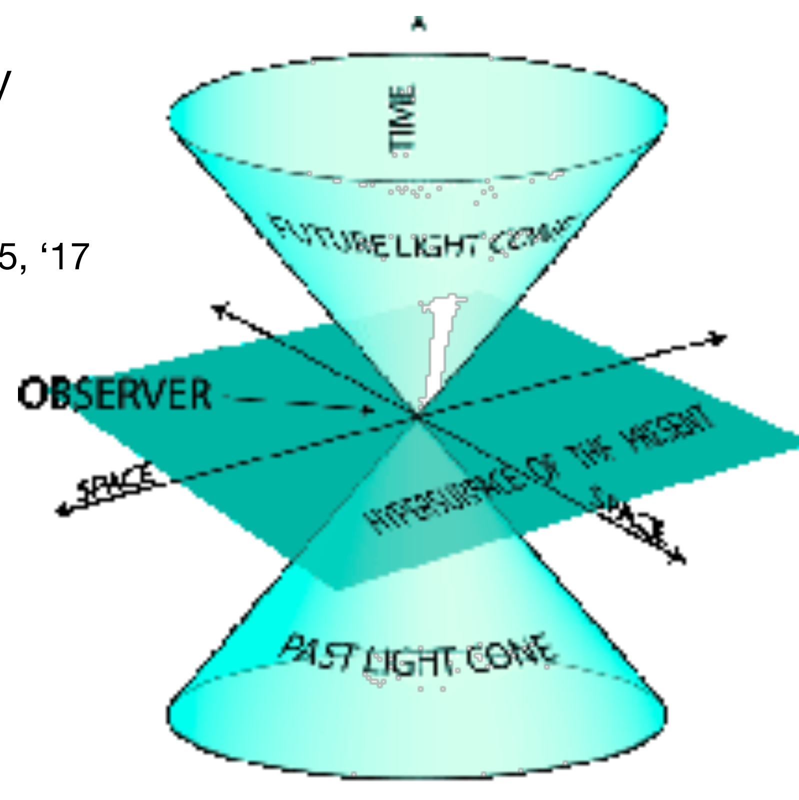
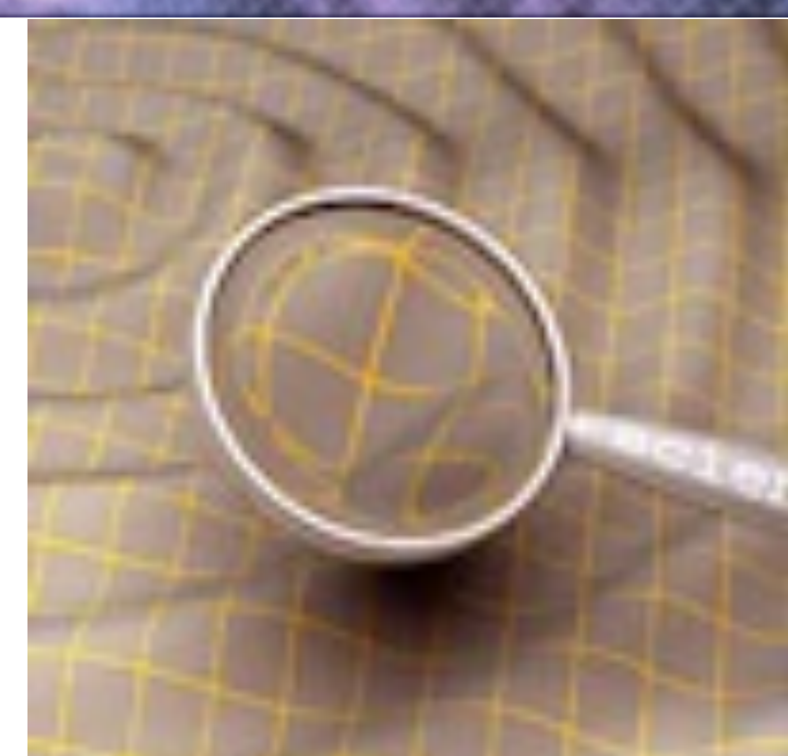


also:

geometric quantities (distances, time intervals, volumes, ….) may be discretized

C. Rovelli, L. Smolin, 1995



minimal length, volume, ..? what is left of continuum intuition?

further issues and possibilities open up in "emergent spacetime" scenarios

besides quantum effects of spacetime, we will have collective effects of "spacetime constituents"

which may manifest in new (or newly explained) spacetime features

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main conceptual point:

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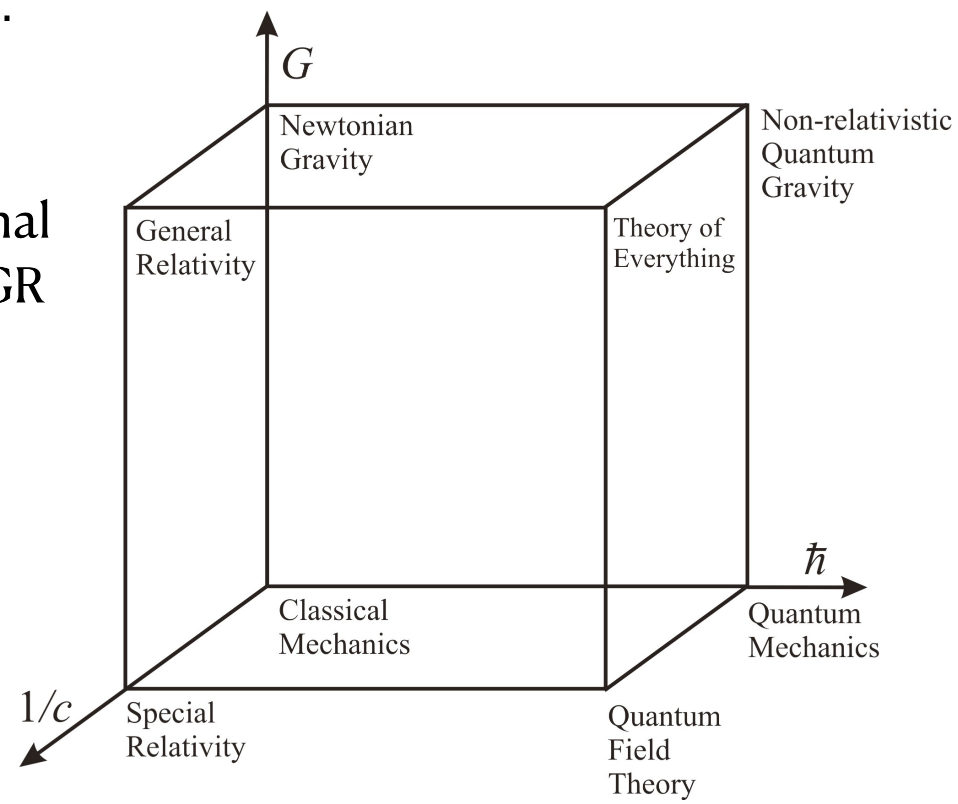
main conceptual point:

the Bronstein cube .....

corresponds to traditional view of QG = quantum GR



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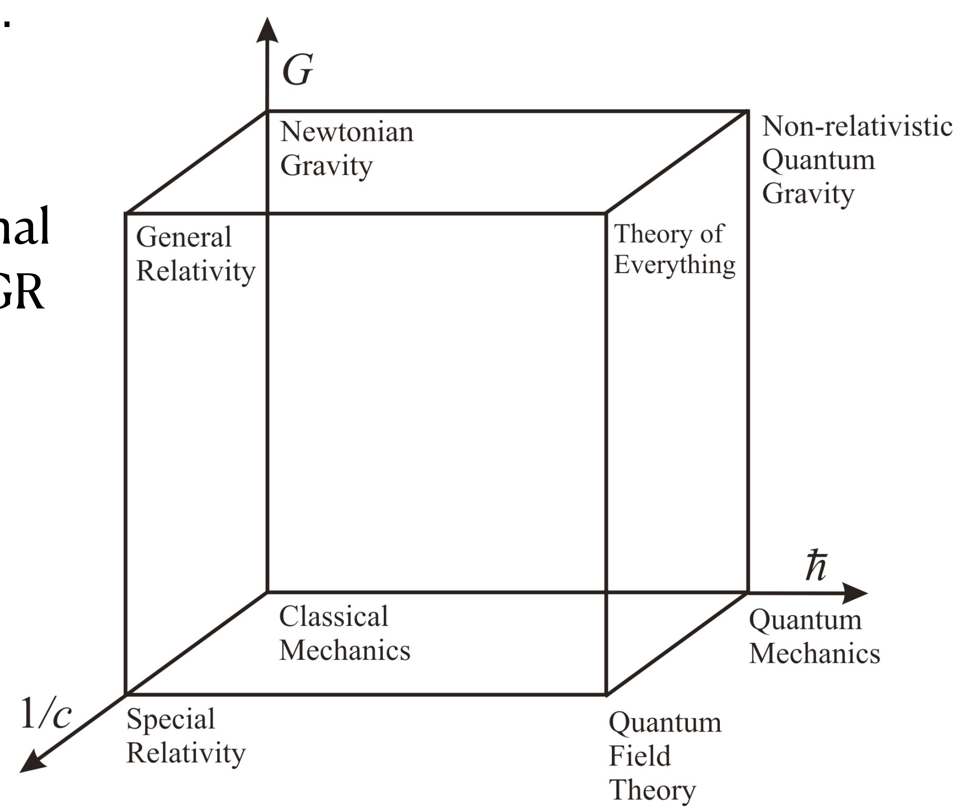
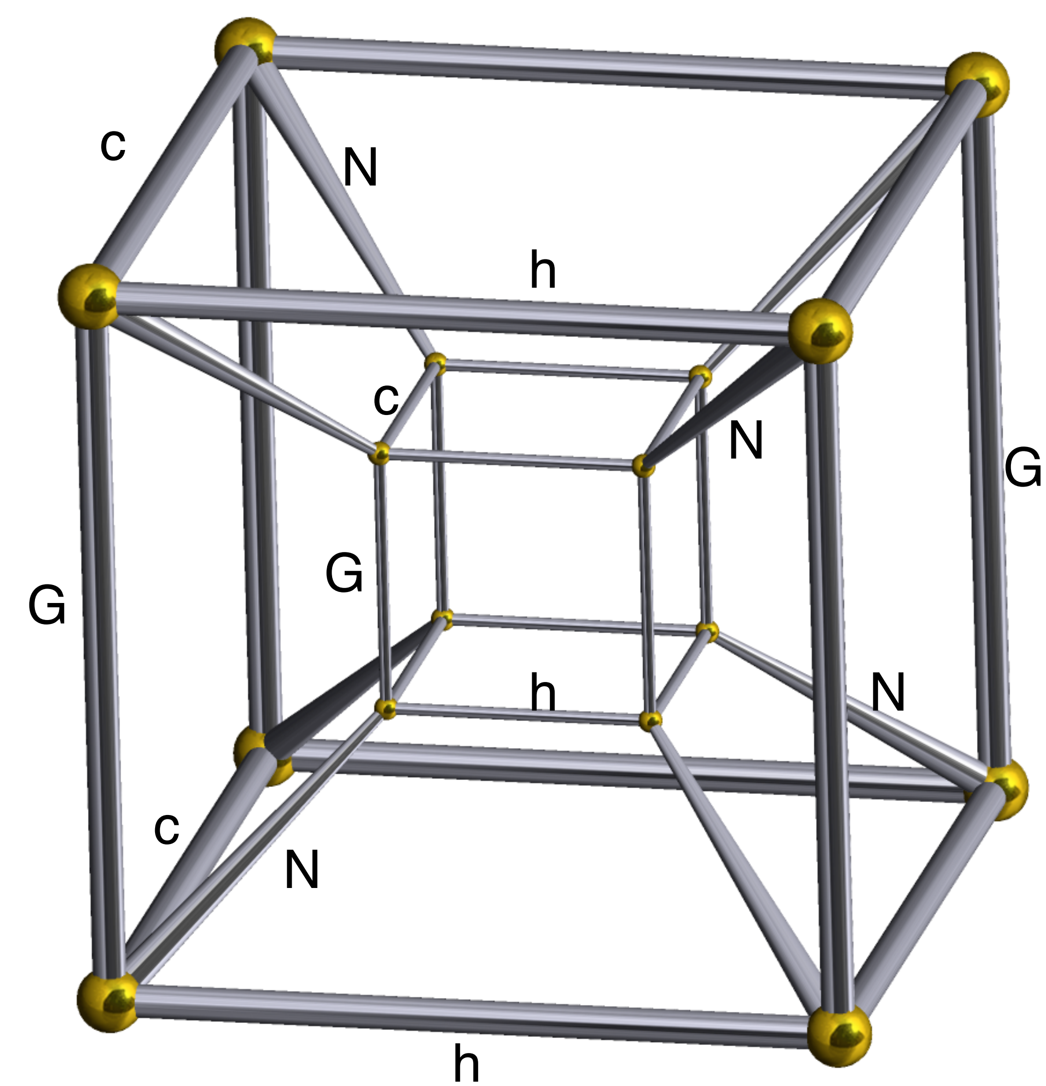
but if fundamental d.o.f.s are not smooth spacetimes (geometries) ……

the Bronstein cube …..

corresponds to traditional view of QG = quantum GR



the Bronstein hypercube of Quantum Gravity



further issues and possibilities open up in "emergent spacetime" scenarios

besides quantum effects of spacetime, we will have collective effects of "spacetime constituents"

which may manifest in new (or newly explained) spacetime features

main conceptual point:

but if fundamental d.o.f.s are not smooth spacetimes (geometries) ……

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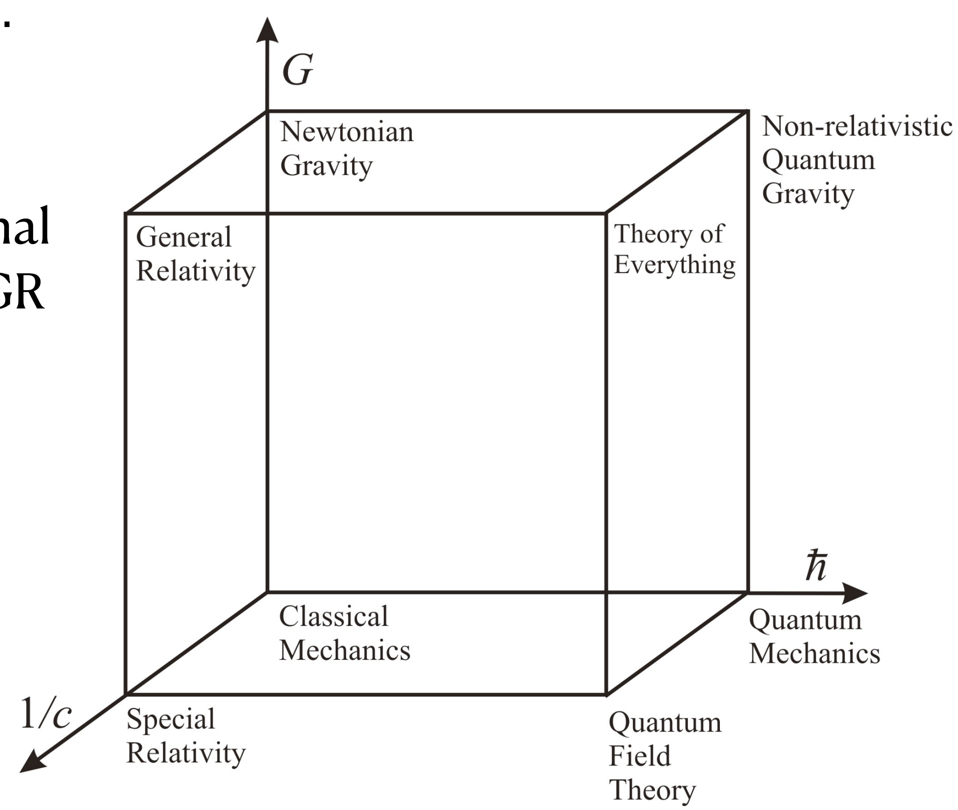

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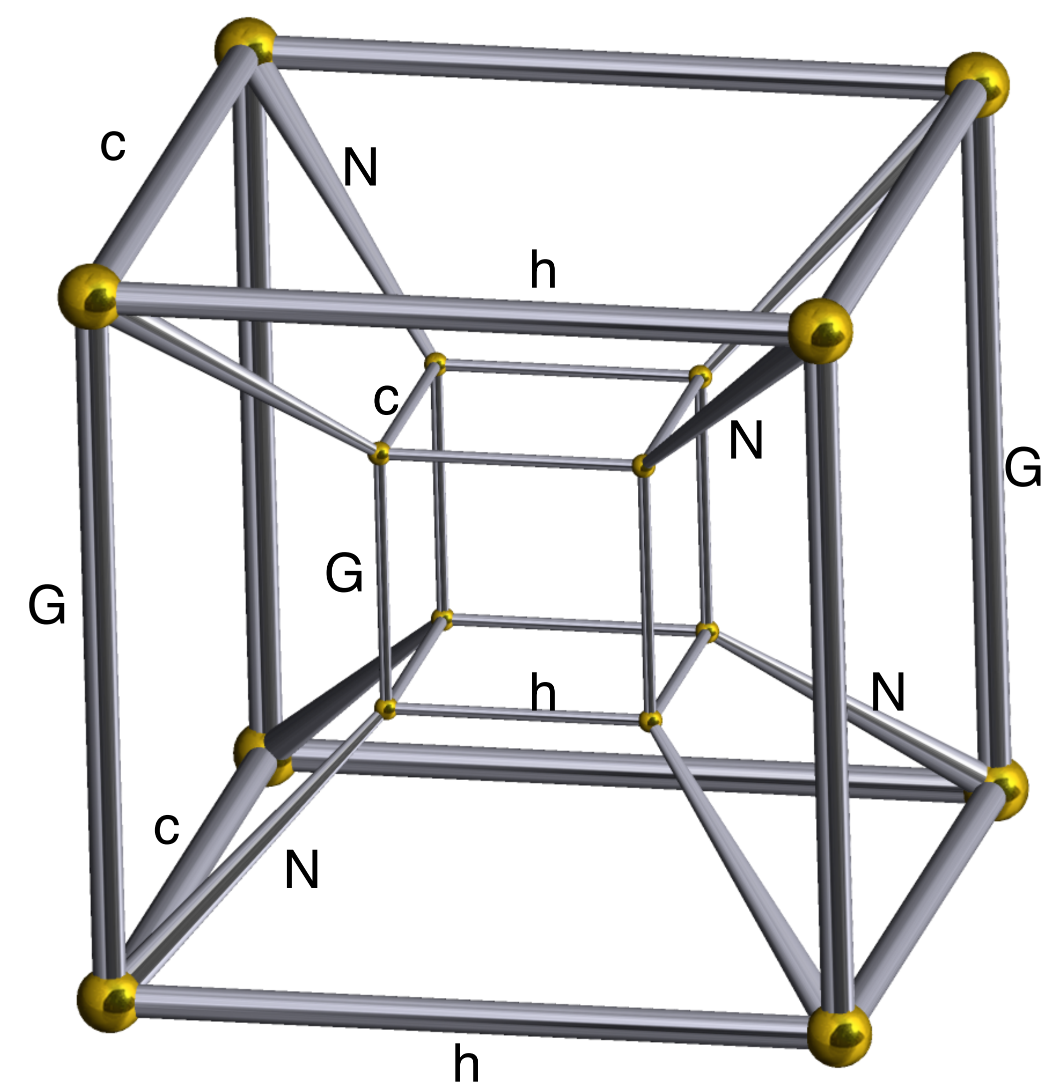
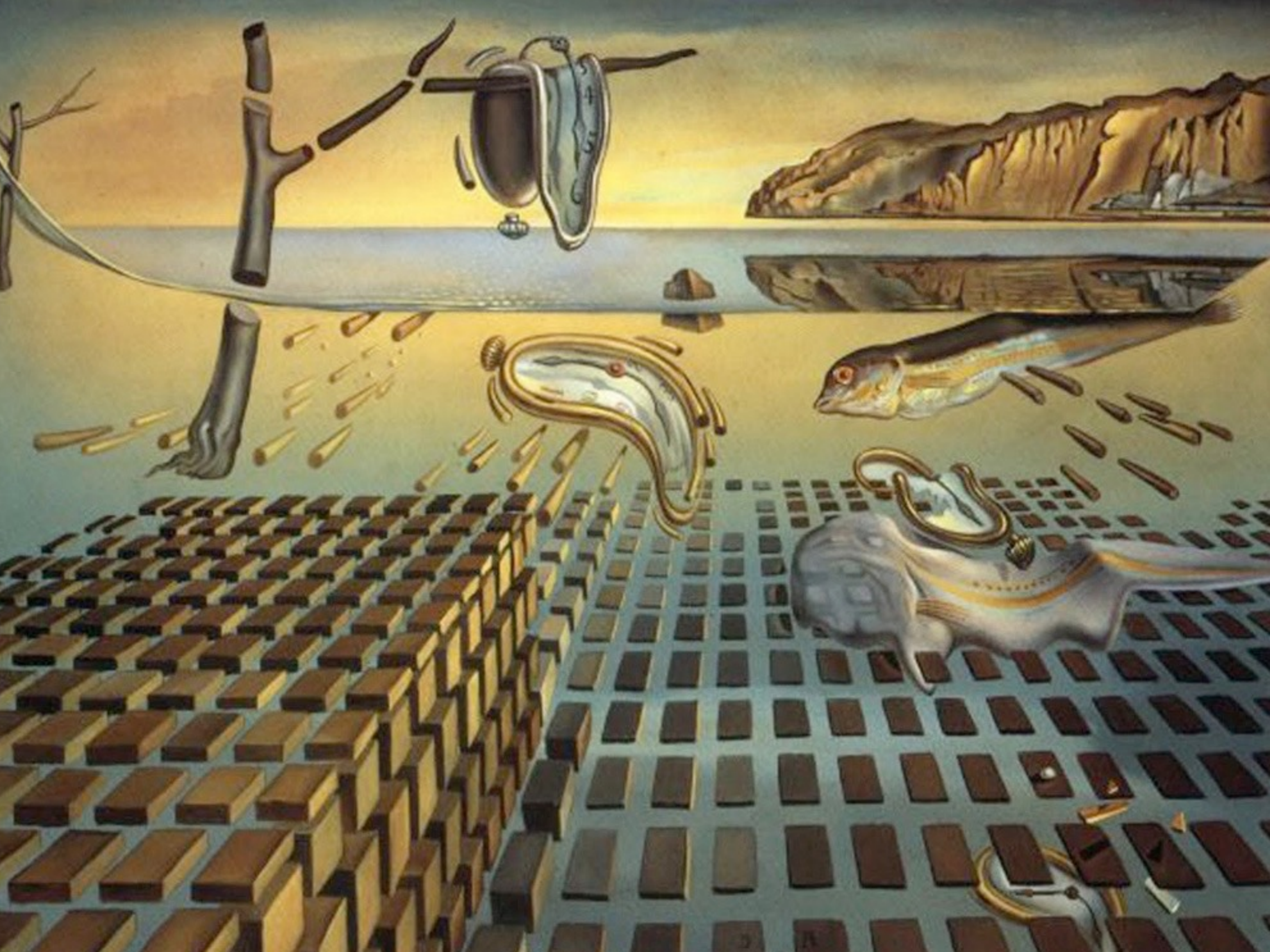


adding a new direction to our understanding of the world….
…. understanding the physics of many QG d.o.f.s

N-direction is where emergent behaviour takes place:
"More is different"



key issues of QG formalisms

identify (candidate) fundamental dofs

define and control their quantum dynamics

control their semi-classical approximation and spacetime interpretation

if fundamental entities are not directly spatiotemporal

control their collective dynamics

control their continuum approximation showing how spacetime emerges

(including phase transitions, emergent physics beyond EFT, etc)

but is it physics?

