GENERALISATION OF THE EQUIVALENCE PRINCIPLE TO QUANTUM REFERENCE FRAMES

Flaminia Giacomini

ETH Zürich

Image credits: J. Palomino

Solstice of Foundations
University of Zurich, 16-20 June 2025

THE EQUIVALENCE PRINCIPLE

"The principle of equivalence has great power. With it one can generalize all the special relativistic laws of physics to curved spacetime. And the curvature need not be small."

Misner, Thorne, Wheeler "Gravitation"

- worldlines of massive test bodies are time-like geodesics

- photons propagate along null geodesics

- there is a link […] between the proper time registered by an ideal clock and the metric

Brown, Read, Am. J. Phys. (2016)

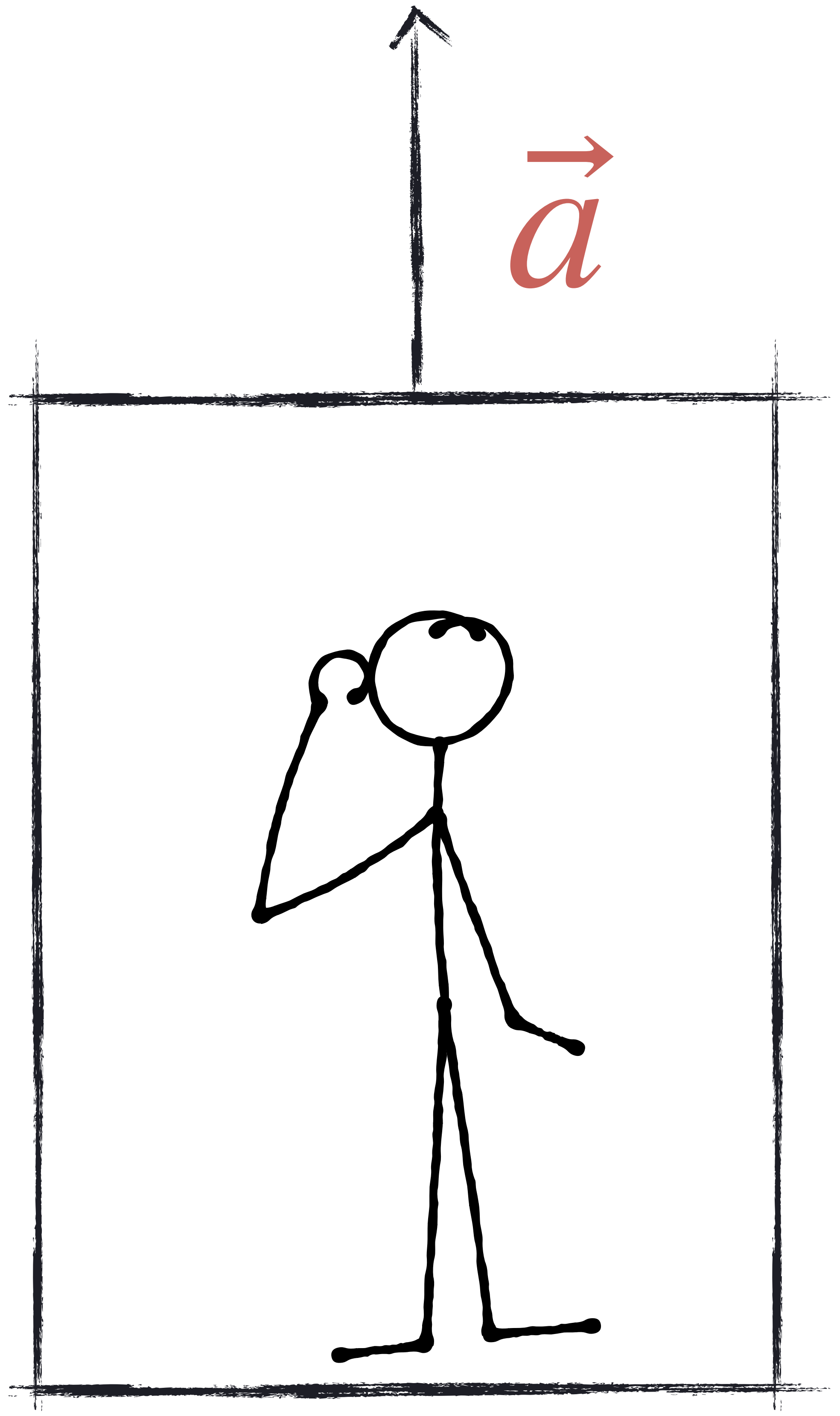
CONCEPTUAL FOUNDATIONS OF GENERAL RELATIVITY

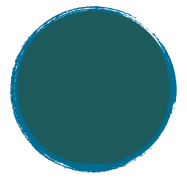
TOOL FOR CONSTRUCTING THE THEORY

The (Einstein) Equivalence Principle encodes the metric character of the theory.