QG radically challenges the very foundations of our physical understanding of the world

- good for QG phenomenology (the more radical the change, the more room for new physics)

plethora of possible QG effects:

- purpose-built phenomenological models/scenarios trying to incorporate QG ideas
- modelling of extreme physical systems inspired by specific QG approaches
- altogether new QG ideas implemented in toy models, waiting for realization in full QG formalisms

- minimal length scenarios
- modified uncertainty principle
- symmetry violation/deformation
- regular black holes
- exotic compact objects
- QG signatures in CMB spectrum
- non-local gravity
- dissipative effects of spacetime atoms
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- danger of EFT intuition: Planck scale, separation of scales, which principles do we rely on?

in principle, Quantum Gravity from cosmological scales to Planck scale

caution: very notion of Planck scale as (only) relevant scale of QG effects results from current physics and EFT intuition, tested only up to very different scales and based on concepts that we do not expect to be fundamental

What could be the relevant scale for QG effects?

based on current theories, i.e. GR and QFT, and on straightforward QG = quantum GR: Planck scale

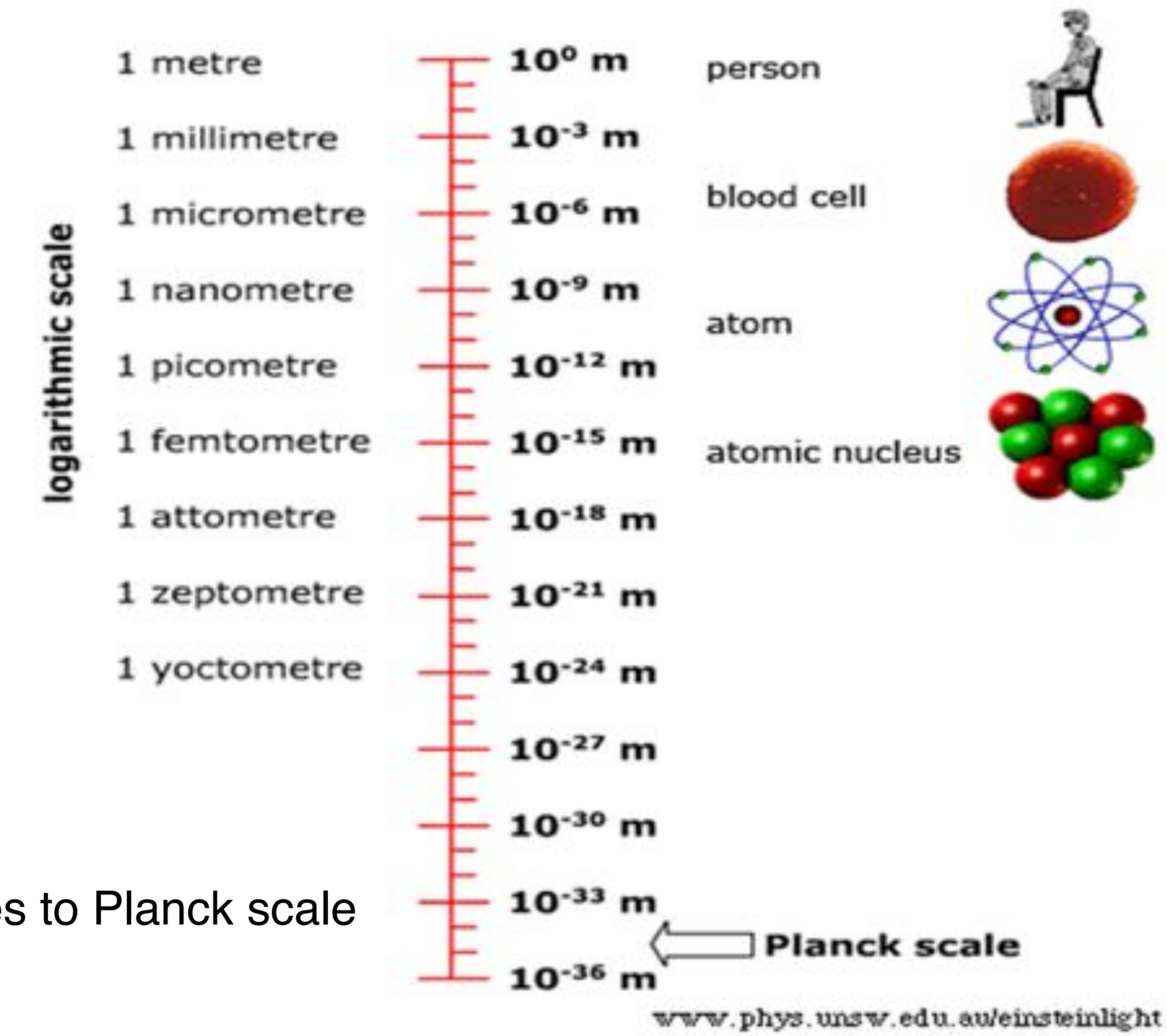
~ where both GR and QFT are relevant

$$l = \sqrt{\frac{Gh}{c^3}} = 3.99 \times 10^{-33}\, cm = 3.99 \times 10^{-35}\, m,$$

$$m = \sqrt{\frac{ch}{G}} = 5.37 \times 10^{-5}\, g = 5.37 \times 10^{-8}\, kg,$$

$$t = \sqrt{\frac{Gh}{c^5}} = 1.33 \times 10^{-43}\, \text{sec},$$

$$T = \frac{1}{k}\sqrt{\frac{c^5 h}{G}} = 3.60 \times 10^{32}\, \text{deg}\, K.$$



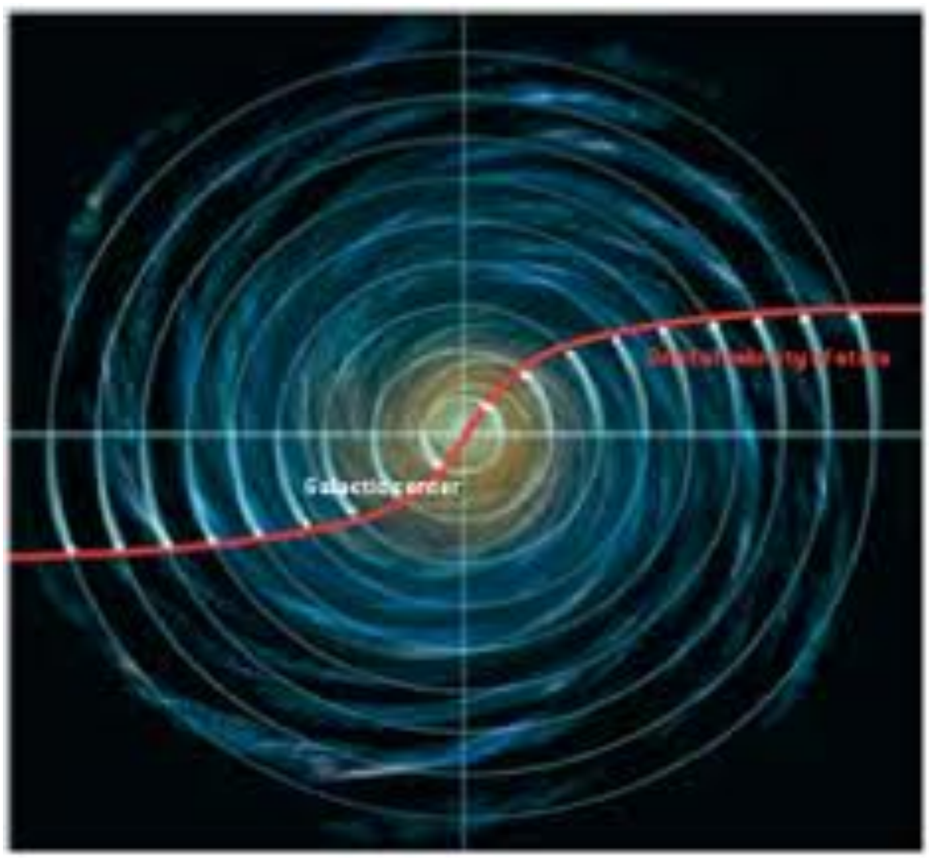
in principle, Quantum Gravity from cosmological scales to Planck scale

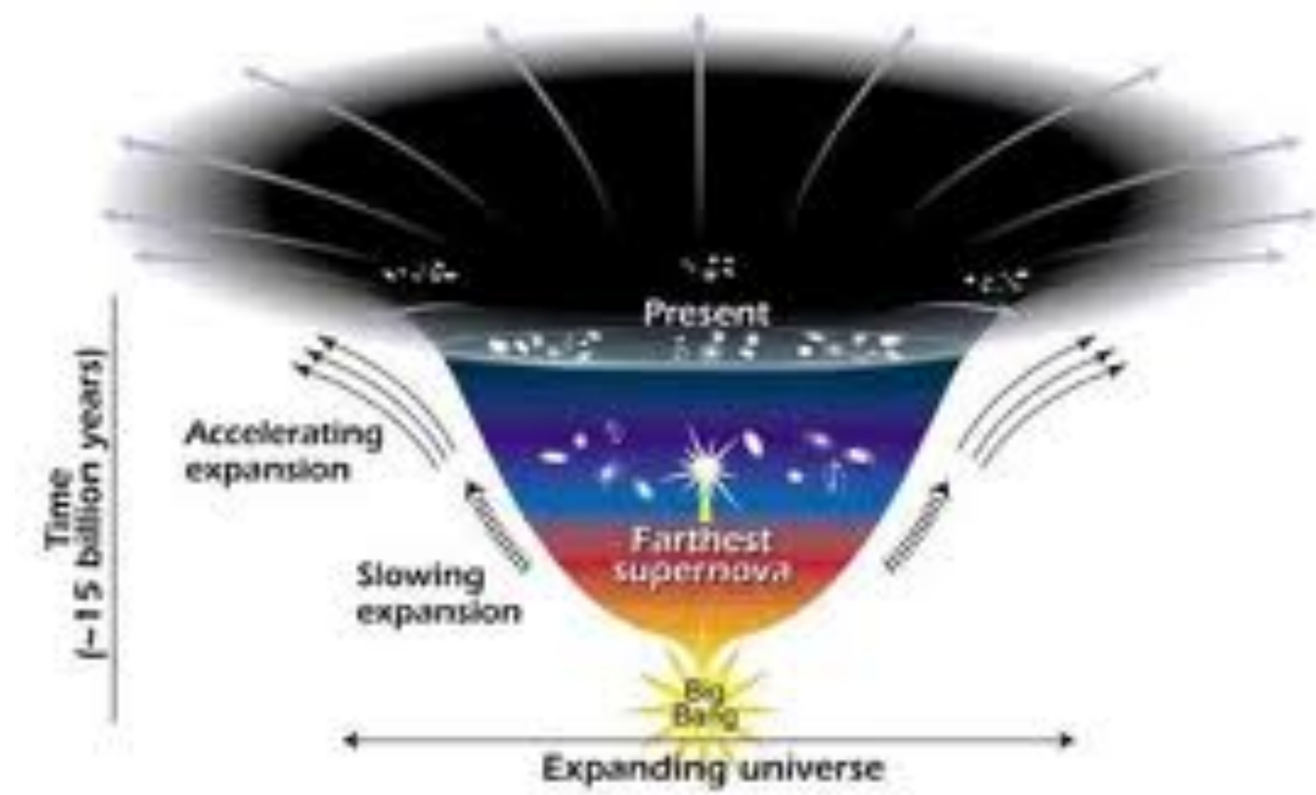
cautionary remark: this is on the basis of current physics, tested only up to very different scales (compared to Planck scale) and based on concepts that may not be valid beyond such scales if spacetime (with its continuum structures, metric, matter fields, topology) is emergent,

even large scale features of gravitational dynamics can (and maybe should) have their origin in more fundamental ("atomic") theory

cannot trust most notions on which effective quantum field theory is based (locality, separation of scales, etc)

e.g. : dark matter (galactic dynamics), dark energy (accelerated cosmological expansion) - either 95% of the universe is not known, or we do not understand gravity at large scales





e.g. cosmological constant as possible large scale manifestation of microscopic (quantum gravity) physics

QG effects (potentially) testable

despite possible suppression by Planck scale

Main theoretical problem:

most testable effects obtained within simplified models and phenomenological frameworks

very weak link with fundamental theory

no real control over approximations and assumptions

pressing issue:
connect simplified models with fundamental formalisms

QG requires abandoning/generalizing (one or more) basic principles of QM and QFT

locality, unitarity, local Lorentz symmetry?

probably worse in "emergent spacetime" scenarios

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→ Quantum Gravity meets Quantum Foundations

even if we focus mostly on spacetime (gravitational) aspects of QG, our understanding of QM needs to be re-assessed

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two directions:

- how to generalize QM in presence of key (expected) aspects of QG?
- which generalization of QM give best framework for QG?

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topics in quantum foundations of interest for QG

indefinite causality

quantum reference frames (see later discussion on relational observables in QG)

generalised probability theories

beyond unitary quantum evolution

Quantum Gravity:

diffeomorphism invariance,
background independence,
spacetime observables

key aspect of classical GR, that we expect to be maintained in QG

- diffeo invariance and background independence
- "problem of time" and QG observables

D. Giulini, '06

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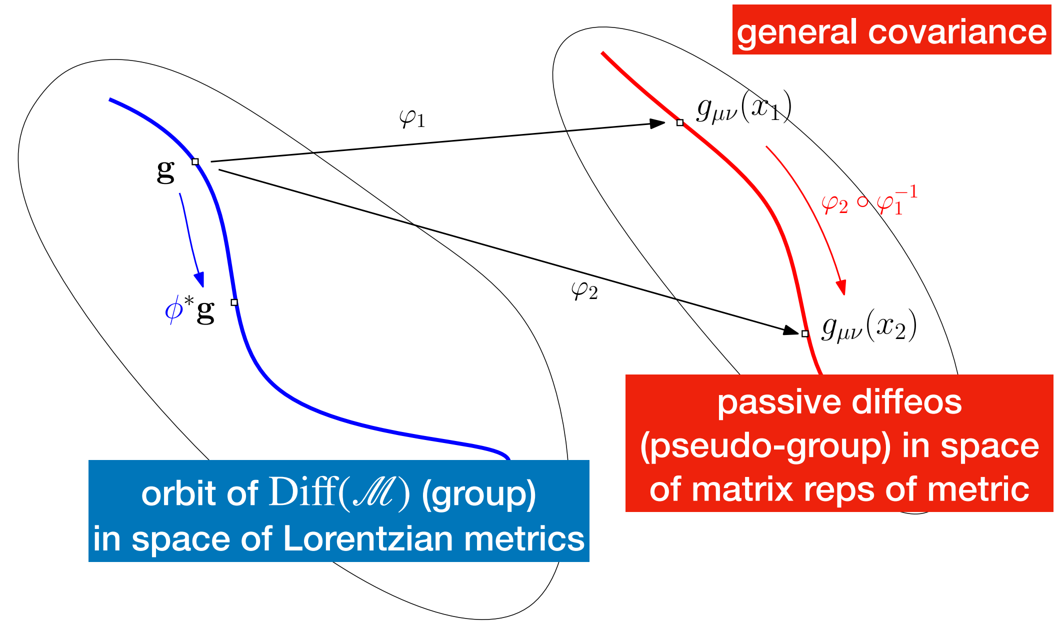
Action $S_{\mathrm{EH}} + S_{\mathrm{matter}} + S_{\mathrm{boundary}}$ invariant under
(i) active and
(ii) passive diffeos

Observables should be invariant under symmetries



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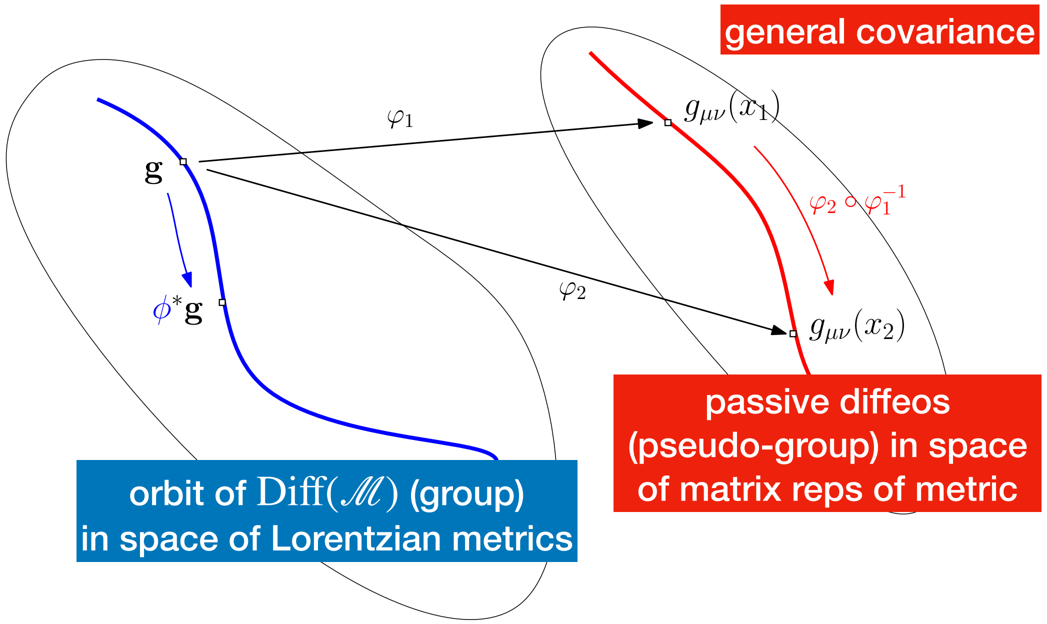

- the diffeomorphism group acts on geometric objects defined on the manifold, i.e. all tensor fields (metric + matter)

$$\boxed{Diff\,(\mathcal{M}) = \{f : \mathcal{M} \to \mathcal{M}\,,\, f \in C^\infty(\mathcal{M})\,,\, f^{-1} \in C^\infty(\mathcal{M})\}}$$

$$\boxed{\varphi' \equiv f \cdot \varphi = D(f_*) \cdot \varphi \cdot f^{-1}}$$

-different- tensor field

$GL(4,\mathbb{R})$ irrep for φ

- dynamics is specified by equations of motion, for given background structures $\boxed{\mathcal{F}\,[\{\varphi\}\,,\,\Sigma] = 0}$

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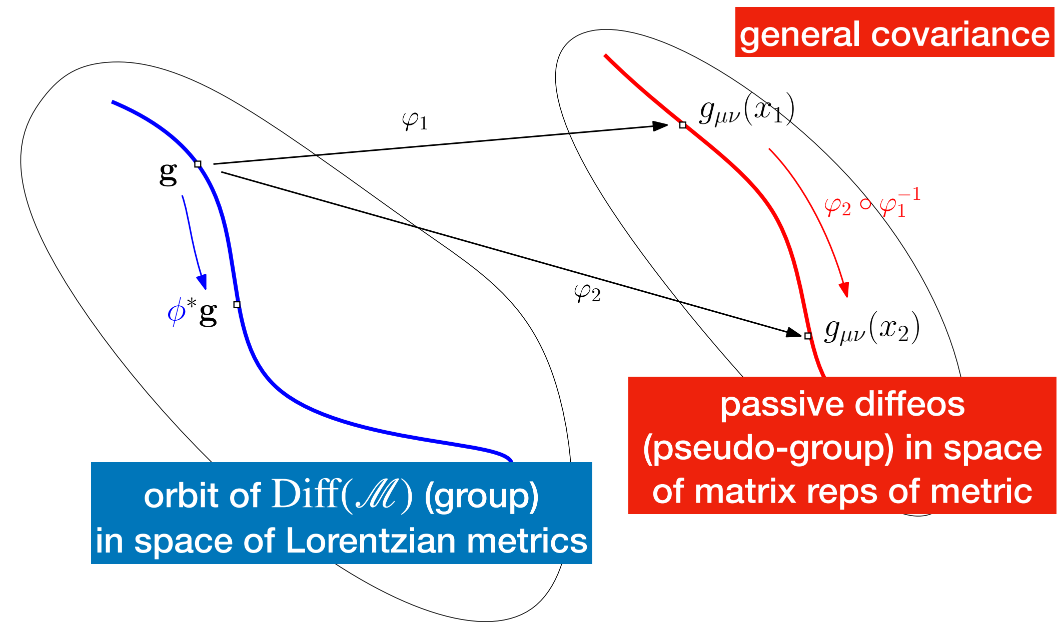

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- <u>Def:</u> $\mathcal{F}\,[\{\varphi\}\,,\,\Sigma] = 0$ is (GENERALLY) COVARIANT under $G \subseteq Diff(\mathcal{M})$ if and only if
$$\forall f \in G \qquad \mathcal{F}\,[\{\varphi\}\,,\,\Sigma] = 0 \quad \Longleftrightarrow \quad \mathcal{F}\,[\{f\cdot\varphi\}\,,\,f\cdot\Sigma] = 0$$

simply requirement that eqns are geometrically well-defined - any theory can be written as such

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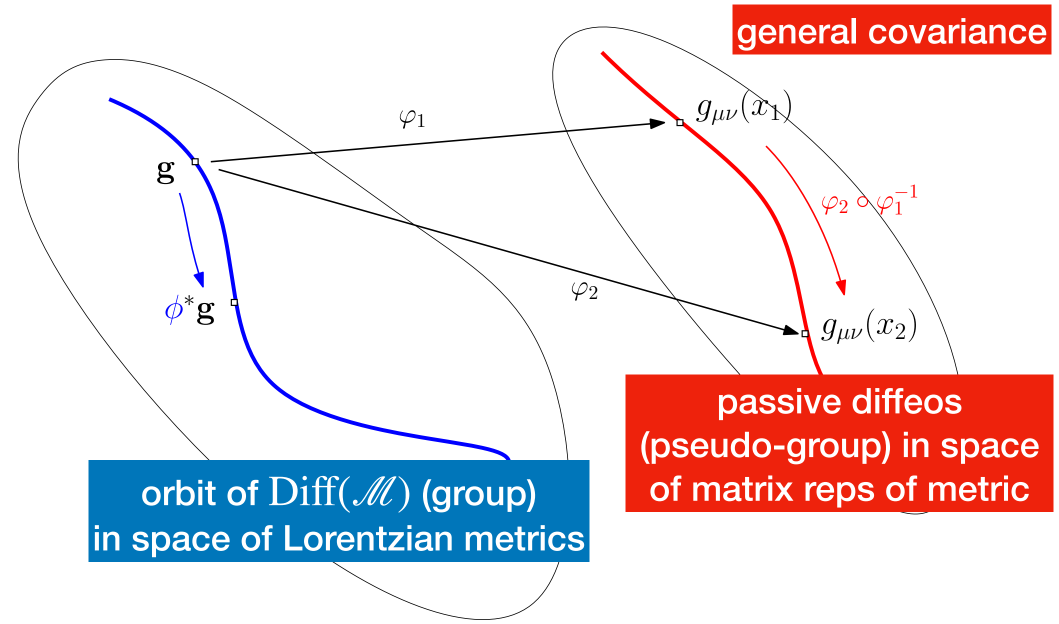

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equivalently: $\quad \forall\, f\,\in\,G \qquad \mathcal{F}\,[\{\varphi\}\,,\,\Sigma]\,=\,0 \qquad\Longleftrightarrow\qquad \mathcal{F}\,[\{\varphi\}\,,\,f\cdot\Sigma]\,=\,0$

non-trivial: solutions are mapped to solutions of <u>same</u> equations

- thus, largest invariance group is stabilizer of background structures:

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example: Kretschmann-Sorkin

real scalar field in Minkowski spacetime $\Box_\eta \phi = 0$ $\quad \Box_\eta = \eta^{ab} \nabla_a \nabla_b$ $\quad$ not diffeomorphism invariant (flat metric is background structure)

can be turned into a diffeomorphism invariant theory with no background structure as:

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the differentiable manifold, its points, directions, atlases and associated coordinate systems are unphysical

spacetime is not the manifold, events are not manifold points
time is not a timelike direction on the manifold
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fields themselves (as functions of manifold points) are unphysical objects $g_{\mu\nu}(x)$ $\quad A_\mu(x)$ $\quad \varphi(x)$

GR, as usually formulated, is written in a (useful) highly redundant language

physical notions of events, space and time will have to be defined in terms of diffeomorphism-invariant observables, constructed using dynamical fields, and compatible with dynamical equations of the theory

problem of "time" and "space"

dynamics and observables do not refer to any (specific, preferred) time or space direction (nor coordinate, of course)

in particular, quantum dynamics is fully constrained:

WdW eqn: $\widehat{\mathcal{D}}\,|\Psi\rangle\;=\;0$ (counterpart of Schroedinger eqn)

$$\langle h_2|h_1\rangle \;=\; \sum_{g_{\mu\nu}|h_1,h_2} \mathcal{A}(g)$$ (counterpart of transition amplitudes)

time more problematic than space because of key role in (quantum) mechanics

key issue in QG, but already present in classical GR

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scalars

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however, *a priori* not particularly interesting or useful:

- **may diverge (e.g., $O(x) = R^2(x)$)**
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- **definition indep. of dynamics ⇒ kinematical diff-inv observables, apply to *any* diff-inv. gravity theory**

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relational strategy

Localization in space and time (and dynamics) relational

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Relational observables: “functions on reference fields”

correlations on superspace (space of fields)

what is a reference system?

- As non-invariant/asymmetric under gauge symmetries as possible (invariants worst possible reference systems)
- As many DoFs as there are indep. gauge directions (want to parametrize orbits with dynamical reference DoFs)

$\Rightarrow$ **reference DoFs are gauge DoFs**

How do we describe physics relative to dynamical reference systems?

identify some dynamical fields as clock/rods and use their values to label evolution/localization of other dynamical fields

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reduction to coordinate frames: idealized clock/rods behaving like (global) test fields

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reduction to coordinate frames: idealized clock/rods behaving like (global) test fields

relational perspective: physics is in the relations between dynamical fields $g_{\mu\nu}(x)$ $A_\mu(x)$ $\varphi(x)$

(complete, Dirac) observables = correlations on superspace (space of fields)

what are -local- diffeo-invariant observables, those with a spacetime interpretation?

relational strategy **Localization in space and time (and dynamics) relational**

Rovelli, Dittrich, Ashtekar, Bojowald, Gambini, Giddings, Giesel, Kaminiski, Lewandowski, Marolf, Pullin, Thiemann, Chataignier, PH, Husain, Pons, Salisbury, Singh, Sunderymeyer, Tambornino, Tsobanjan, …

Relational observables: "functions on reference fields" correlations on superspace (space of fields)

what is a reference system?

- As non-invariant/asymmetric under gauge symmetries as possible (invariants worst possible reference systems)
- As many DoFs as there are indep. gauge directions (want to parametrize orbits with dynamical reference DoFs)

$\Rightarrow$ **reference DoFs are gauge DoFs**

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(complete, Dirac) observables = correlations on superspace (space of fields)

example: $R(g(x)), \varphi_a(x) \to R(g(\varphi_a))$ very difficult to define and compute such observables

no known formulation of GR purely in terms of diffeomorphism invariant quantities

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simplest example: parametrized pendulum

classical single 1d pendulum

physical quantities: pendulum position as function of physical time Q = $Q(T)$ (value of some clock)

dynamics: $\dfrac{d^2Q}{dT^2} = -\omega\, Q \longrightarrow$ general solution: $Q(T) = A\,\sin\left(\omega\, T + \phi\right)$

true physical system is pendulum + clock physics is in the relation Q(T)

Q and T can be measured (partial observables); what can be predicted is only Q(T) (complete observable)

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turn dynamical variables into functions of new "time parameter" (i.e. scalar fields in d=1): $Q(\tau)$ $T(\tau)$

$$\frac{dQ}{d\tau} = P_Q \quad \frac{dT}{d\tau} = P_T \qquad H(Q, P_Q, T, P_T) = P_T(\tau) + \frac{1}{2}P_Q^2(\tau) + \frac{1}{2}\omega^2 Q^2(\tau)$$

$$\frac{dQ}{d\tau} = P_Q = \frac{dH}{dP_Q} \quad \frac{dT}{d\tau} = P_T = \frac{dH}{dP_T} = 1 \quad \frac{dP_Q}{d\tau} = P_Q = -\frac{dH}{dQ} = -\omega^2 Q \quad \frac{dP_T}{d\tau} = -\frac{dH}{dT} = 0$$

+ invariance (covariance of equations) under 1d diffeos: $\tau \rightarrow f(\tau)$ 1d manifold not physical

only diffeo-invariant observable, evaluated on solutions on the dynamics, is: $Q(T) = A\sin(\omega T + \phi)$

$Q(\tau)$ $T(\tau)$ are neither measurable nor predictable (as functions of affine parameter)

only Q(T) (complete observable) can be predicted - Q and T are only "physical" in relational sense

diffeomorphism invariance indicates what is physical and what is not

no known formulation of GR purely in terms of diffeomorphism invariant quantities

relational strategy implemented by:

- introduce matter fields as material reference frames (they "cover" manifold just like coordinate frames)
- express theory in terms of -relations- between values of metric field and values of material reference frames

Brown, Kuchar ’95

[see also Kuchar, Torre ’91; Rovelli ’91; Husain, Pawlowski ’11]

typical choices: free scalar matter, pressureless dust, ...

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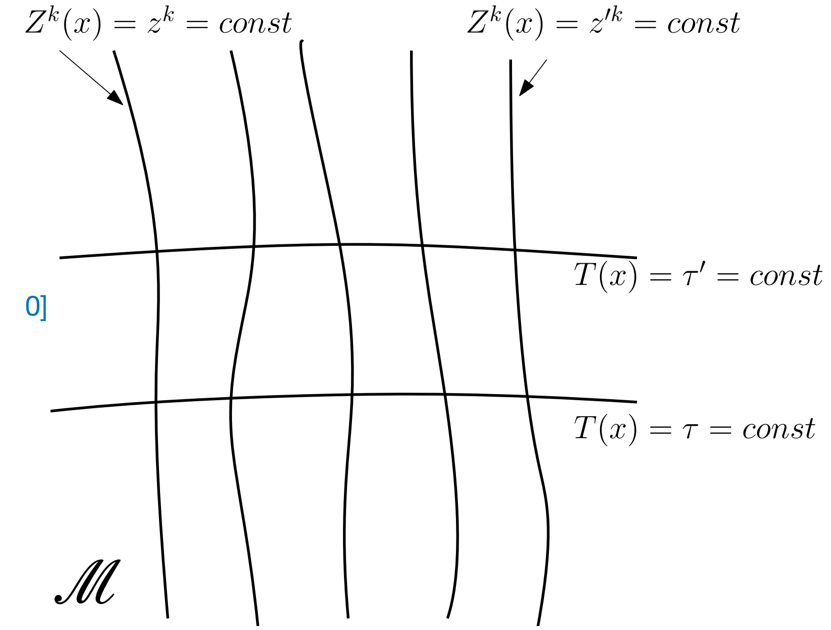

P. Hoehn, ..., '21,'22, '23, '24 ....

C. Goeller, P. Hoehn, J. Kirklin, '22

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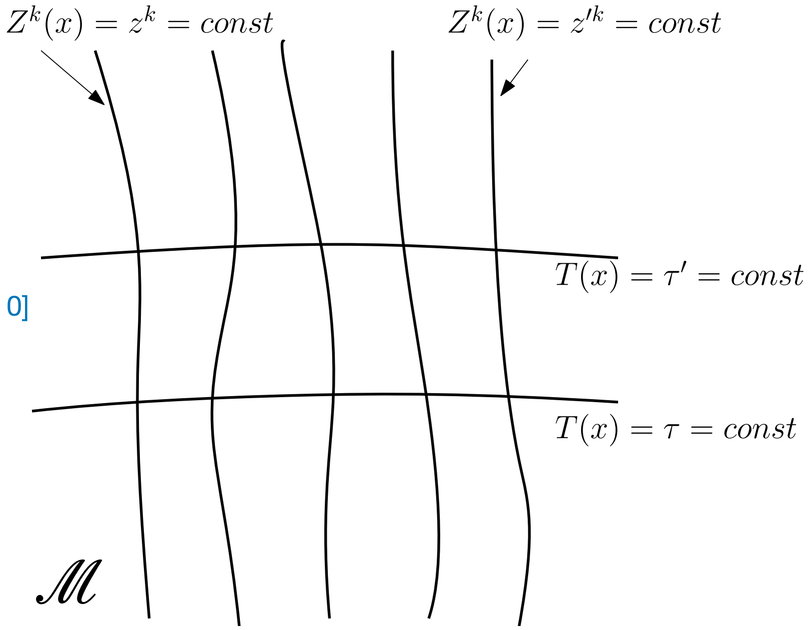

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many obstacles to full deparametrization of GR

- in general: global material frames not physical, realistic material frames not global
- relational formulation with realistic matter can only be local, approximate; need to use several physical frames

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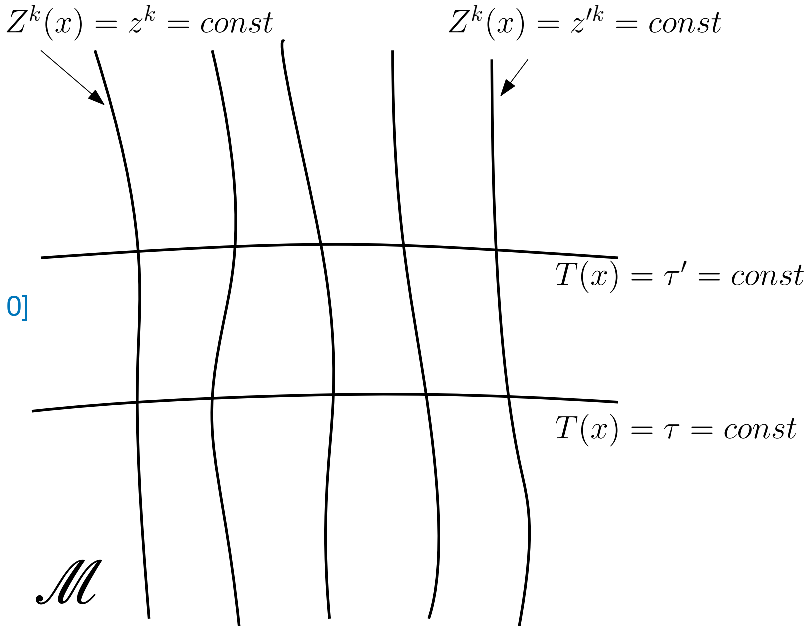

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even more difficult to identify suitable clock

internal clock subject to quantum fluctuations

no reason to expect exact unitary evolution wrt internal clock

exact constructions often require to solve full dynamics

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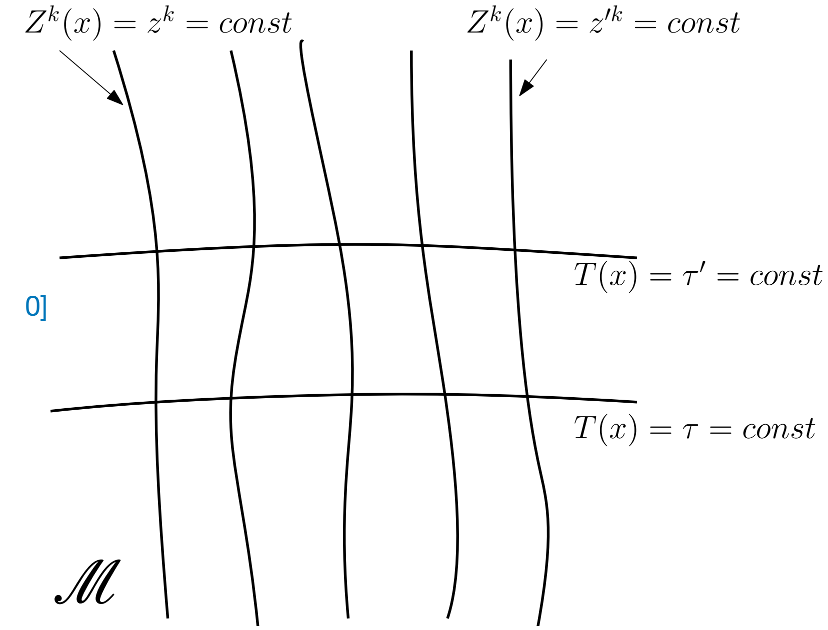

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three approaches

- **Relational observables in Dirac quantization**
  1. **Quantize *all* DoFs on kin. Hilbert space $\mathscr{H}_{\text{kin}}$**
  2. **Solve constraints $\hat{C}_H|\psi_{\text{phys}}\rangle = \hat{C}_a|\psi_{\text{phys}}\rangle = 0$**
  3. **Turn set of solutions $|\psi_{\text{phys}}\rangle$ into phys. Hilbert space $\mathscr{H}_{\text{phys}}$**
  4. **Construct relational observables $\hat{F}_{f,T_i}(\tau_i)$ on $\mathscr{H}_{\text{phys}}$**
- **Reduced phase space quantization**
  1. **Solve constraints classically, remove gauge DoFs**
  2. **Quantize only gauge-inv. observables (no constraints in the QT)**
- **Page-Wootters formalism**
  1. **Steps 1-3 from Dirac quantization**
  2. **Condition $|\psi_{\text{phys}}\rangle$ on reference system ‘orientation’ (conditional probabilies)**

general point: physics is on superspace (space of field configurations), not manifold (only auxiliary structure)

difficult to express/extract it in general QG case

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things much simpler in cosmological context

restriction to global features of universe: (approximately) homogeneous fields

example: flat Friedmann universe (homogeneous, isotropic) $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ab}\mathrm{d}x^a\mathrm{d}x^b$

dynamical variables = scale factor (universe volume) and massless scalar field

GR action reduces to: $S = \frac{3}{8\pi G}\int \mathrm{d}t\, N\left(-\frac{aV_0\dot{a}^2}{N^2} + \frac{V}{N}\frac{\dot{\chi}^2}{2N}\right)$ $\quad V \equiv V_0 a^3$ invariant under 1d diffeos

minisuperspace is 2d flat manifold $\{a, \chi\}$ only relational observable $V(\chi)$

can be fully deparametrized to give relational evolution: $\left(\frac{1}{3V}\frac{\mathrm{d}V}{\mathrm{d}\chi}\right)^2 \equiv \left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}$

quantum cosmology wavefunction on minisuperspace $\Psi(a, \chi)$

satisfying Hamiltonian constraint: $\mathcal{H}\left(a, \partial_a; \chi, \partial_\chi\right)\Psi(a, \chi) = 0$ that can be turned into evolution eqn wrt to χ

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summary

to identify "spacetime = manifold" or "spacetime physics = physics on manifold" is approximation at best

(corresponds to case in which set of four scalar fields behave like test fields covering manifold, and can be used as coordinates for manifold points)

do not expect to find manifold etc neither at fundamental QG level, nor in its effective description

Quantum Gravity challenges to objectivity: fundamental perspectivalism and approximate-only objectivity

H. Gomes, S. Langenscheidt, DO, '24

Thesis

Once we take seriously the distinction between open and closed systems (encoded in the presence of finite spacetime boundaries), and the need for physical reference frames (encoded in gauge invariance and nonlocality with respect to the supporting manifold), we conclude that there are serious obstacles to a framework that is completely intersubjective, and there are fundamental limits to strong objectivity.

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Presuppostions:

a) Modeling starts with a system/agent or system/observer split. If the system is to be also modeled in its spatiotemporal properties, existence of a system/observer split is encoded then in a boundary for such region

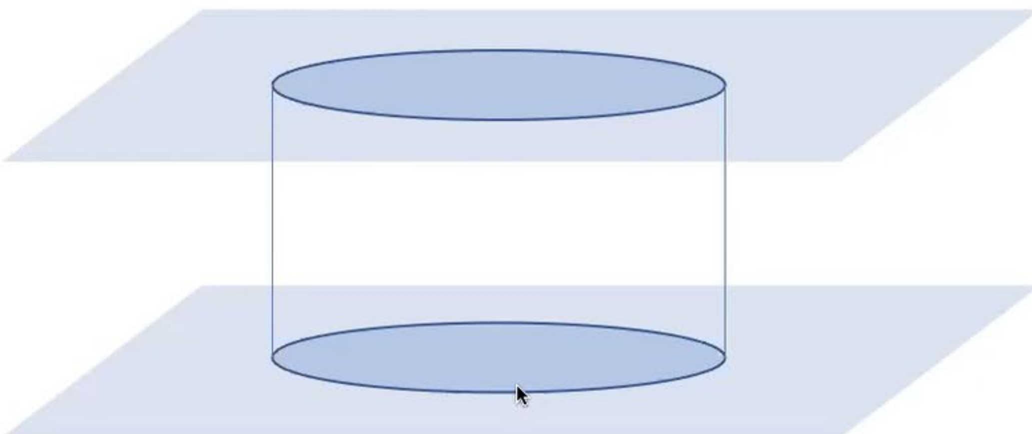
b) We can include aspects of observers/agents in our models (as specific physical systems, boundary conditions, external potentials, ...). In particular, observers/agents come with reference frames (clock+rods) to localize systems in spacetime

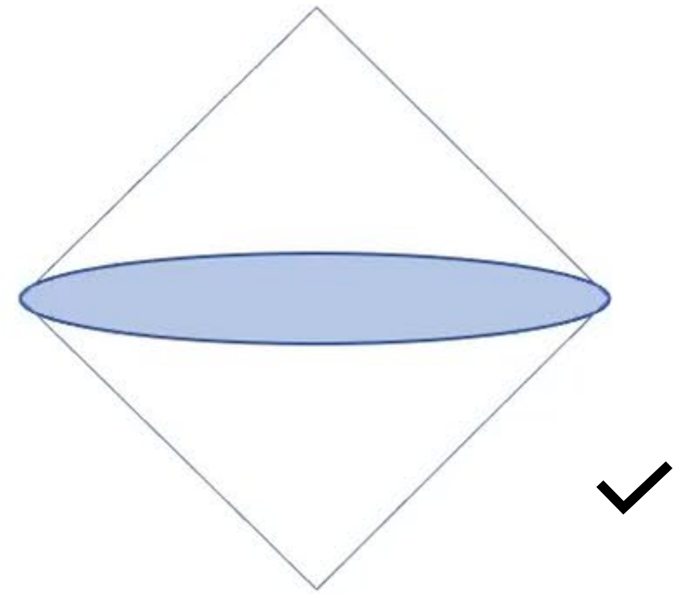
c) If reference frames are to be considered part of physical reality, their physical properties, incl. interaction with other physical systems, backreaction on spacetime geometry, quantum properties, should be modeled too

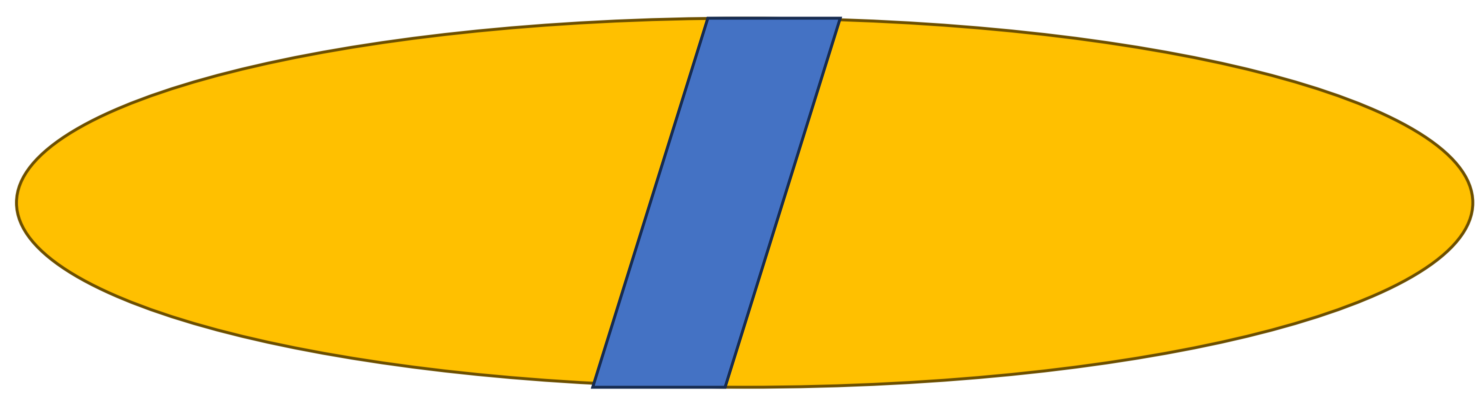
Much recent work on (quantum) gravity in finite, bounded regions

and on proper way to "glue" together finite bounded regions of spacetime

Strominger, Harlow, Donnelly, Freidel, Donnay, Pranzetti, Geiller, Gomes, Riello, Carrozza, Hoehn, ...







Challenges and surprises

- diffeos are "broken" at boundaries
- new "edge modes" appear as dynamical dofs
- they are needed to fully characterize bulk physics
- they are needed to properly glue regions to form extended ones

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A 10-steps argument:

0. Starting point - The localization of physical systems (and their evolution) in spacetime are modeled in terms of points and trajectories in a differentiable manifold, made explicit in terms of coordinate frames. Such coordinate systems are taken to refer to specific observers, and their domain of definition is often assumed to run up to infinity in time and/or space.

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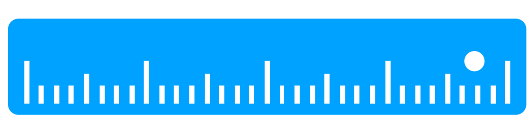

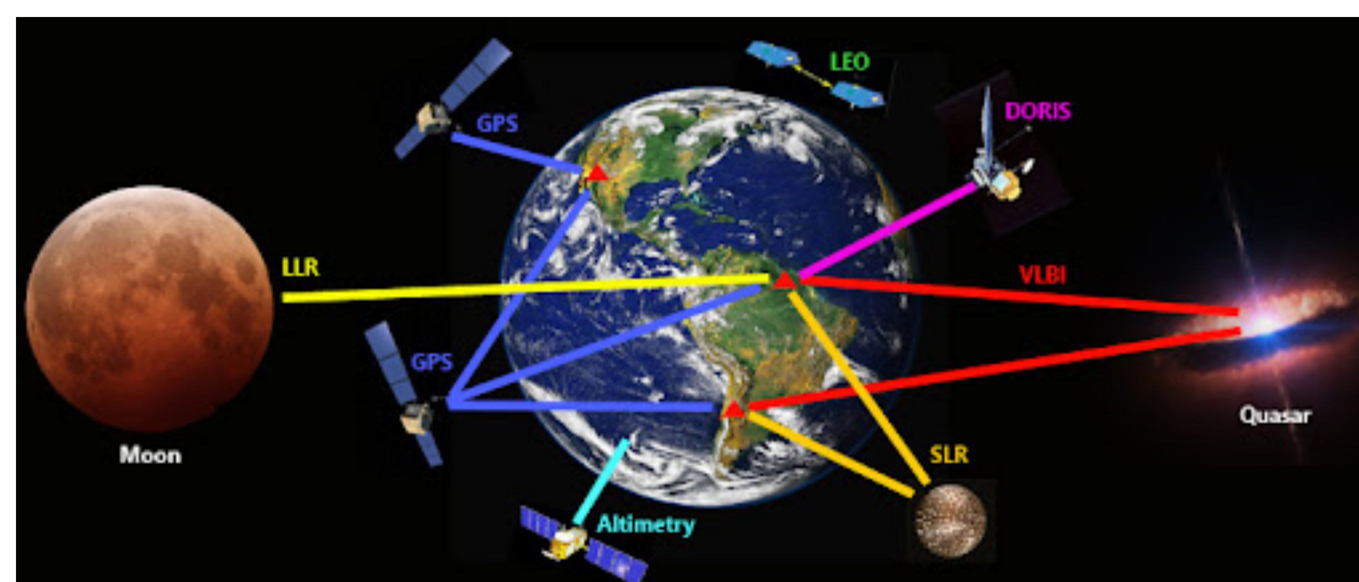

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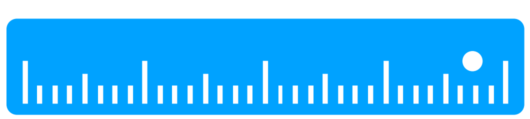
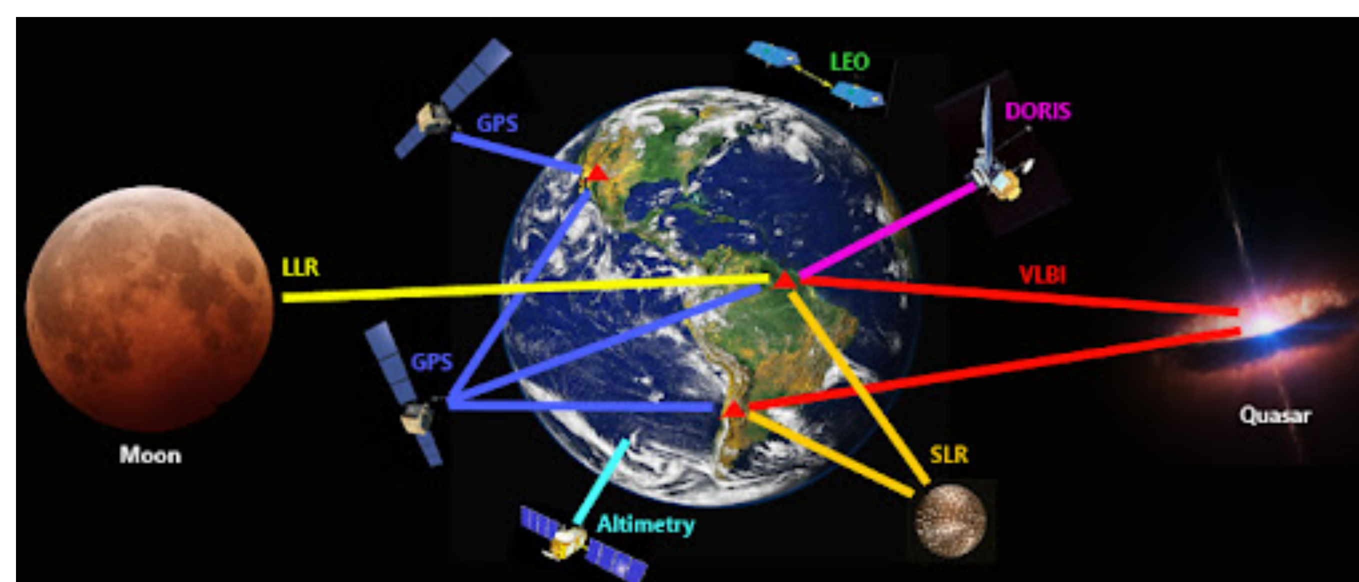
3. Truly physical frames are very different from coordinate frames - Physical frames are fully dynamical, interacting with other dynamical entities, backreacting on geometry. They are also quantum: generically in a superposition of definite values of the clock/rods observables, subject to quantum fluctuations, entanglement, contextuality.





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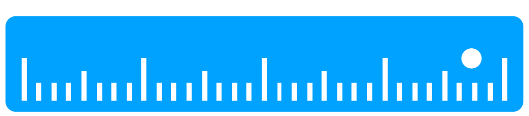
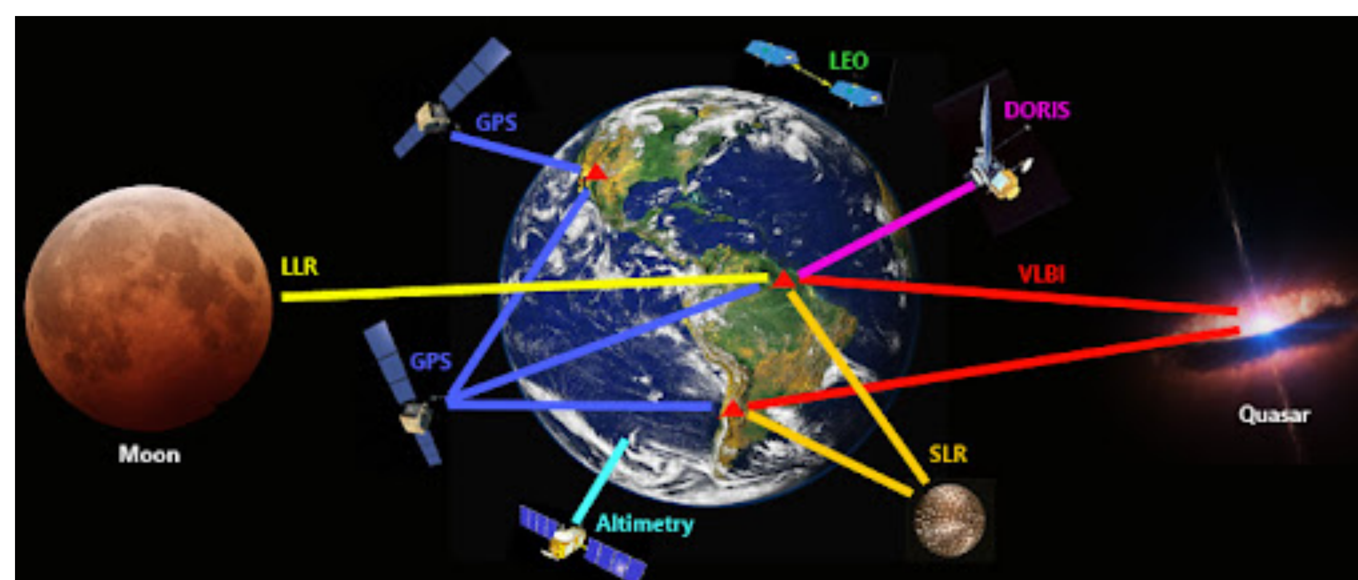


4. Coordinate frames are physical only in (useful) idealizations, which have to be ultimately removed - They correspond to specific physical frames in idealized case (not gravitating, not interacting and classical).



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fundamental perspectivalism and approximate-only objectivity

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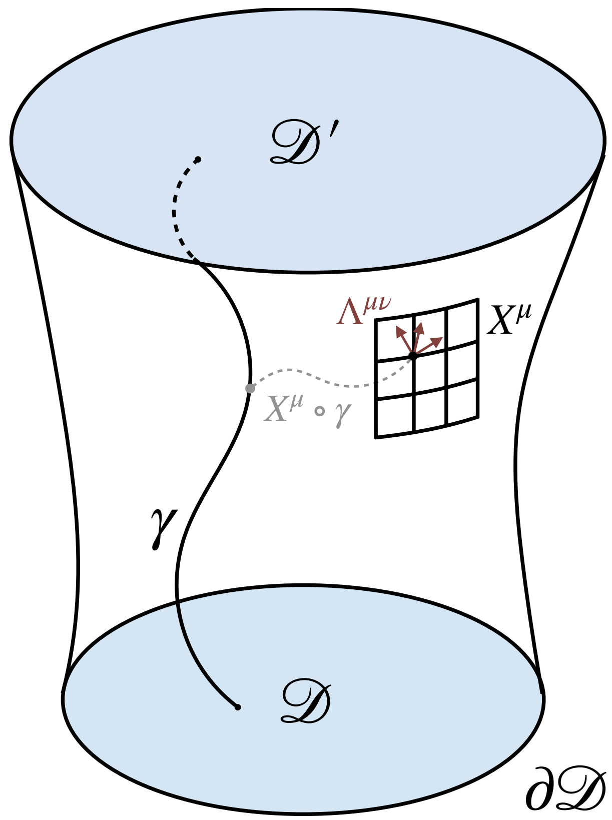

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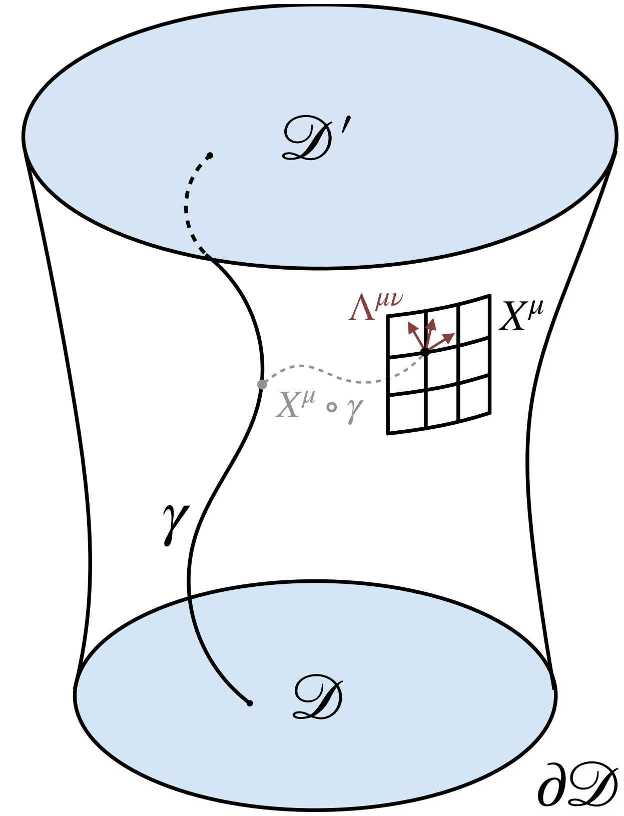

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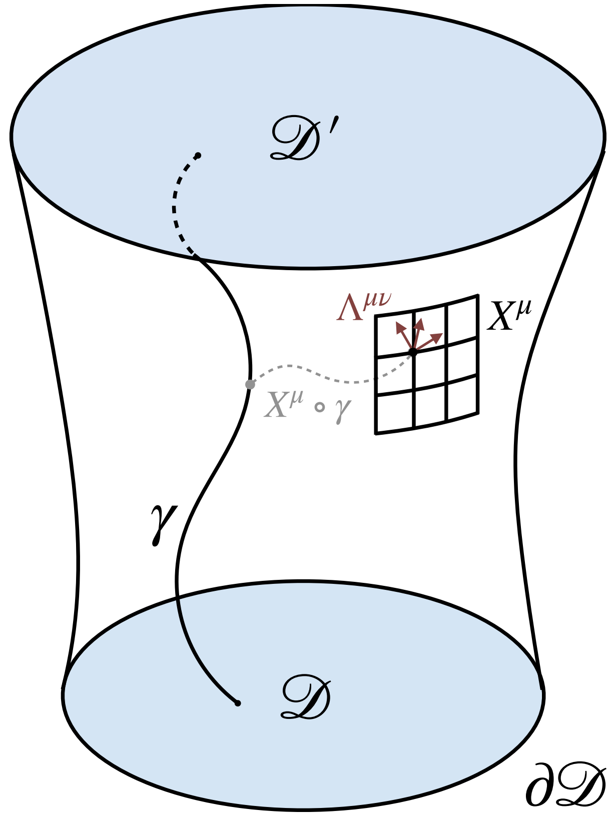

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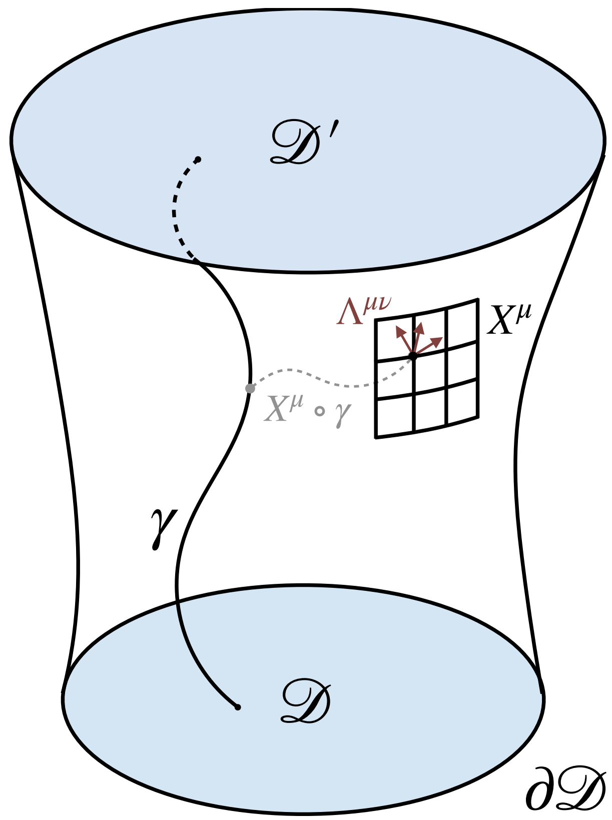

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8. Gravitational (spacetime) physics is intrinsically perspectival - According to the correspondence between realistic agents and their physical reference frames, any set of relational observables is expressed, by definition, in the perspective of some agent participating in the dynamics of spacetime.

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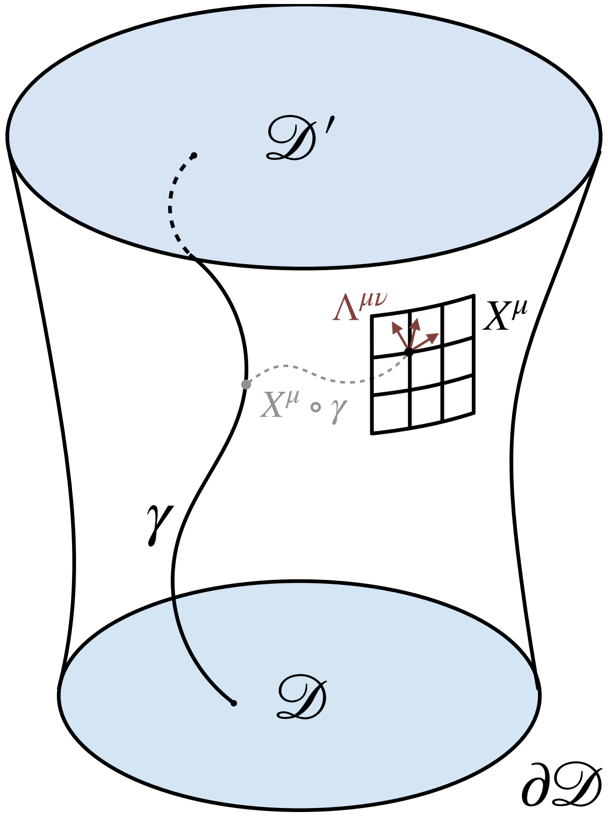


5. Gravitational physics in finite regions requires to endow boundaries with dynamical edge modes, also in order to preserve full gauge invariance, when diffeos are taken to act on boundary degrees of freedom

6. Edge modes define themselves physical frames - so that defining a physical spacetime boundary in all its dynamical aspects is equivalent to selecting a physical frame

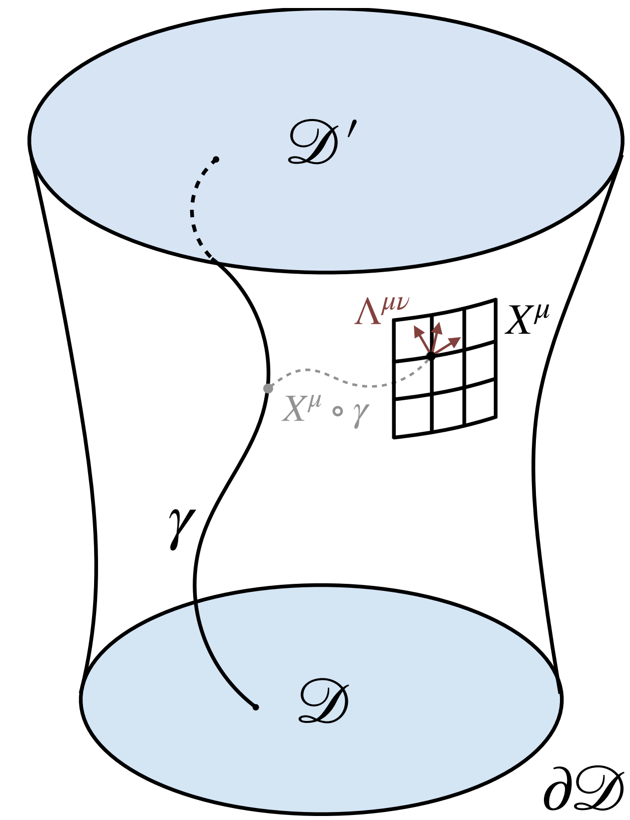
7. We are left with only physical reference frames, but a more realistic (and closer to operational) local physics - The end result of removing idealizations leaves us with gravitational systems defined on finite regions of spacetime with geometric observables expressed as relations between field values (including the metric)

8. Gravitational (spacetime) physics is intrinsically perspectival - According to the correspondence between realistic agents and their physical reference frames, any set of relational observables is expressed, by definition, in the perspective of some agent participating in the dynamics of spacetime.

9. We may have no physical general covariance, and even less invariance - we have at our disposal, in principle, several possible physical frames. The standard notion of general covariance of general relativity is irrelevant, in a relational reformulation. One could argue that some form of physical covariance is expected (general mathematical arguments and explicit examples), at least at the classical level. On the other hand, one should not expect, generically, invariance of all physical properties or dynamical features.

Quantum Gravity challenges fundamental perspectivalism and approximate-only objectivity

H. Gomes, S. Langenscheidt, DO, '24



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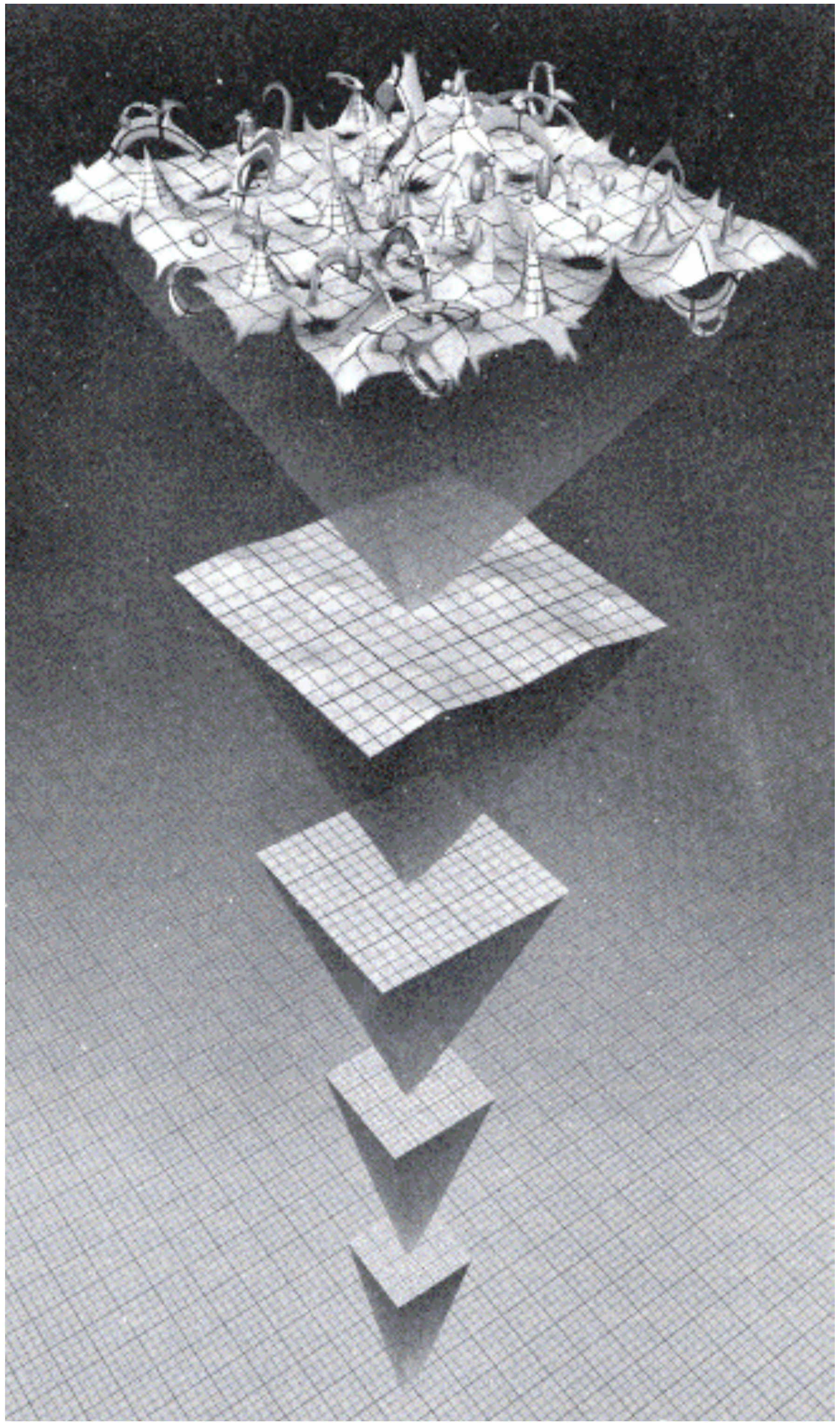
10. This points to a challenge to complete intersubjectivity (identified with physical general covariance among physical frames) and an irreducible limitation to strong objectivity (identified with invariance under change of physical frames)

Quantum Gravity:

more background structures
become dynamical?

background structures in GR and QG?

- GR has some diffeo-invariant background structures, which could be maintained in QG
    - spacetime dimension
    - topology
    - signature



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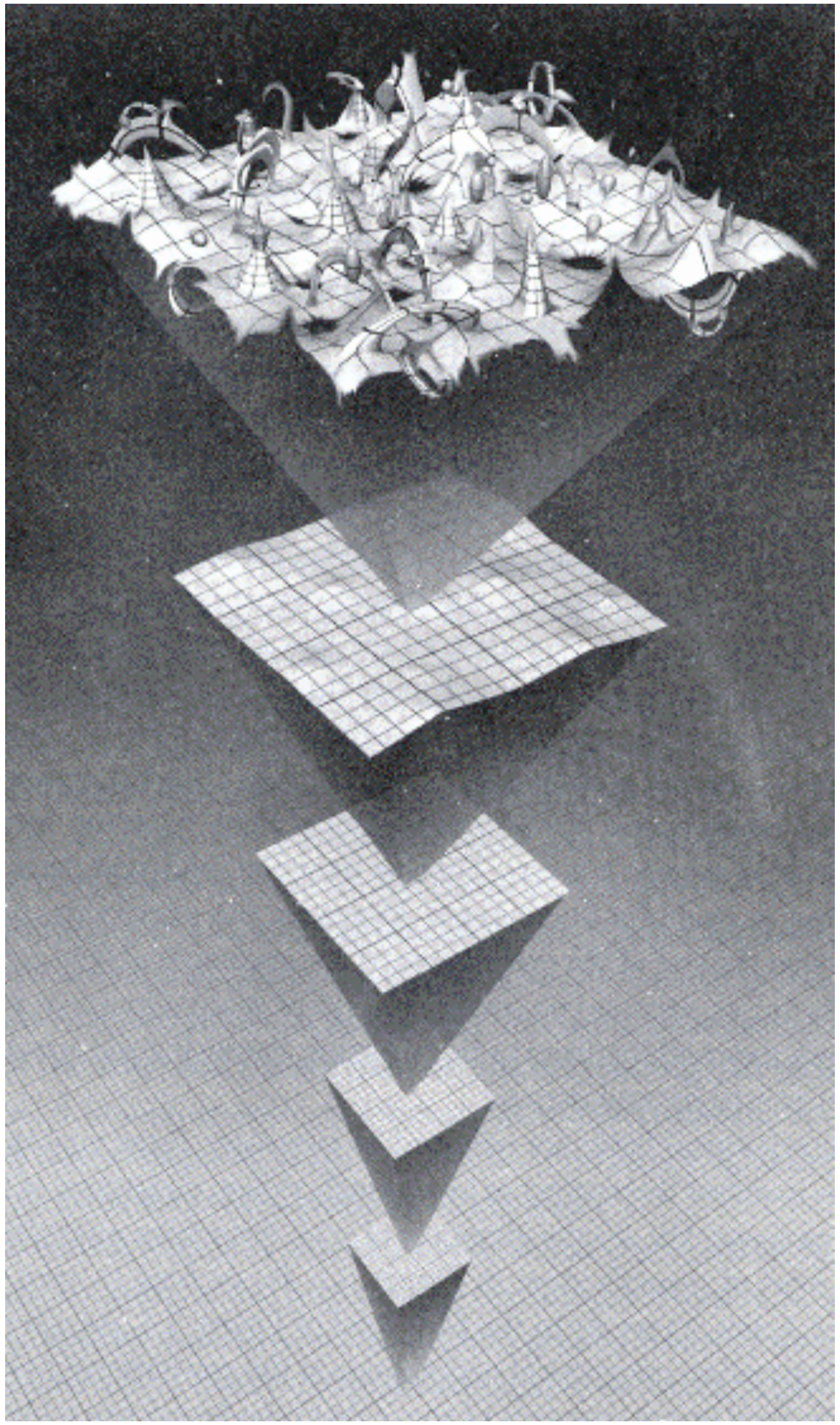

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can they become dynamical to, in full QG?

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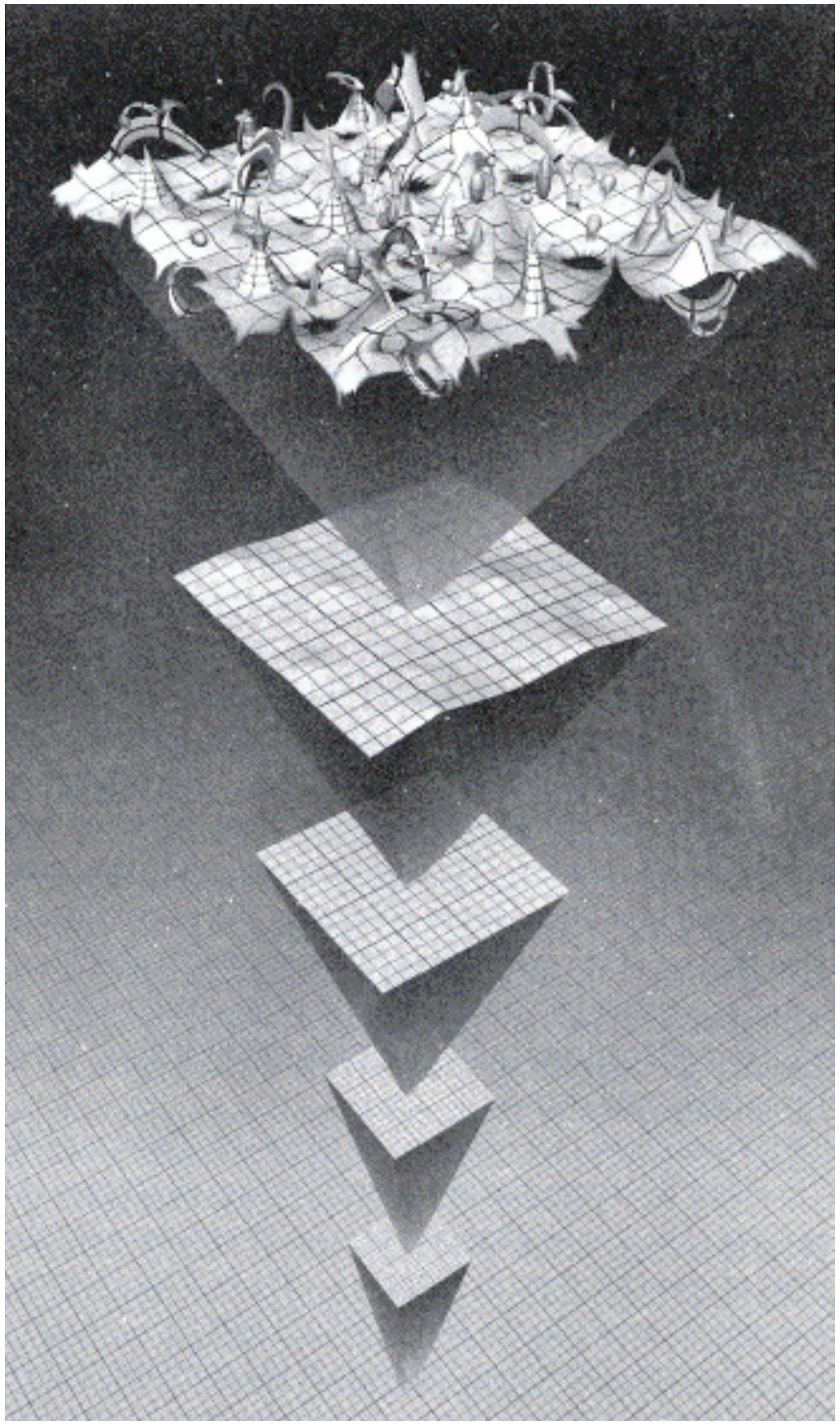
indeed, we have examples of QG formalisms
where they all become dynamical



dynamical dimension

dynamical dimension

causal dynamical triangulations

lattice gravity path integral = sum over equilateral lorentzian triangulations with global foliation

$$Z(G_N,\Lambda) := \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\, e^{iS^{\text{Regge}}_{G_N,\Lambda}[T]}$$

|Aut(T)|

strong indications of (at least) one continuum phase with nice geometric properties (e.g. emergent de Sitter universe)

- in such phase, can evaluate spectral dimension of effective continuum geometry, by studying diffusion processes on the triangulations summed over, characterized by the return probability:

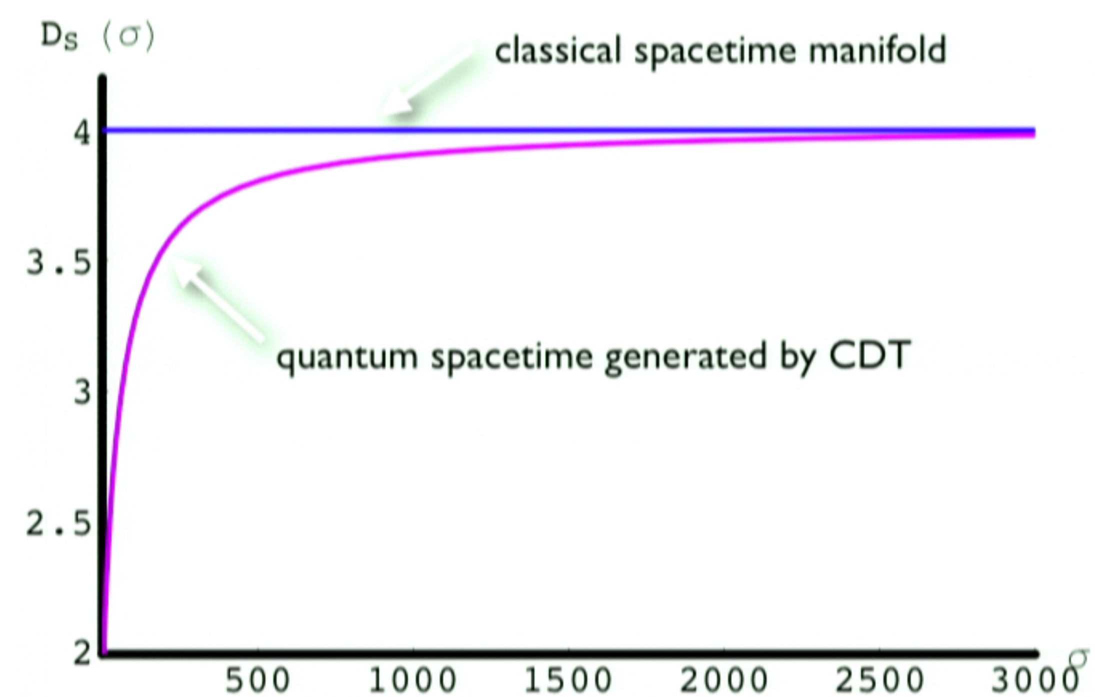
$$\mathcal{R}_V(\sigma) := \frac{1}{V(M)}\int_M d^dx\, P(x,x;\sigma) \propto \frac{1}{\sigma^{D_s/2}}$$

diffusion time

sol.n to heat equation

spectral dimension

- in the continuum limit, one finds:
  - correct large scale dimension (equal to topological dimension of simplices (i.e. D = d)
  - dimensional reduction to D = 2 at high energies/ small scales



qualitative behaviour of $D_s(\sigma)$

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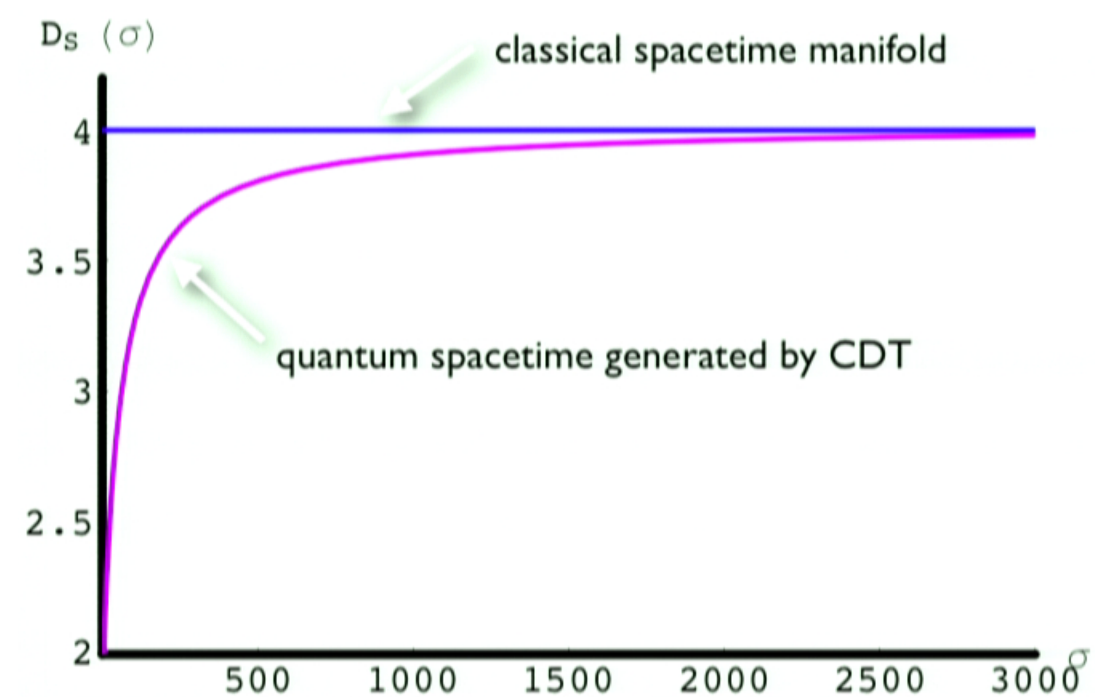
similar results (but with less dynamical control) in canonical LQG, spin foam models, GFT



qualitative behaviour of $D_s(\sigma)$

dynamical signature

canonical loop quantum cosmology   loop quantization of symmetry-reduced (cosmological) sector of GR

- can define full Hilbert space of quantu geometry states + Hamiltonian constraint operator (quantum dynamics)
- can compute (anomaly-free) algebra of constraints
- quantum corrections appear to lead to bouncing scenario replacing big bang singularity

classical GR constraint algebra

$$\begin{aligned}
\{D[N^a], D[M^a]\} &= D[\mathcal{L}_{M^a} N^a] \\
\{H[N], D[M^a]\} &= H[\mathcal{L}_{M^a} N] \\
\{H[N_1], H[N_2]\} &= D[q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)]
\end{aligned}$$

deformed by QG corrections

$$\{H_{(\beta)}[N_1], H_{(\beta)}[N_2]\} = D[\beta q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)]$$

phase space function encoding quantum corrections

- computing algebra along cosmological evolution, close to would-be big bang or big bounce, one has $\beta < 0$
- negative values correspond to constraint algebra for euclidean signature
- thus no evolution across large density/high curvature region, but quantum regime with no notion of evolution at all

result probably has more general validity (beyond symmetry-reduced case)

dynamical topology

can we implement and control sum over topologies?

old (formal) idea: third quantization of gravity

Coleman, Strominger, Giddings, McGuigan, Rubakov, ...

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idea: QFT on superspace

(mathematically ill-defined, but conceptually useful)

canonical QG wavefunction ----> field on superspace (space of geometries on 3-sphere)

$$\Psi(q(x)) \longrightarrow \varphi(q(x)) : \mathcal{S} \to \mathbb{C}$$

with action

$$S[\varphi(q)] = \int_{\mathcal{S}} \mathcal{D}q\, \varphi^*(q)\, H\left(q, \frac{\delta}{\delta q}\right) \varphi(q) + \lambda \int_{\mathcal{S}^3} \mathcal{D}q\mathcal{D}q'\mathcal{D}q''\, \varphi^*(q)\, \varphi^*(q')\, \mathcal{V}(q, q', q'')\, \varphi(q'') \quad + \quad c.c.$$

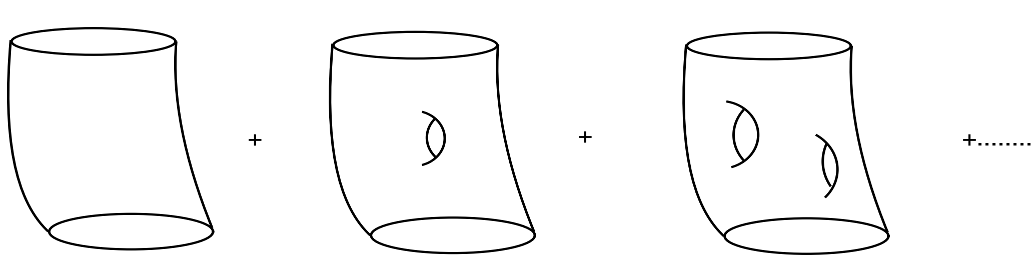
Hamiltonian constraint

non-local interaction on superspace

quantize the theory via path integral, defined perturbatively in sum over "Feynman diagrams"

$$Z_\lambda = \int \mathcal{D}\varphi(q) e^{-S[\varphi(q)]} = \sum_{\mathcal{M}} \mathcal{A}[\mathcal{M}]$$

Feynman diagrams = manifolds of different topologies



Feynman amplitudes = sum over geometries
(i.e. gravitational path integral) for given topology

$$\mathcal{A}[\mathcal{M}] = \int_{\{g|\mathcal{M}\}} \mathcal{D}g \quad e^{i\, S^{EH}_{\mathcal{M}}(g)}$$

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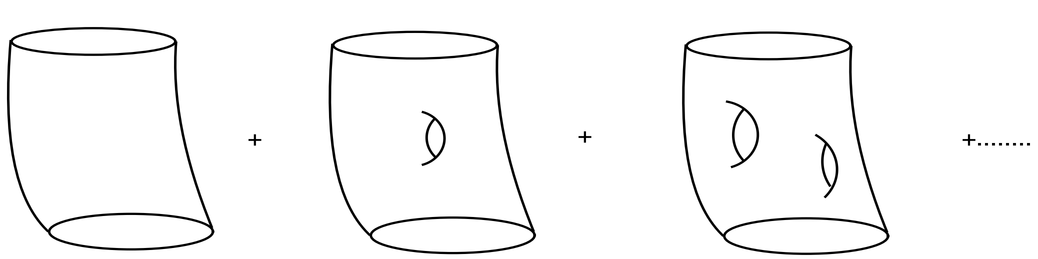

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discrete (mathematically well-defined) realization: matrix models, tensor models, group field theory

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matrix models: $$S(M) = \frac{1}{2} tr M^2 - \frac{g}{\sqrt{N}} tr M^3 = \frac{1}{2} M^i{}_j K^{jl}{}_{ki} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V^{jnl}{}_{mki}$$

$$Z = \sum_{\Delta} g^{t_\Delta} N^{\chi(\Delta)} = \sum_{\Delta} g^{t_\Delta} N^{2-2h} = \sum_h N^{2-2h} Z_h(g) = N^2 Z_0(g) + Z_1(g) + N^{-2} Z_2(g) + ...$$

dominated by spheres in large-N regime

other topologies included in double scaling (large-N, critical g)

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(colored) tensor models $T^a_{ijk} : \mathbb{Z}_N^{\times 3} \to \mathbb{C}$ $\quad a = 0, 1, 2, 3$

rank-3 example, results valid in any dimension

$$S(T) = \frac{1}{2} \sum_a \sum_{i,j,k} T^a_{ijk} \bar{T}^a_{ijk} - \frac{\lambda}{4!\sqrt{N^3}} \sum_{ijklmn} T^0_{ijk} T^1_{klm} T^2_{mjn} T^3_{nli} \quad + \quad c.c.$$

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(topological) Group Field Theories

random tensor models enriched with group-theoretical data (QFT on group manifold)

same techniques from tensor models

dominance of melons in large-cutoff limit (Gurau, '11)

suppression of pseudo-manifolds (Carrozza, DO, '12)

detailed scaling behaviour (Bonzom, Smerlak, '10, '11)

The emergence of spacetime in Quantum Gravity

3 (+1) levels of emergence for space and time in QG

DO, '17, '21

several challenges for the emergence of spacetime in QG (valid for all QG formalisms)

each characterized by new different conceptual and physical issues to be tackled

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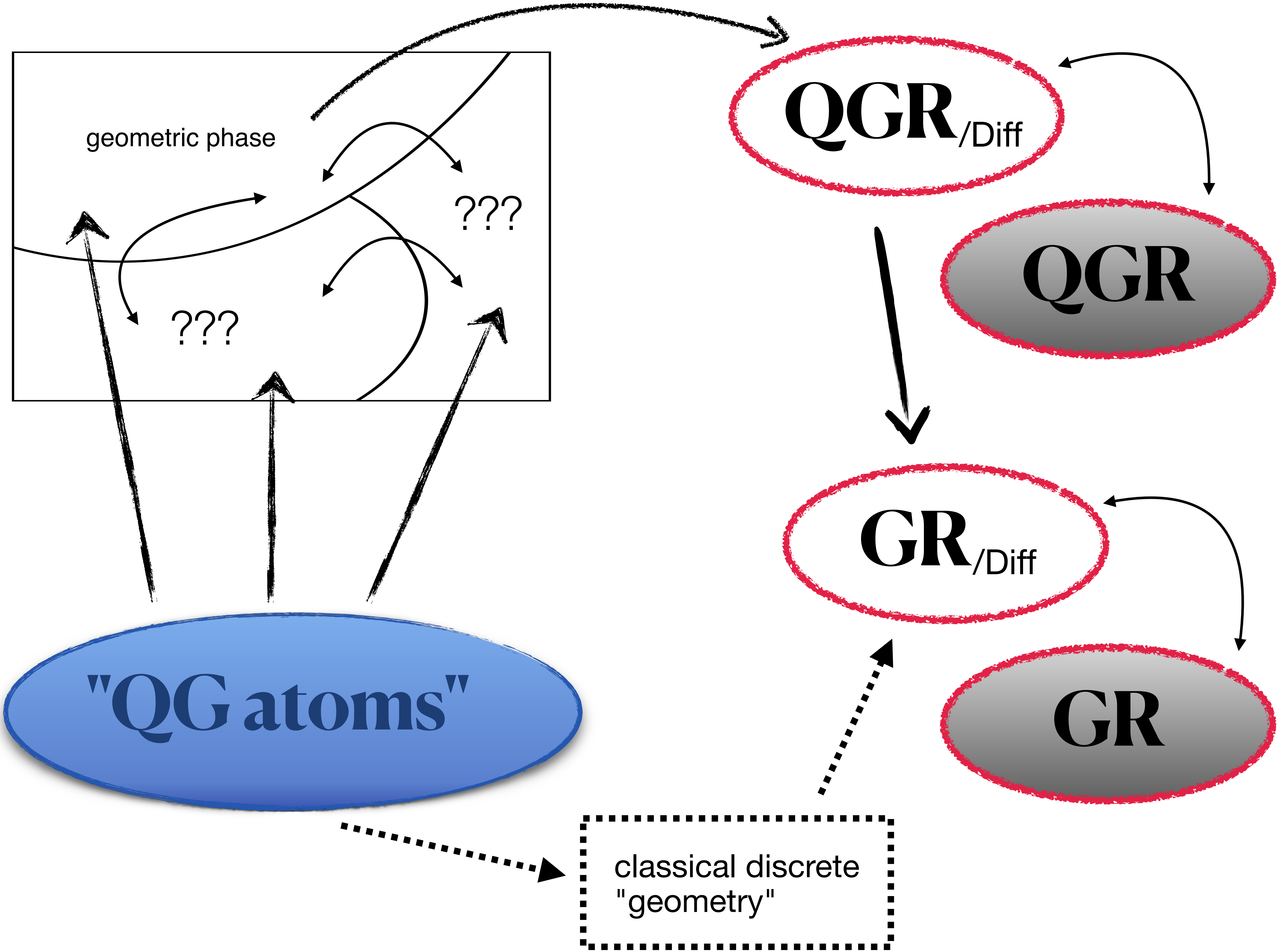
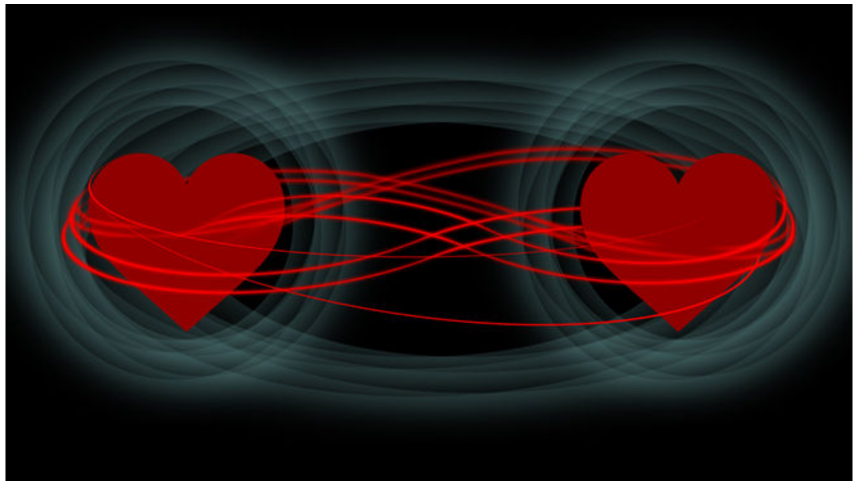
and what is the physics (and observational signatures) of the geometrogenesis phase transition?


geometric phase
???
???
QGR/Diff
QGR
GR/Diff
GR
"QG atoms"
classical discrete "geometry"

entanglement/geometry correspondence

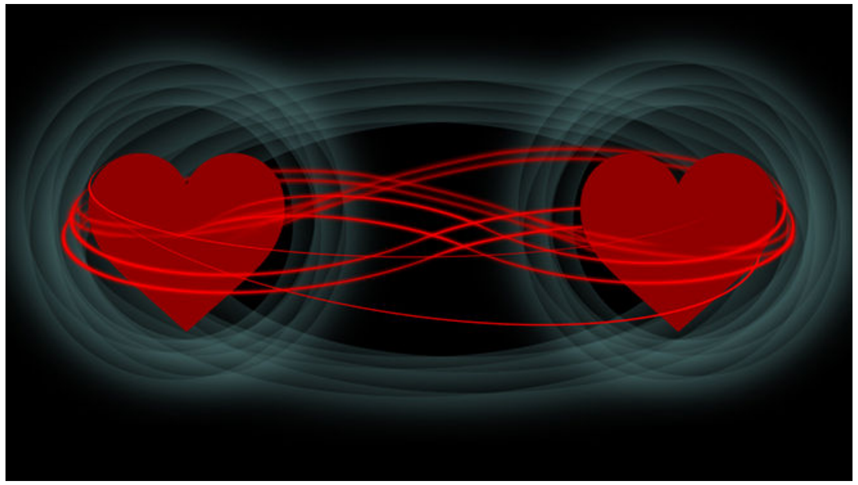


If spacetime is emergent, which quantum features of the fundamental entities are responsible for its geometric properties?

Recent results put in correspondence geometric quantities (e.g. distances, areas) with entanglement between constituents of non-gravitational systems.

Is the world “made of entanglement”? Is geometry just quantum information?

entanglement/geometry correspondence



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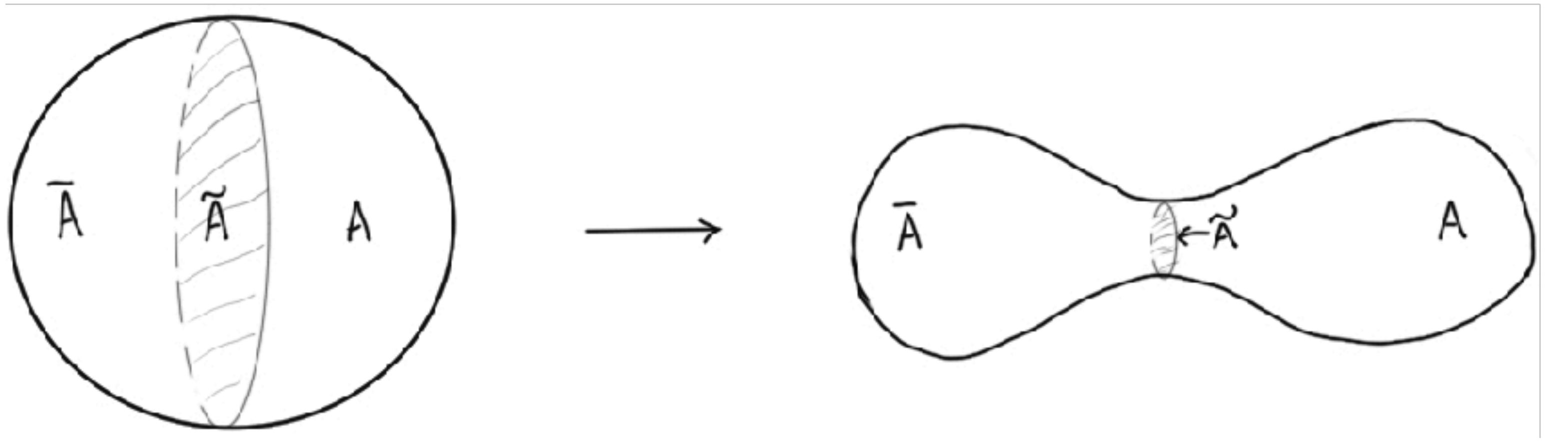
- spacetime bulk reconstruction from CFT quantum correlations between boundary regions

many results in the context of AdS/CFT correspondence but suggestion is more radical than that

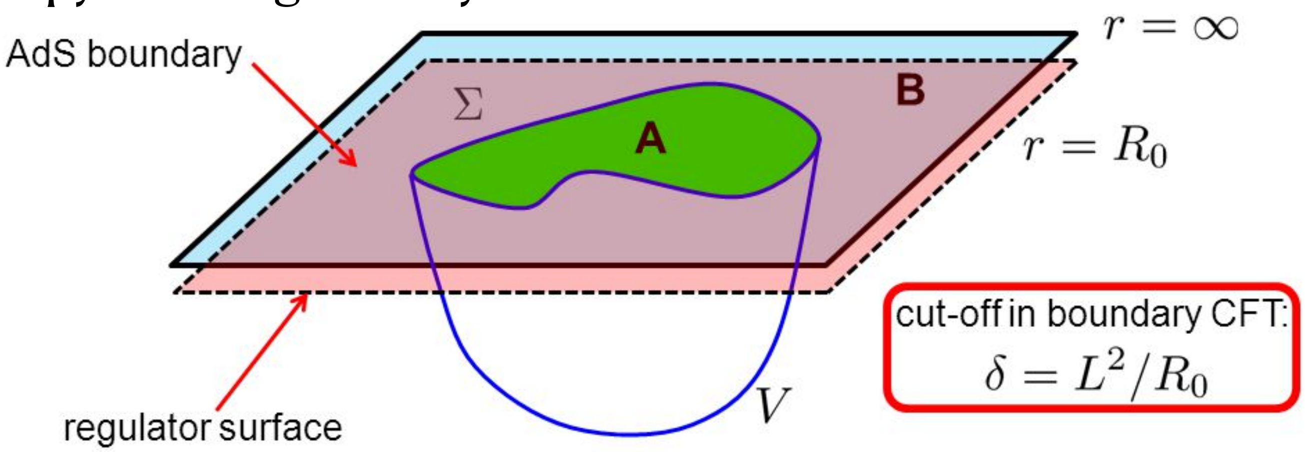
e.g. (mutual information) entanglement ~ spacetime connectivity



- holographic entanglement entropy - CFT entanglement entropy as bulk geometry

e.g. Ryu-Takayanagi entropy formula

Ryu-Takanayagi, ’06, ’12;
Miyaji-Takayanagi ’15



$$S(A) = \underset{\partial V = \Sigma}{\text{ext}} \frac{A_V}{4G_N}$$

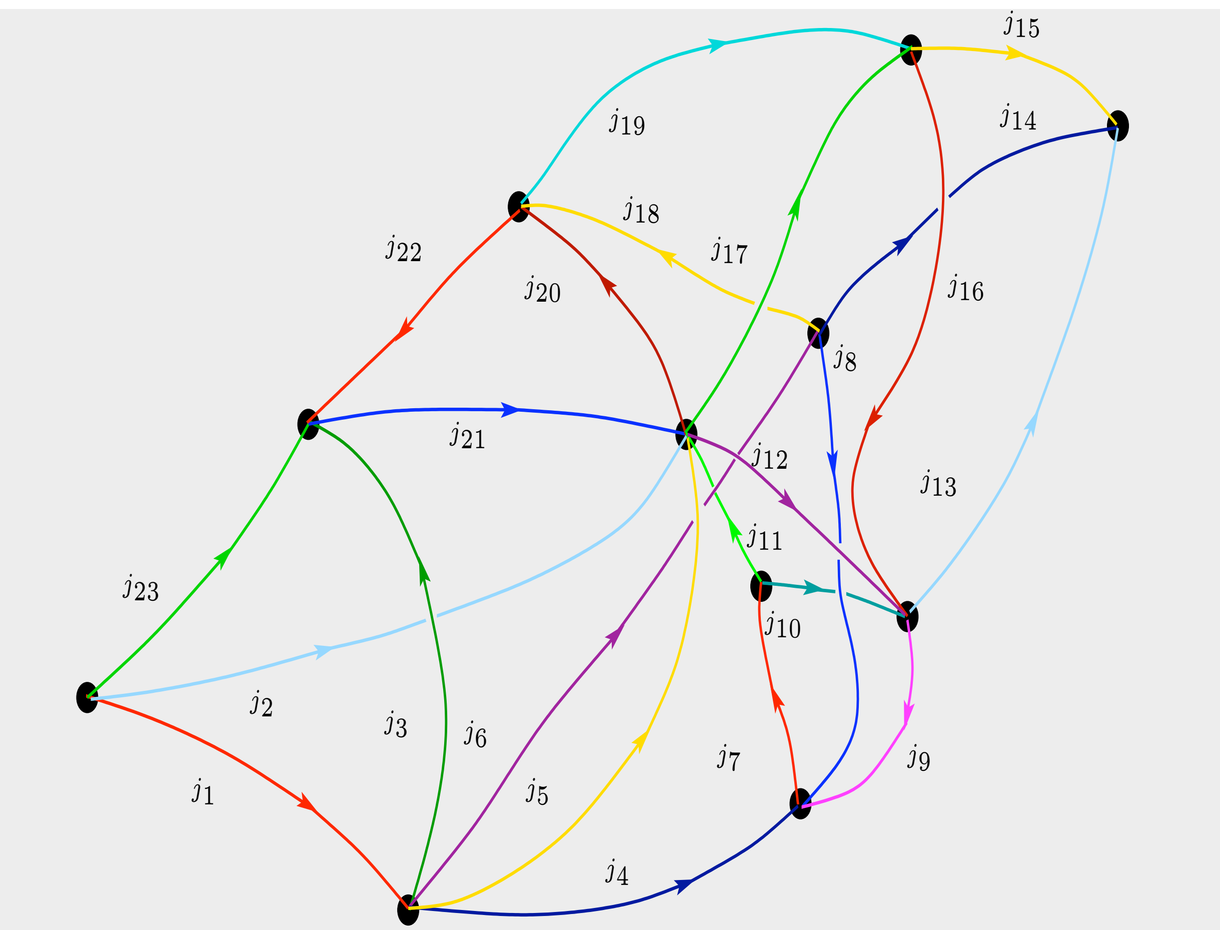
suggests generalization of BH entropy to other (arbitrary?) surfaces in several QG approaches:

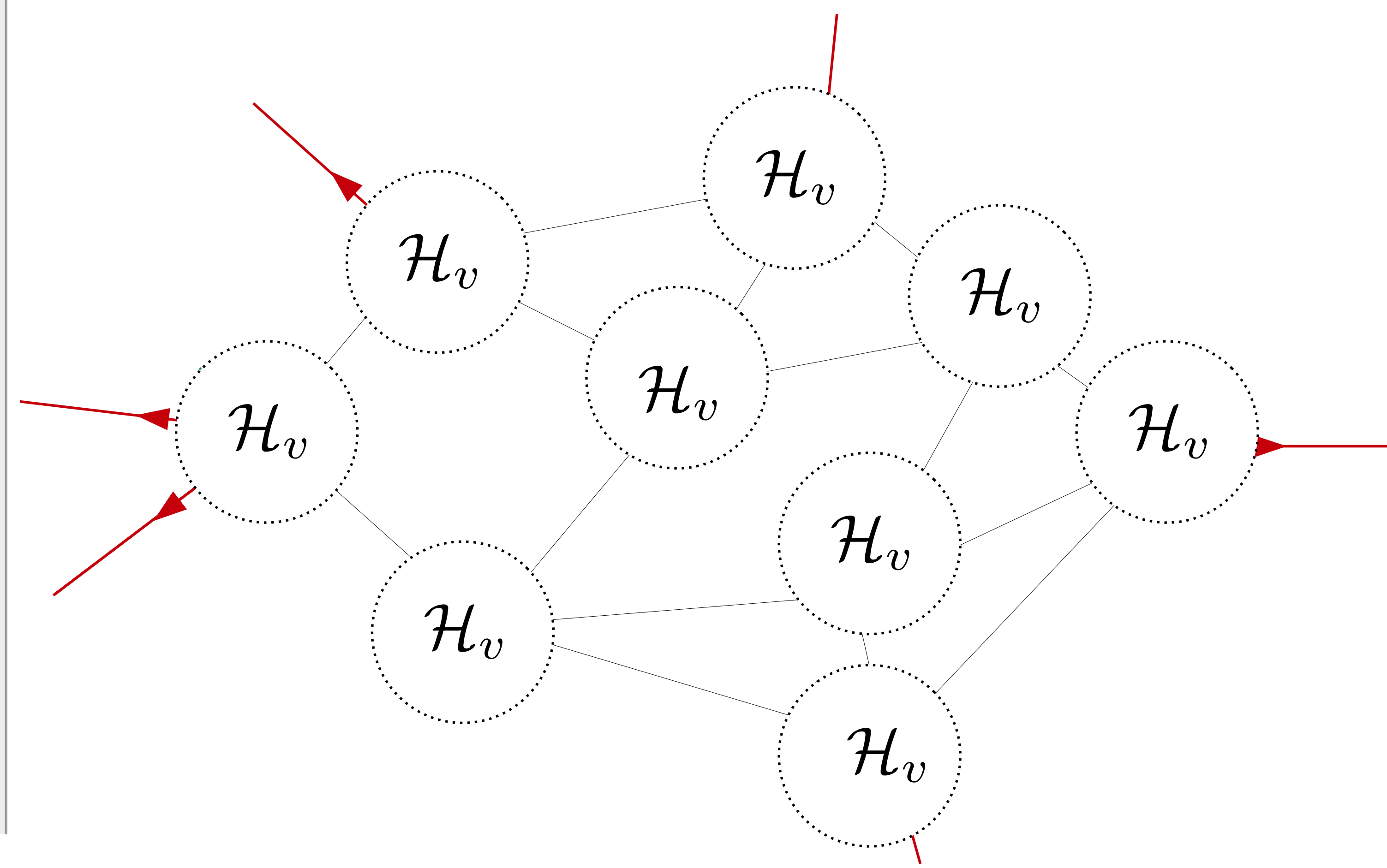
QG states = entanglement networks of quantum geometric blocks

algebraic data on graph

elementary quantum systems on nodes

graph ~ pattern of entanglement across nodes





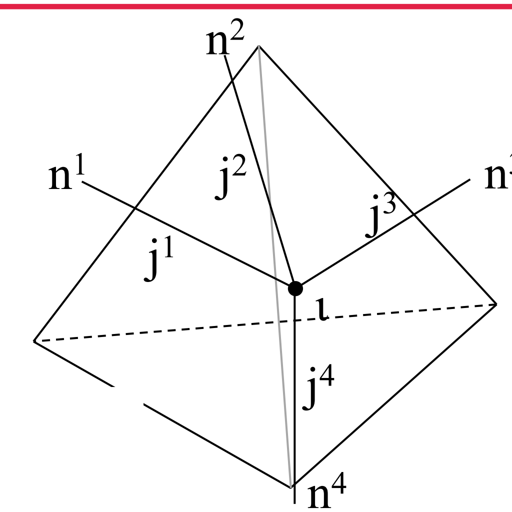
one-body Hilbert space and quantum simplicial geometry

Hilbert space of quantum tetrahedron

(in terms of SU(2) irreps)

spin network vertex ~ quantum tetrahedron

$$\mathcal{H}_v = \bigoplus_{\vec{j}_v} \left(\bigotimes_{i=1}^{d} \underbrace{V^{j^i_v}}_{\text{repr. space}} \otimes \underbrace{\mathcal{I}^{\vec{j}_v}}_{\text{intertwiner space}} \right)$$



quantum geometric operators act on this Hilbert space:

$|j^i n^i\rangle \in V^{j^i}$ diagonalises **area operator**

$|\iota^{\vec{j}}\rangle \in \mathcal{I}^{\vec{j}} = \mathrm{Inv}_G \left[V^{j^1} \otimes ... \otimes V^{j^d} \right]$ diagonalises **volume operator**

note: other constructions using different algebraic data (e.g. SL(2,C) available

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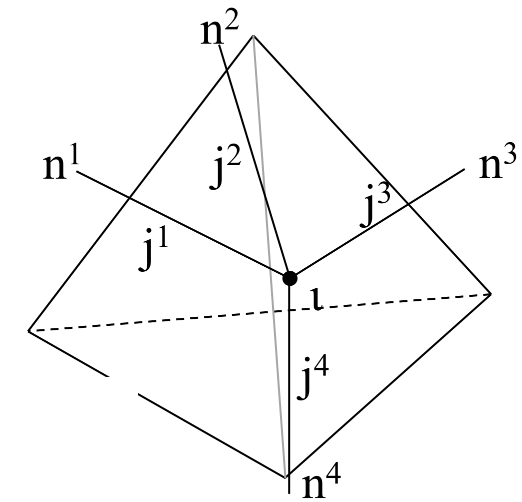

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- encoding of discrete quantum geometry

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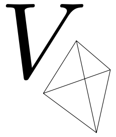
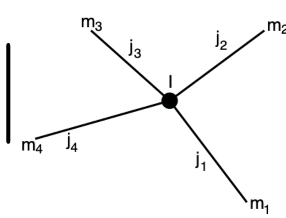
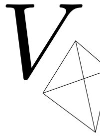
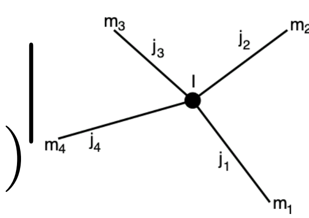
e.g. volume operator $\widehat{V}$ 

$\Big|$  $\Big\rangle$ $=$ $A_{\triangle_i}(j_i)$ / V  $(I(\{j_i\}))$ $\Big|$  $\Big\rangle$

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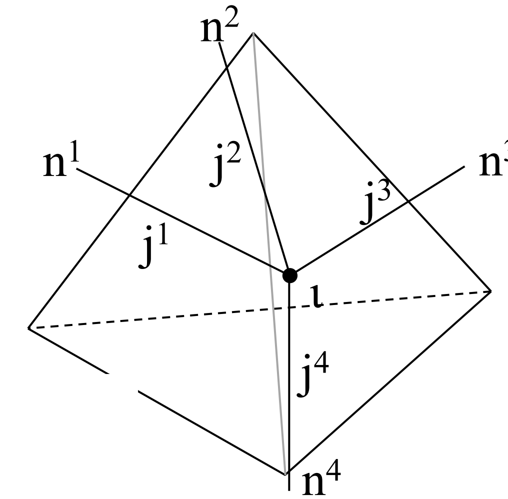

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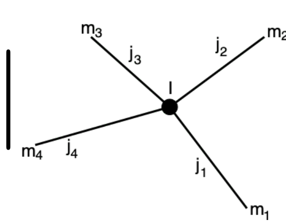
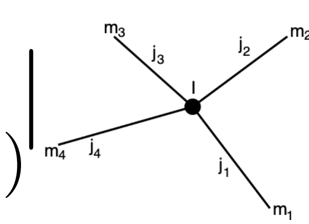
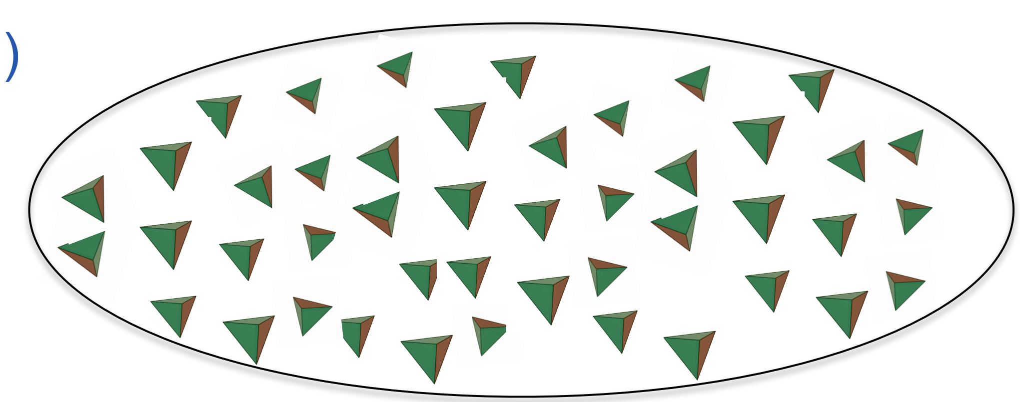
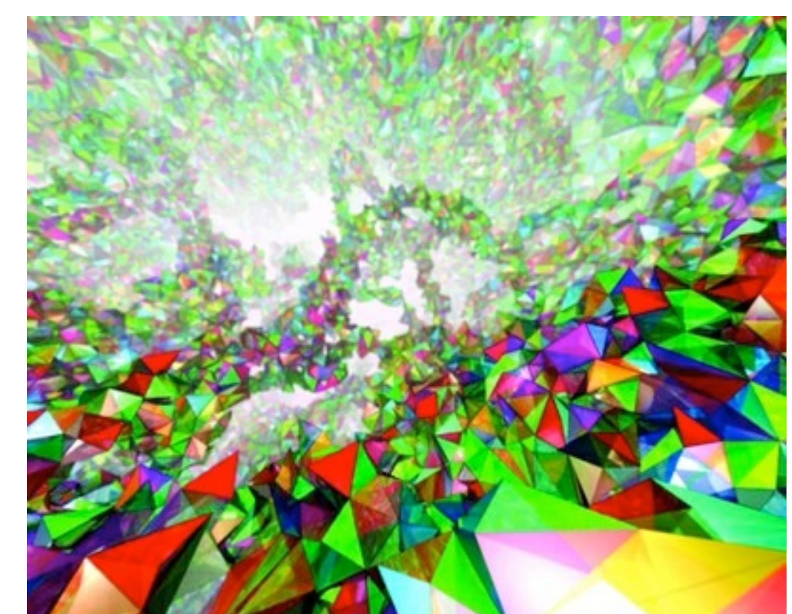
$|$  $\rangle$ = $|$  $\rangle$

atoms of space:

quantized tetrahedra
(decorated with algebraic data)

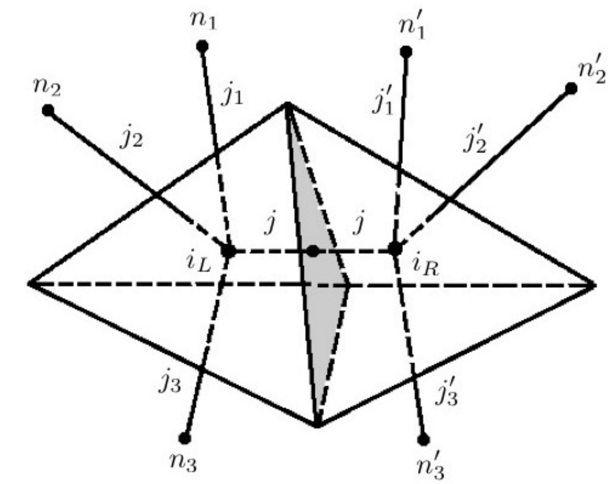




gluing quantum tetrahedra

quantum states for simplicial 3-complexes = entangled many-body states of quantum tetrahedra

- QG graphs as entanglement patterns B. Baytas, E. Bianchi, N. Tokomizo, '18; E. Colafranceschi, DO, '20



gluing among QG atoms of space = invariance under SU(2) group action
= maximal entanglement of link dofs

- can be enforced by "entangling map": $P_i^{x\otimes y} : \mathcal{H}_i^x \otimes \mathcal{H}_i^y \;\rightarrow\; \mathrm{Inv}(\mathcal{H}_i^x \otimes \mathcal{H}_i^y)$

  which entangles x and y along link i - by tracing over SU(2) labels

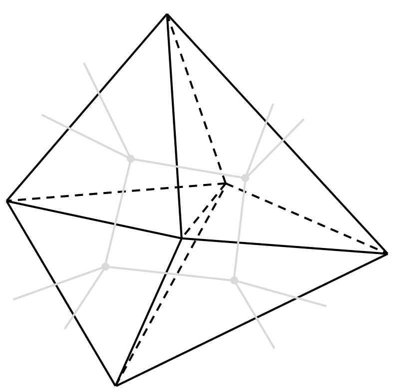
- can generalize to arbitrary graph:

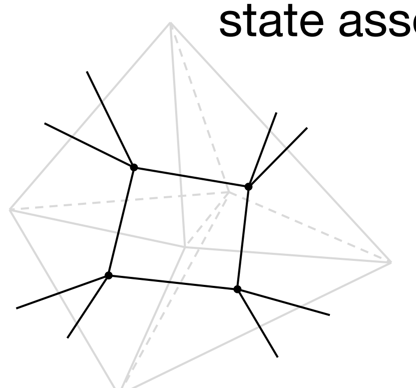
$$|\psi_\gamma\rangle = \prod_{A^i_{xy}=1} P_i^{x\otimes y}|\psi\rangle$$

state associated to graph

adjacency matrix of graph

"disconnected" state
of N GFT quanta
(spin network
vertices ~ tetrahedra)





simplicial complex ~ spin network graph

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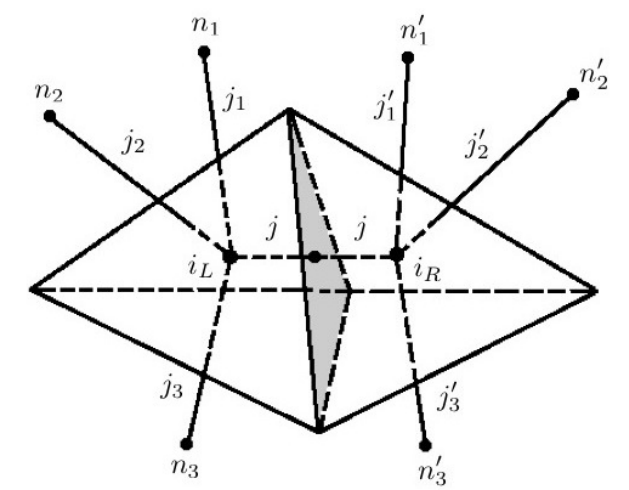

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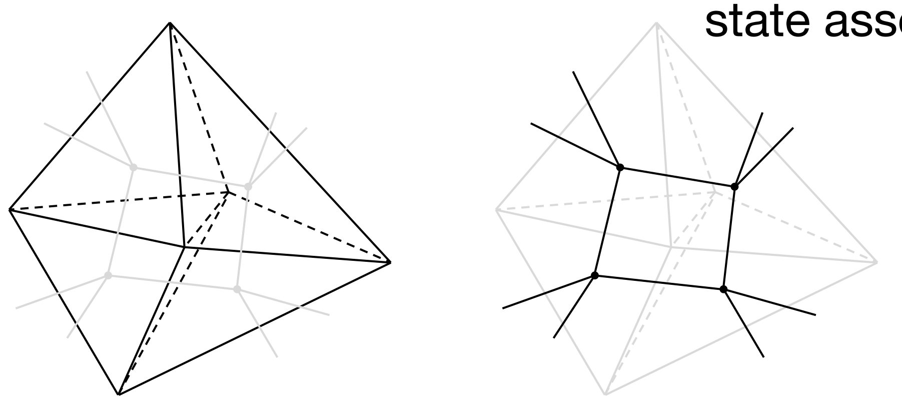
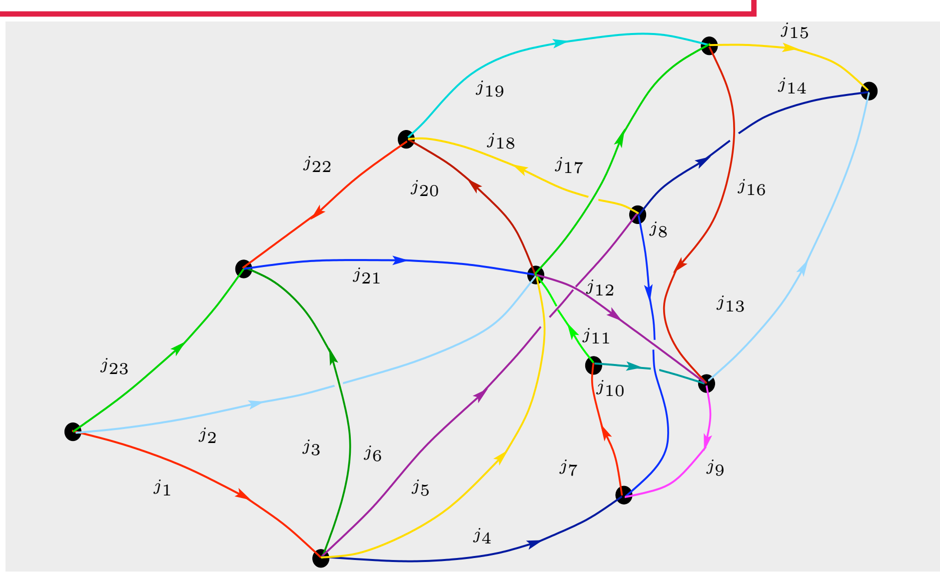


simplicial complex ~ spin network graph

spin network state on given graph:

$$|\varphi_\gamma\rangle := \bigoplus_{\mathbf{j}^1\ldots\mathbf{j}^V} \sum_{\mathbf{n}^1\ldots\mathbf{n}^V} \sum_{\iota^1\ldots\iota^V} \varphi^{\{\mathbf{j}^1\ldots\mathbf{j}^V\}}_{\{\mathbf{n}^1\ldots\mathbf{n}^V\}\{\iota_1\ldots\iota_V\}} \prod_{a^i_{vw}=1} \delta_{j^v_i, j^w_i}\delta_{n^v_i, n^w_i} \bigotimes_v |\mathbf{j}^v\mathbf{n}^v\iota_v\rangle$$

- SN states = QG states in several related QG formalisms
  - canonical LQG
  - spin foam models
  - Tensorial Group Field Theory



Entanglement/geometry correspondence

Van Raamsdonk, '09, Swingle, '10, ….

spacetime geometry (and, possibly, topology) from entanglement of fundamental quantum constituents

- "primitive" entanglement/geometry correspondence E. Colafranceschi, DO, '20
  - graph underlying QG state = entanglement pattern:

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"disconnected" state of N quanta

area operator

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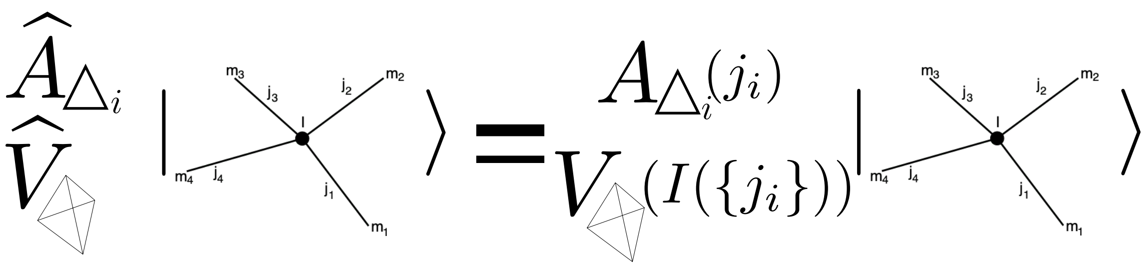

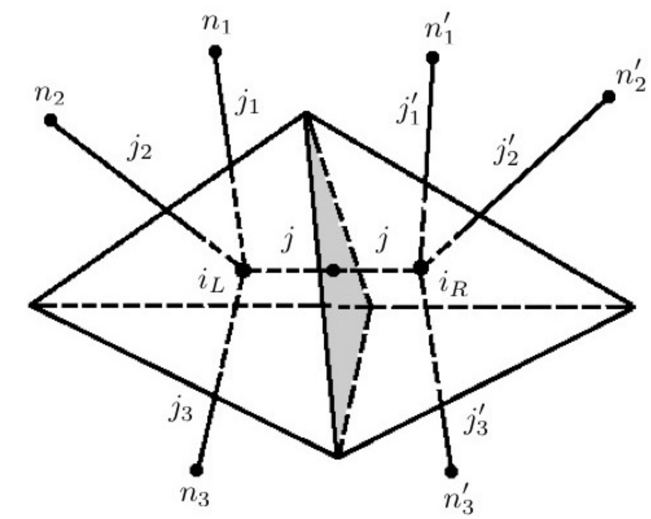

- Primitive entanglement/connectivity (topology) correspondence:

entanglement between entangled vertices = adjacency relations of dual simplices

- Primitive entanglement/area correspondence:

link dual to surface shared by simplices; spin attached to link = eigenvalue of area operator of dual surface

local measure of entanglement prop. to D = dim(H_j) = 2j+1, thus prop. to surface area

- Primitive entanglement/volume correspondence:

vertex/simplex to gluing of links by gauge projection; intertwiner label = volume of simplex

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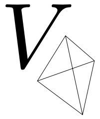
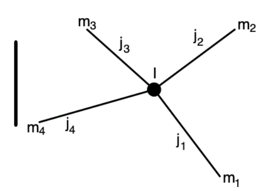
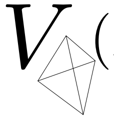
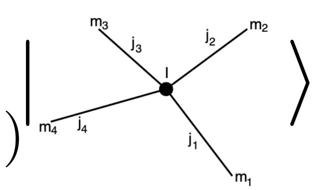
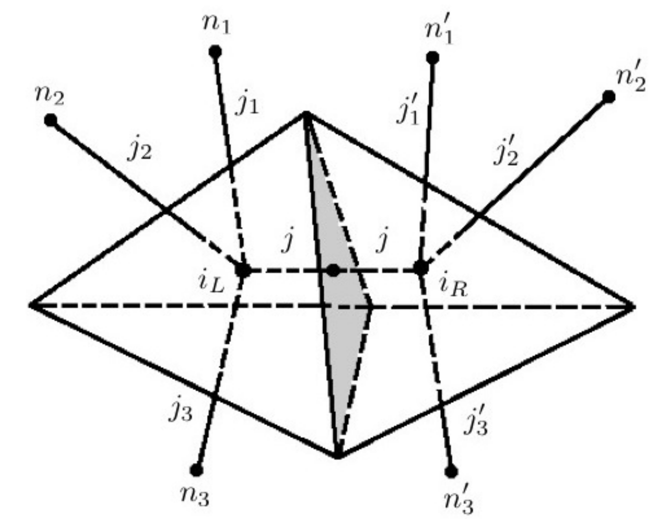
volume operator $\widehat{V}$ 



$= A_{\triangle_i}(j_i)$ / V  $(I(\{j_i\}))$





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- key for obtaining:
  - Ryu-Takanayagi entropy formula
  - holographic bulk-boundary maps
  - holographic boundary-boundary maps

G. Chirco, DO, M. Zhang, '17; G. Chirco, E. Colafranceschi, DO, '21; E. Colafranceschi, S. Langenschedit, DO, '22

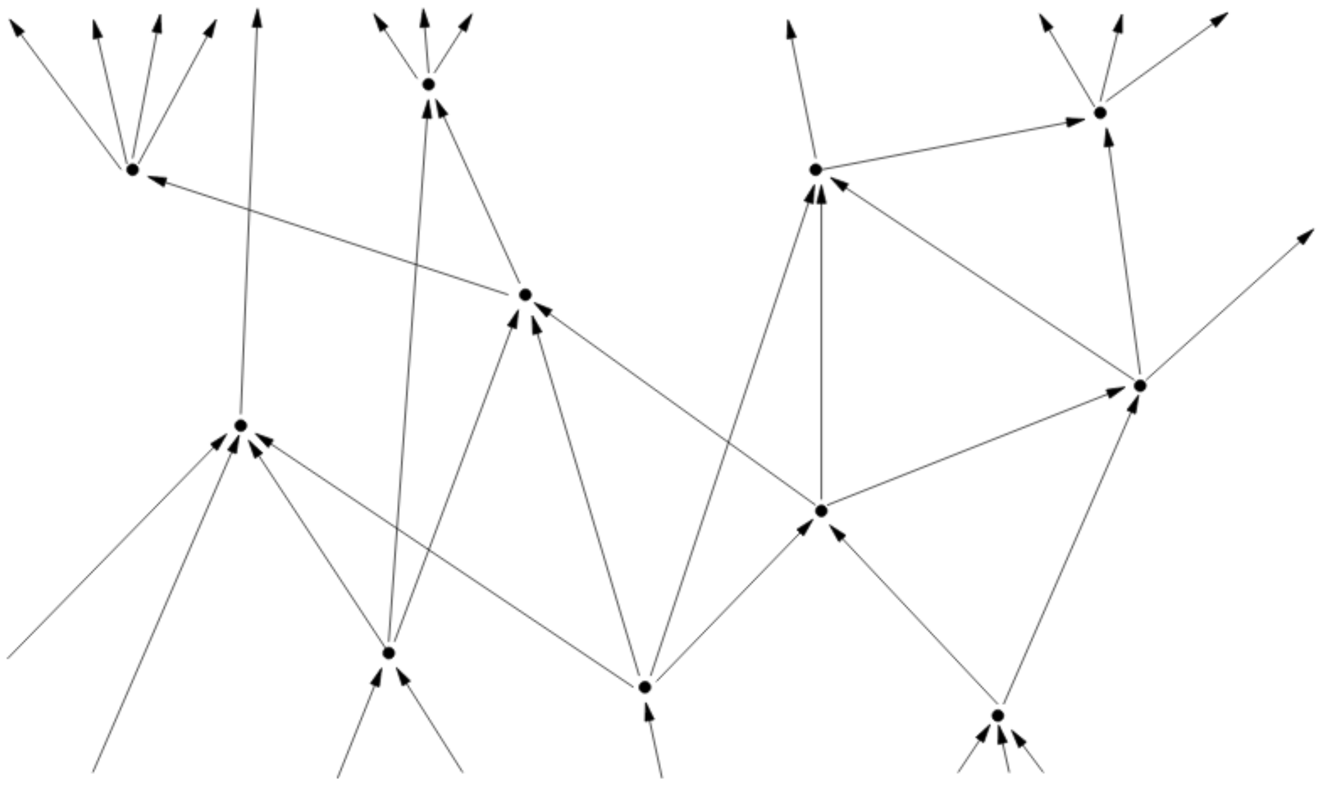
via random tensor-networks techniques

Quantum processes of atoms of space (~ quantum causal histories)

F. Markopoulou, '99; E. Livine, DO, '02; E. Hawkins, F. Markopoulou, H. Sahlmann, '03

- possible process:

set of "events" + order relations between (pairs of) them = directed graph



- simplicial geometric setting --> directed graph = dual 1-skeleton of oriented (simplicial) 4-complex

(simplicial complexes --> only 5-valent nodes)

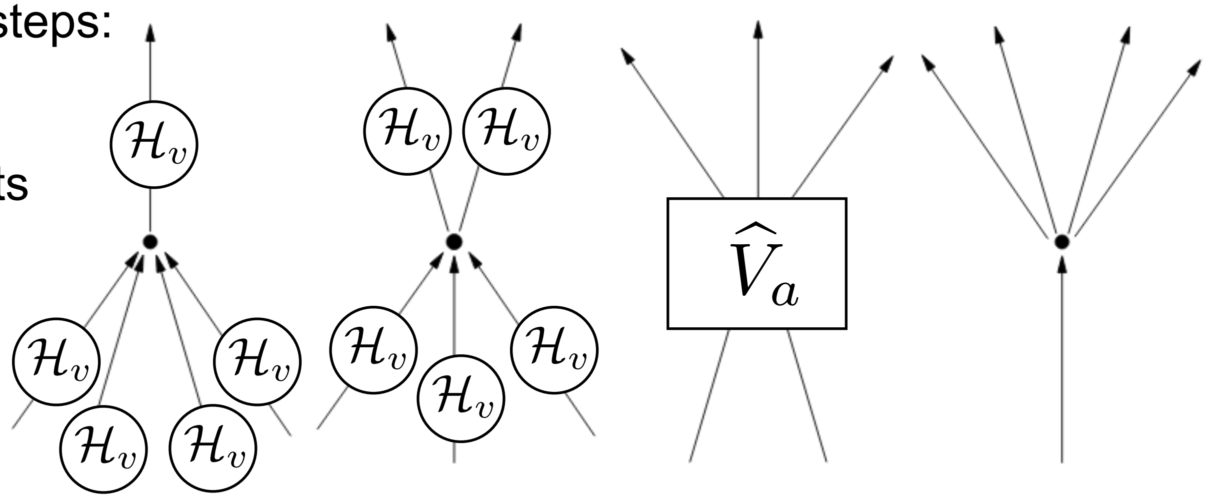
- for Lorentzian models, possible causal interpretation for order relations

- irreflexive directed graph (no closed causal loops) = poset (causal set)

- can be decomposed into building blocks ~ elementary "evolution" steps:

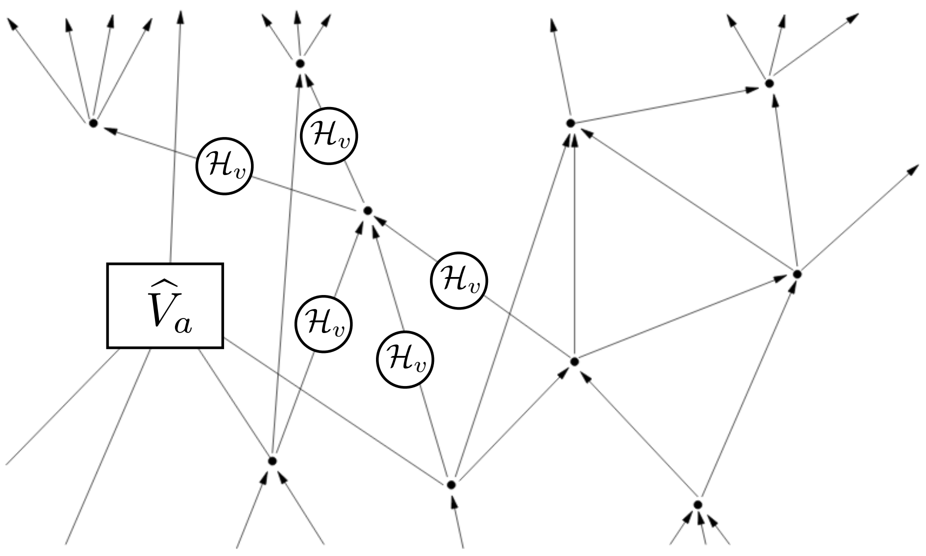
- quantum theory: Hilbert spaces on links/edges (and tensor products for unordered (acausal) links)

- quantum dynamics: elementary "evolution" operators on nodes (+ additional "gluing" operators on links)



Quantum "transition amplitudes" for QG processes - spin foam models (discrete gravity path integrals)

- quantum dynamics: assignment of quantum amplitude to each possible process (directed graph ~ cellular complex)



- full amplitude obtained from elementary operators (kernels):

$$\mathscr{V}_{\mathfrak{a}} \; : \; \bigotimes_{\mathfrak{p}\in\partial\mathfrak{a}} \mathcal{H}_{\mathfrak{p}} \longrightarrow \mathbb{C}$$ node operator

$$\mathscr{K}_e \; : \; \mathcal{H}_{\mathfrak{p}_1} \otimes \mathcal{H}_{\mathfrak{p}_2} \longrightarrow \mathbb{C}$$ gluing operator

- amplitude associated to the whole spin foam complex):

$$\mathcal{A}(\mathfrak{m}) = \mathrm{Tr}_{\mathfrak{p}\in\mathfrak{m}} \left(\prod_{e|\mathfrak{m}} \mathscr{K}_e \prod_{\mathfrak{a}\in\mathfrak{m}} \mathscr{V}_{\mathfrak{a}} \right)$$

where the trace is defined over any complete basis in each link Hilbert space

- sum over histories: complete spin foam model given by sum over all complexes/processes (within chosen class):

$$\boxed{Z = \sum_{\mathfrak{m}} w(\mathfrak{m}) \; \mathcal{A}(\mathfrak{m})}$$

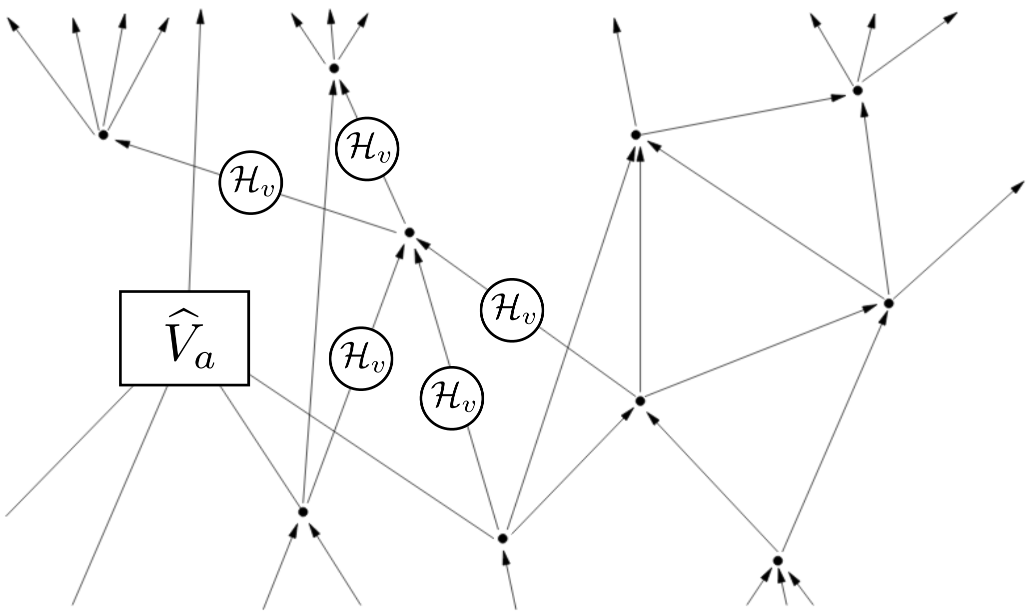
then generalised to the presence of boundary data

- different quantum gravity models (spin foam, LQG, lattice path integrals, TGFT) = different choices of elementary operators (and Hilbert spaces)

- most models constructed to have good simplicial geometry interpretations: amplitudes ~ lattice gravity path integral

$$Z = \sum_{\mathfrak{m}} w(\mathfrak{m}) \; \mathcal{A}(\mathfrak{m}) = \sum_{\Delta} w(\Delta) \int \mathcal{D}g_\Delta \, e^{i\, S_\Delta(g_\Delta)} \equiv \int \mathcal{D}g \, e^{i\, S(g)}$$

Quantum causal histories interpretation



proper causal structure at quantum level? is quantum evolution unitary?

- usual quantum causal history framework:

  each process is causal unitary evolution

for evolution operator between a-causal subsets (incl. elementary operators)

$$E_{\alpha\beta} : \mathcal{H}_\alpha \to \mathcal{H}_\beta$$

$$\mathcal{H}_\alpha = \otimes_i \mathcal{H}(p_i)$$

reflexive: $E_{\alpha\alpha} = Id_\alpha$

antisymmetric: $E_{\alpha\beta}E_{\beta\alpha} = Id_\alpha \Leftrightarrow E_{\alpha\beta} = E_{\beta\alpha} = Id_\alpha$

transitive: $E_{\alpha\beta}E_{\beta\gamma} = E_{\alpha\gamma}$

unitary

F. Markopoulou, '99; E. Hawkins, F. Markopoulou, H. Sahlmann, '03

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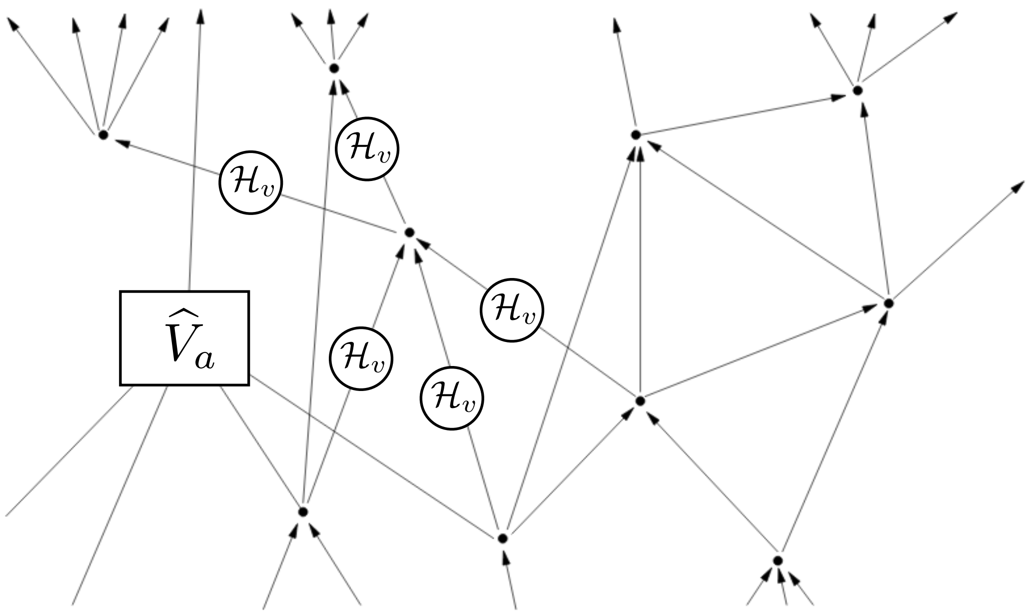

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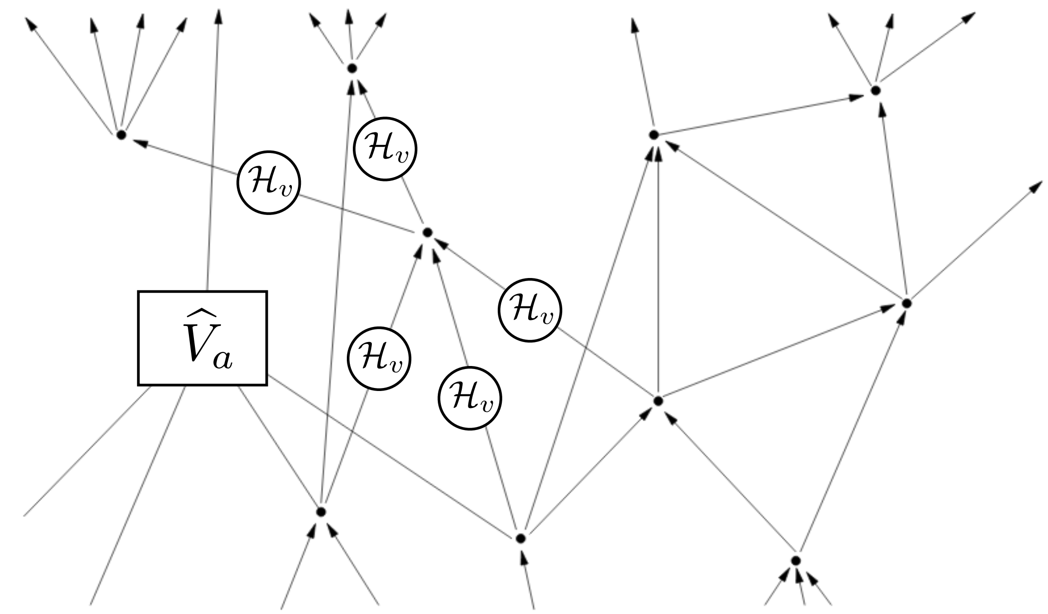

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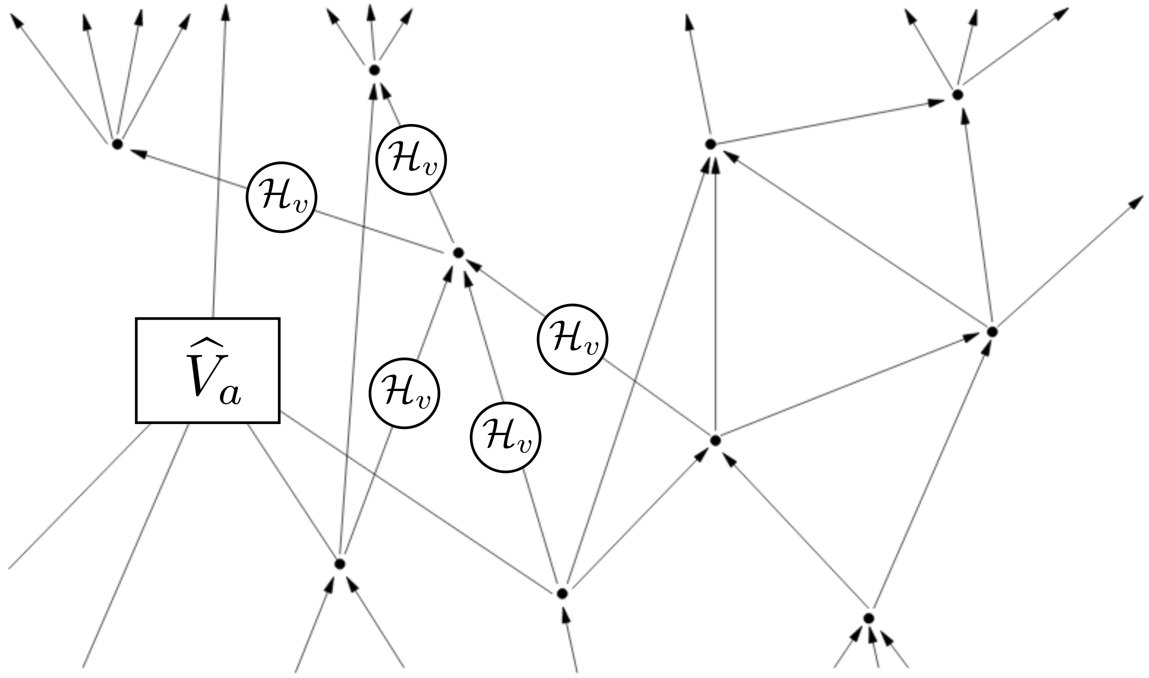
so could also require conditions for "causal unitary evolution" to apply only to full quantum dynamics

E. Livine, DO, '02

- micro reflexive --> full reflexive
- antisymmetry required also on full evolution
- micro transitivity ~ partial triangulation invariance --> too strong in QG
- transitivity for full evolution ~ composition of quantum probability amplitudes $\sum_\beta \mathcal{E}_{\alpha\beta}\mathcal{E}_{\beta\gamma} = \mathcal{E}_{\alpha\gamma}$
- want to impose unitarity of full evolution ---> micro evolution must not be unitary

$$\sum_\beta \mathcal{E}_{\alpha\beta}\,\mathcal{E}^\dagger_{\alpha\beta} = \sum_\beta \mathcal{E}_{\alpha\beta}\bar{\mathcal{E}}_{\beta\alpha} = Id_\alpha \quad \Longrightarrow \quad \sum_\beta E^c_{\alpha\beta}\,E^{c\dagger}_{\alpha\beta} = \sum_\beta E^c_{\alpha\beta}\bar{E}^c_{\beta\alpha} \neq Id_\alpha$$

- proper causal dynamics requires:
  - dependence on orientation (ordering)
  - absence (or "irrelevance") of closed evolution (causal) loops
  - suitable conditions of evolution operators (to ensure unitarity)



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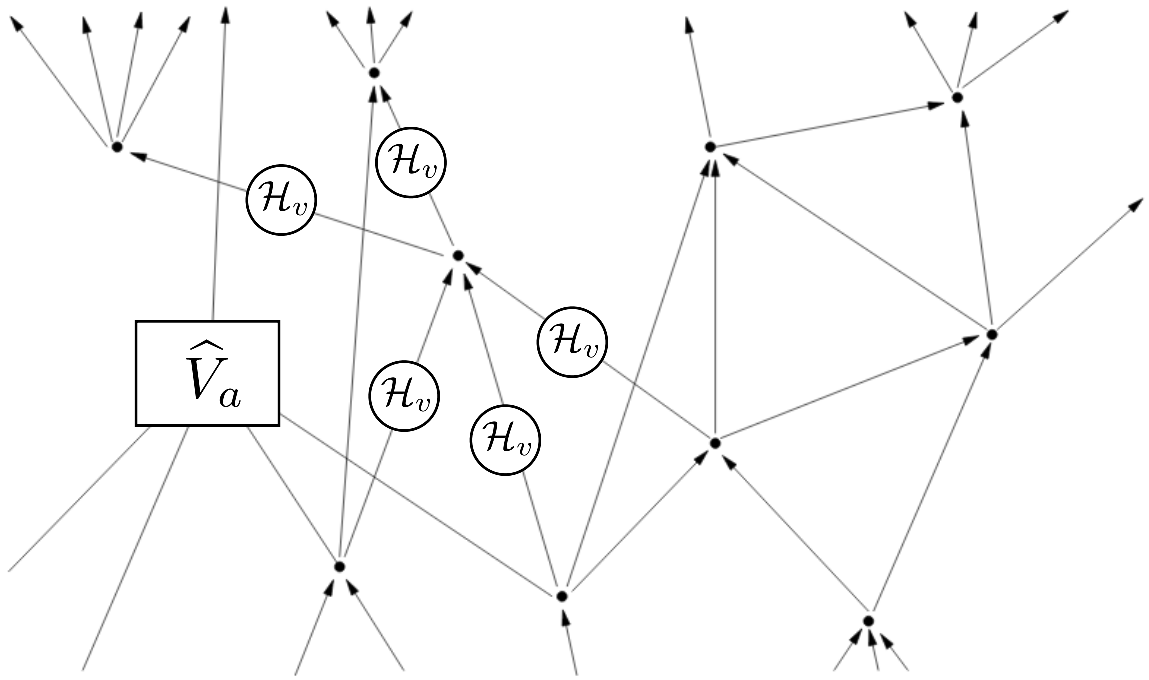

Causal hiccups - closed timelike loops - harmless?

- directed graphs associated to QG dynamics contain, in general, causal loops (CTCs)
  - possible strategies:
    - define (modified) quantum dynamics that eliminates causal loops
    - define (modified) quantum dynamics that suppresses causal loops
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- when is a causal loop harmless?

E. Livine, D. Terno, '06

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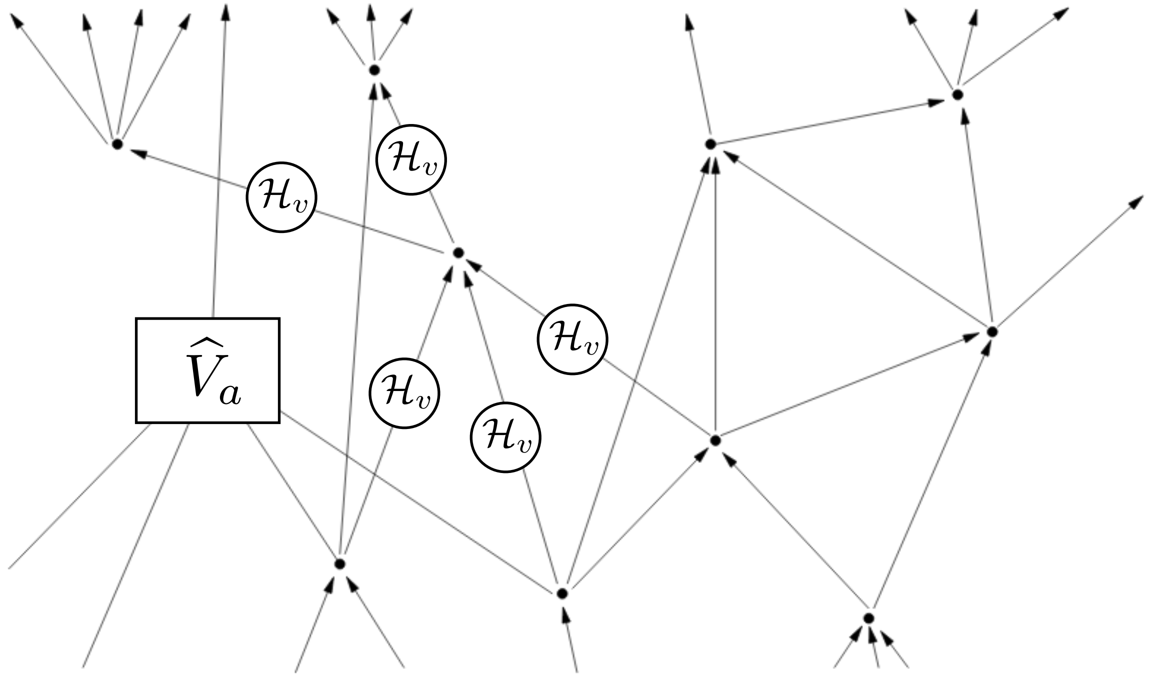

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E. Livine, D. Terno, '06

Causal indifference - inner product vs transition amplitudes in QG dynamics

- all most studied spin foam models are invariant under switch of orientation of simplicial structures
  - they do not "register" the orientation of the complex associated to them

- but orientation of (1-skeleton of) complex ~ order relations in underlying directed graph ~ causal structure in QCH formulation of spin foam models

$\Longrightarrow$ $\boxed{\text{none of the most studied spin foam models defines proper QCH and unitary QG quantum dynamics}}$

- can construct "properly causal" modifications of existing spin foam models

E. Livine, DO, '02

J. Engle, '11, '12; J. Engle, A. Zipfel, '15;
E. Bianchi, P. Martin-Dussaud, '21

by truncating quantum geometric configurations summed over to single orientation

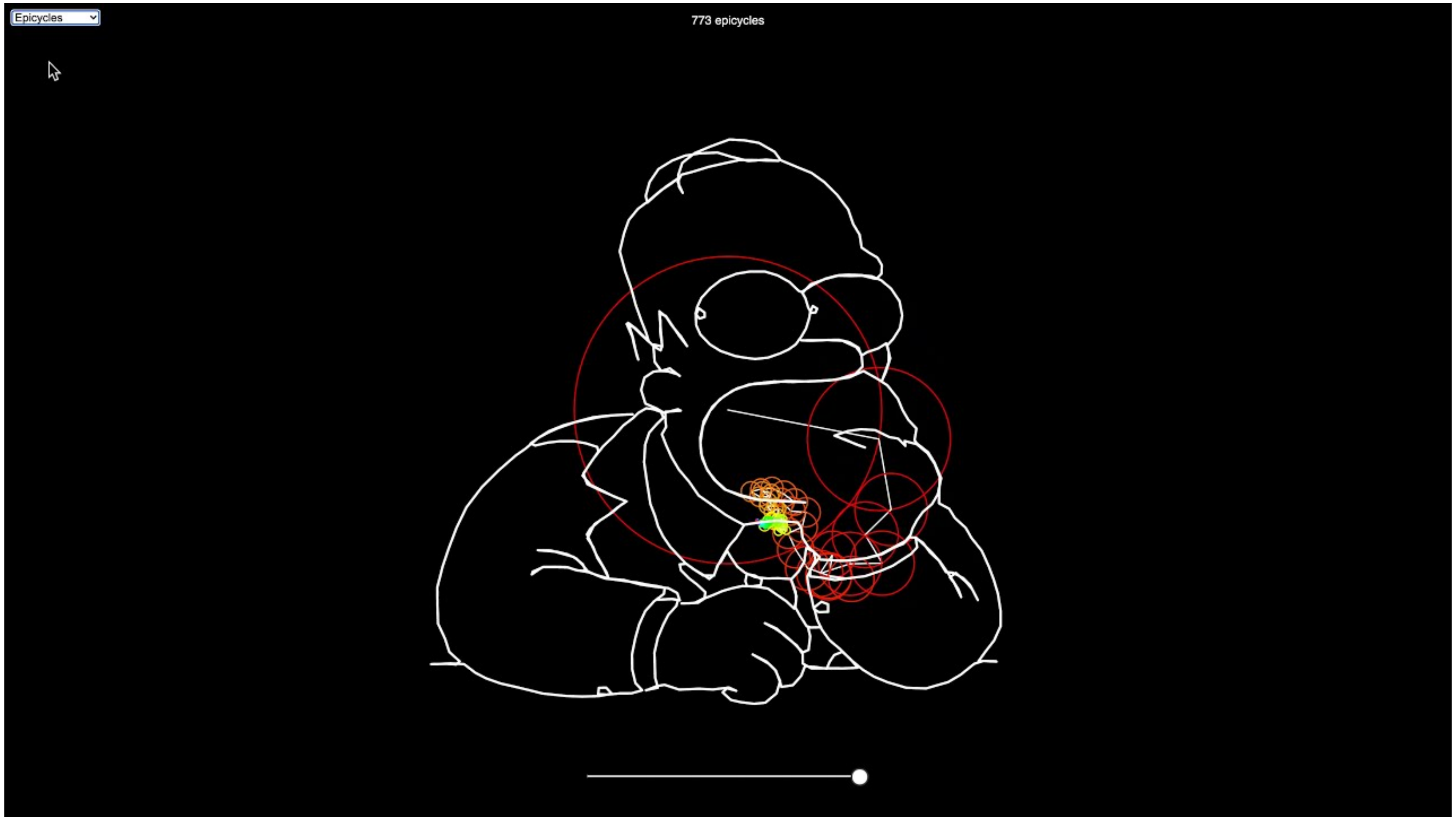
- only such "causally modified" spin foam amplitudes can be formulated as QCH and can define unitary quantum dynamics

but procedure is rather ad hoc, so far

Surprises from Quantum Gravity?

EFT and spacetime-based physics is powerful, and could even be fitting observations successfully

but it could also be very misleading, and reliance on it may hide new underlying principles/mechanisms



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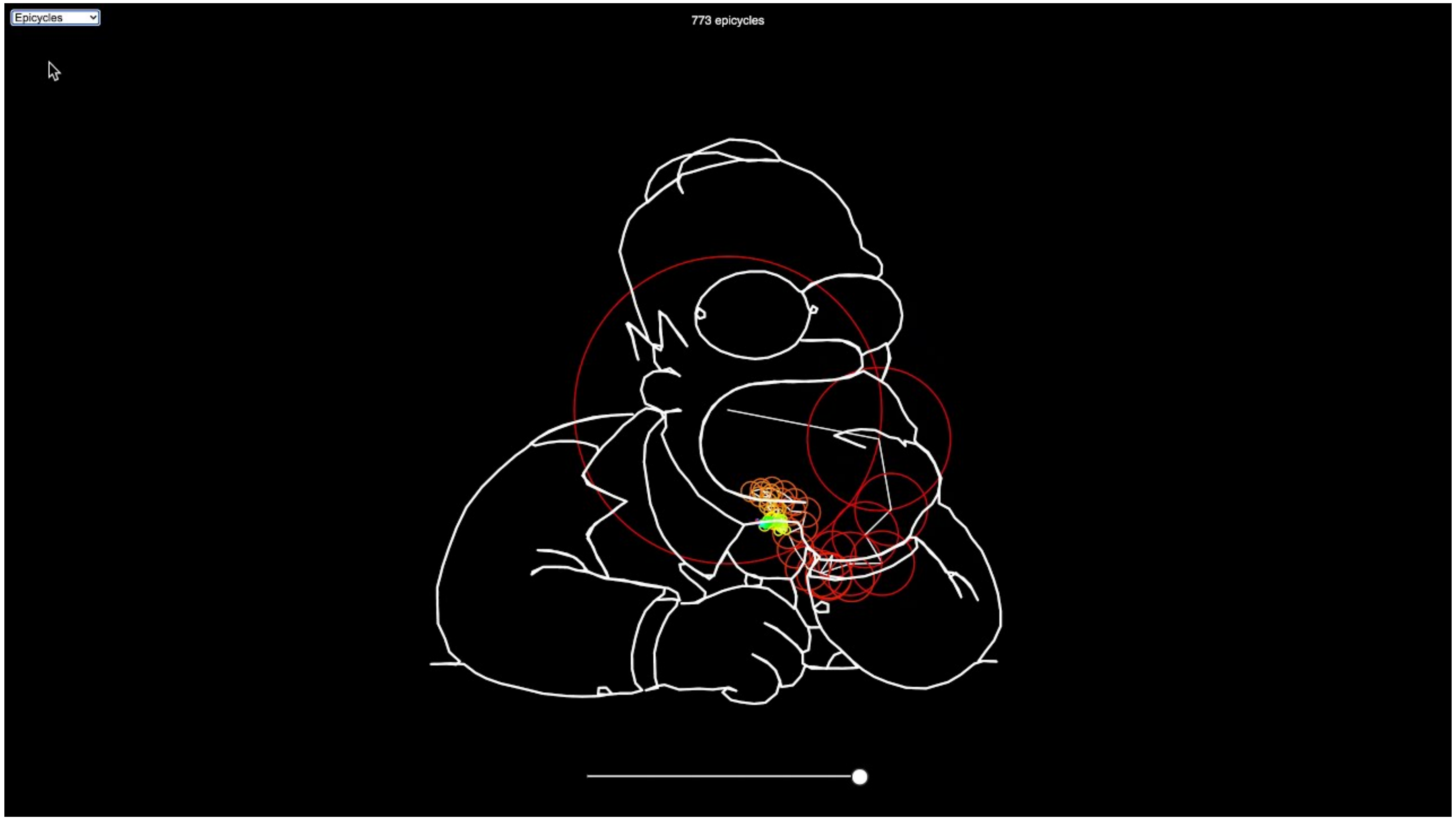
an efficient mathematical framework + enough scalar fields with weird enough potentials can fit observations
.... just like epicycles



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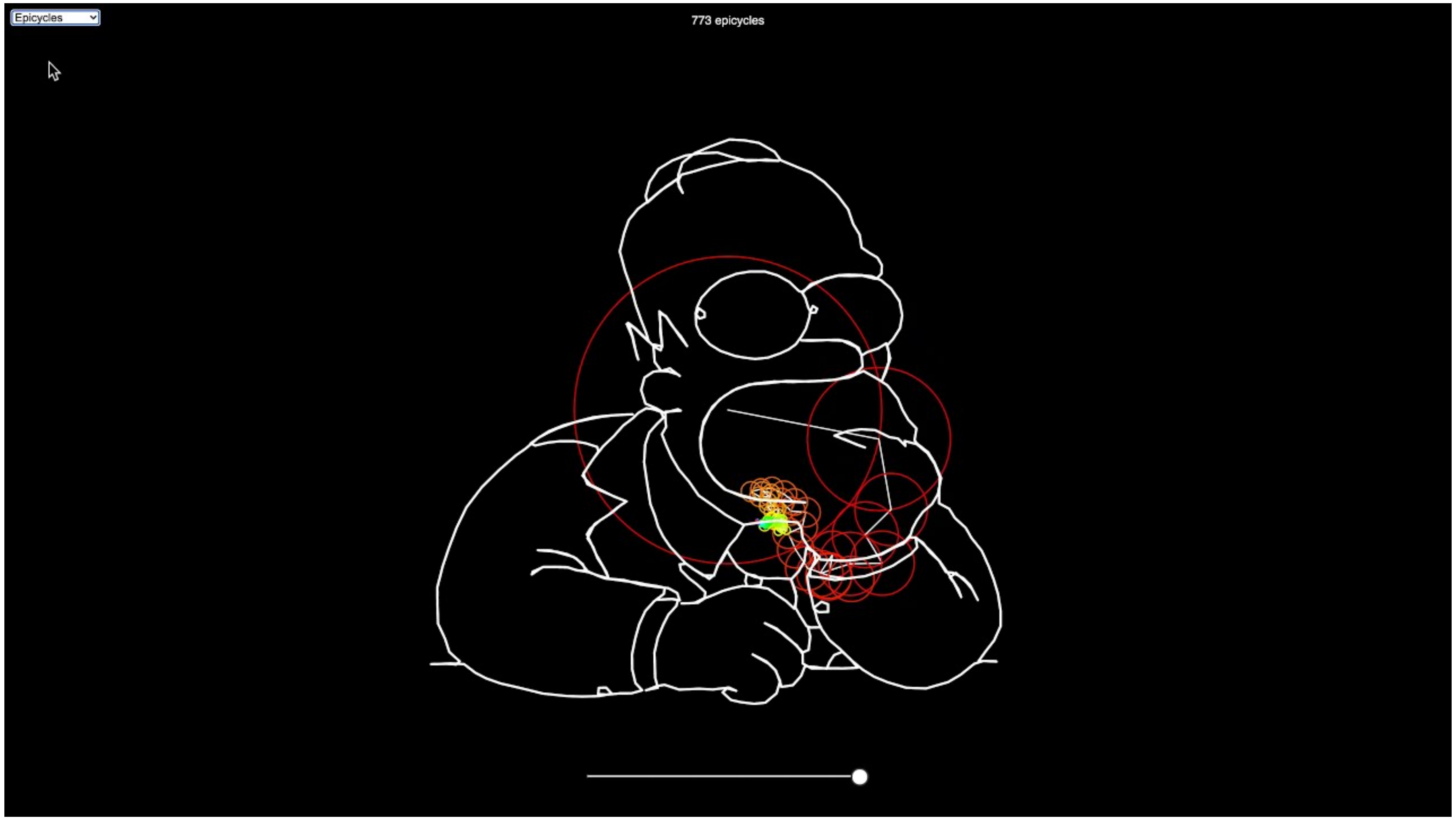

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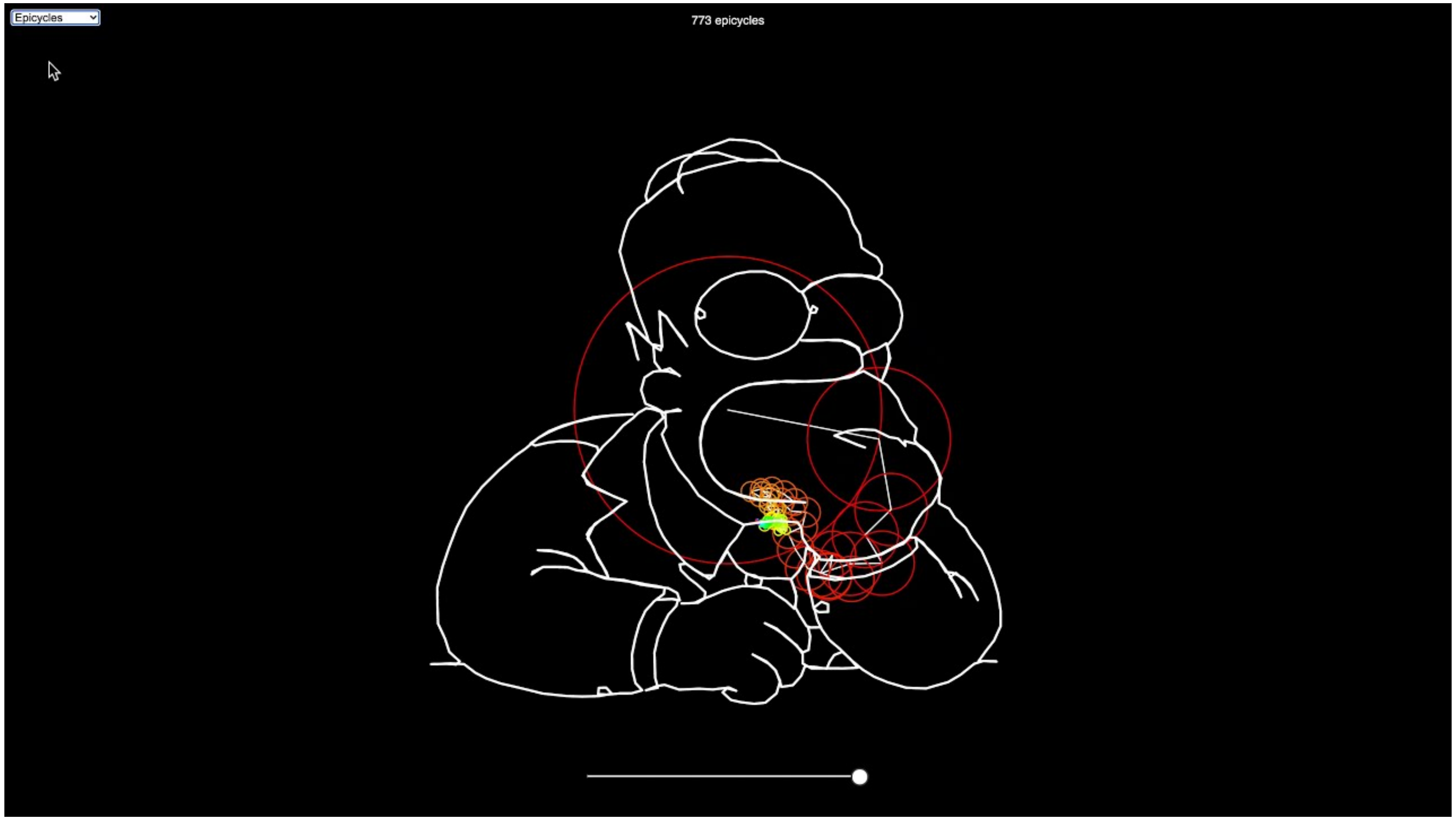

emergent spacetime scenarios (more generally, QG formalisms) introduce new perspective and new tools

QG and cosmology: possible consequences of emergent spacetime scenarios

- if spacetime is emergent, all main ingredient of EFT will eventually break down
- therefore, EFT intuition should be taken with care, if not suspect
- this applies also to phenomena at large distance scales, because the very idea that QG effects are confined at high energies/small distances, is based on EFT intuition and separation of scales
- there may well be underlying QG mechanisms, not captured by EFT techniques nor intuition, that provide natural (if not universal) solutions to cosmological puzzles

an example could be dark energy and
the cosmological constant problem

several emergent spacetime models offer interesting suggestions for concrete mechanisms

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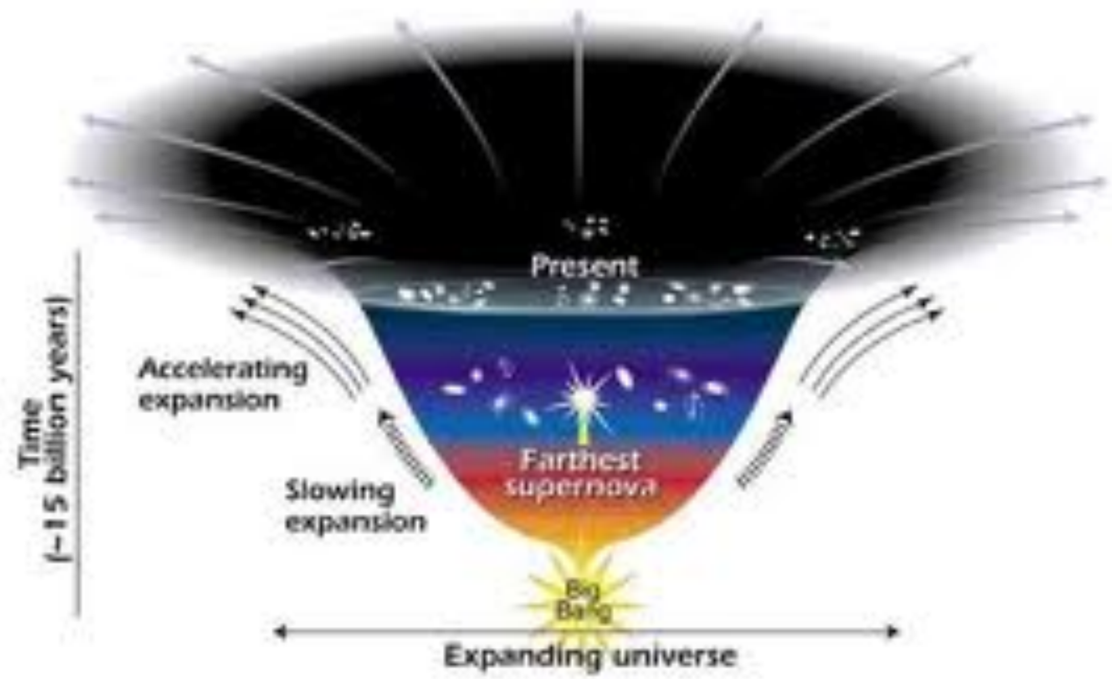

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QG and cosmology: possible consequences of emergent spacetime scenarios

example: late time acceleration in Group Field Theory condensate cosmology

S. Gielen, DO, L. Sindoni, E. Wilson-Ewing, L. Marchetti, ...

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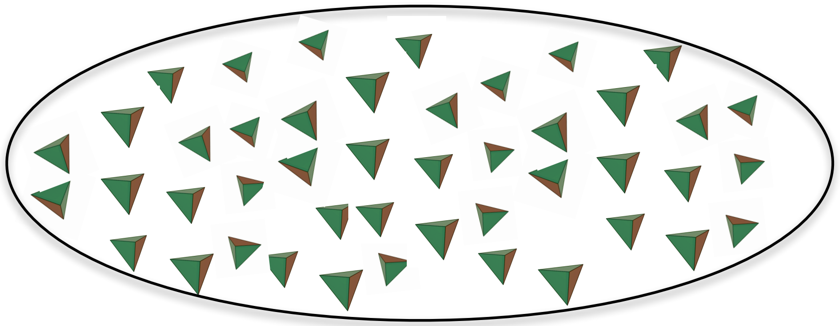
- QFT of quantum simplices - Feynman diagrams = d>2 lattices - Feynman amplitudes = lattice gravity path integrals

$$\varphi : G^{\times d} \to \mathbb{C} \qquad S(\varphi, \overline{\varphi}) = \frac{1}{2} \int [dg_i] \overline{\varphi(g_i)} \mathcal{K}(g_i) \varphi(g_i) + \frac{\lambda}{D!} \int [dg_{ia}] \varphi(g_{i1}) \dots \varphi(\bar{g}_{iD}) \mathcal{V}(g_{ia}, \bar{g}_{iD}) \quad + \quad c.c.$$

- for suitable "quantum geometric" models:
  - consider fluid of quantum tetrahedra ("universe as QG condensate")
  - mean field GFT condensate hydrodynamics ~ ~ non-linear eqn for "wavefunction" on minisuperspace (space of homogenous geom)
  - obtain effective dynamics for universe volume



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- semiclassical Friedmann eqn at large volumes/late times
- quantum bounce replacing big bang singularity

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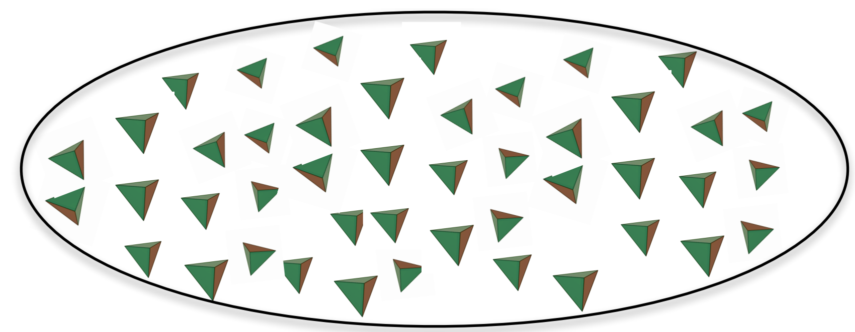

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- late times: as universe expands, interactions end up driving evolution

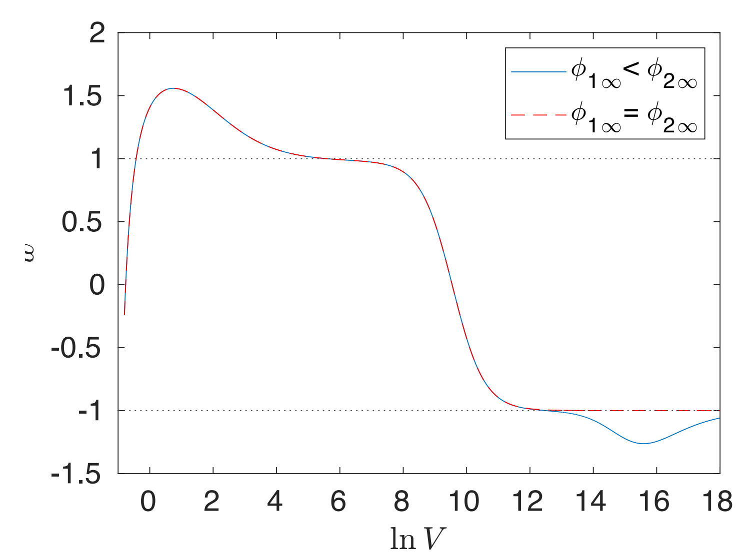
  → accelerated cosmological expansion

X. Pang, DO, '21, '25

- effective cosmological dynamics $\qquad w = 3 - \dfrac{2VV''}{(V')^2}$

  for "emergent matter" component (of QG origin)

→ effective phantom-like dark energy (of pure QG origin)



+ asymptotic De Sitter universe

lesson for QG:

cosmological constant/dark energy is to be understood from full QG theory, from collective behaviour of “quantum atoms of space”, in analogy with condensed matter systems and in a “geometrogenesis” scenario

Conclusions

Beyond spacetime?

… learn to think without space and time ….

Einstein (1936): *"the introduction of a space-time continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. [...] perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature, that is to the elimination of continuous functions from physics. Then, however, we must also give up, by principle, the space-time continuum. It is not unimaginable that human ingenuity will some day find methods which will make it possible to proceed along such a path. At the present time, however, such a program looks like an attempt to breathe in empty space."*

slowly, rather painfully (but still enthusiastically),
we are learning to breathe in empty space....

Thank you for your attention!