THE WEAK EQUIVALENCE PRINCIPLE (WEP)

THE WEAK EQUIVALENCE PRINCIPLE (WEP)

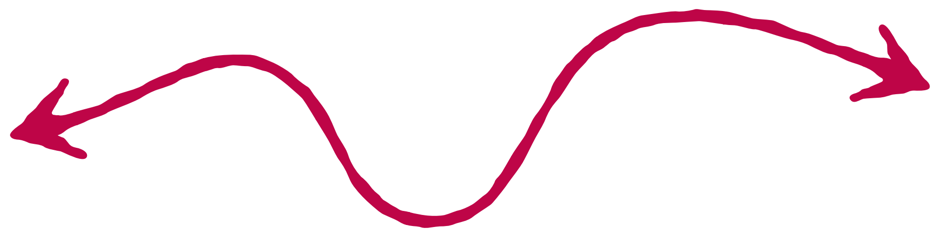


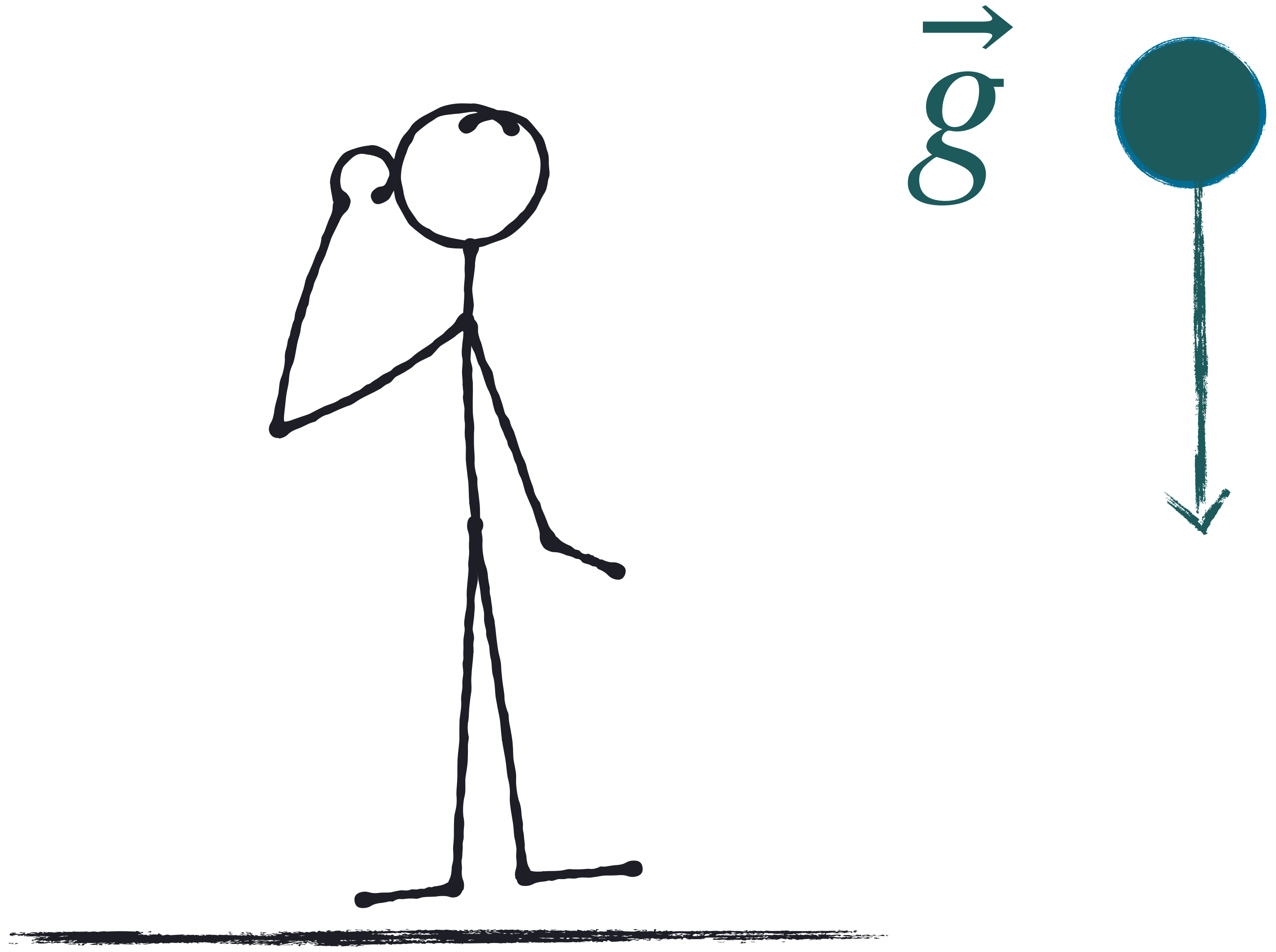


$$\vec{F} = m_I \vec{a}$$

$$\vec{F}_G = m_g \vec{g}$$

$$m_I = m_g$$

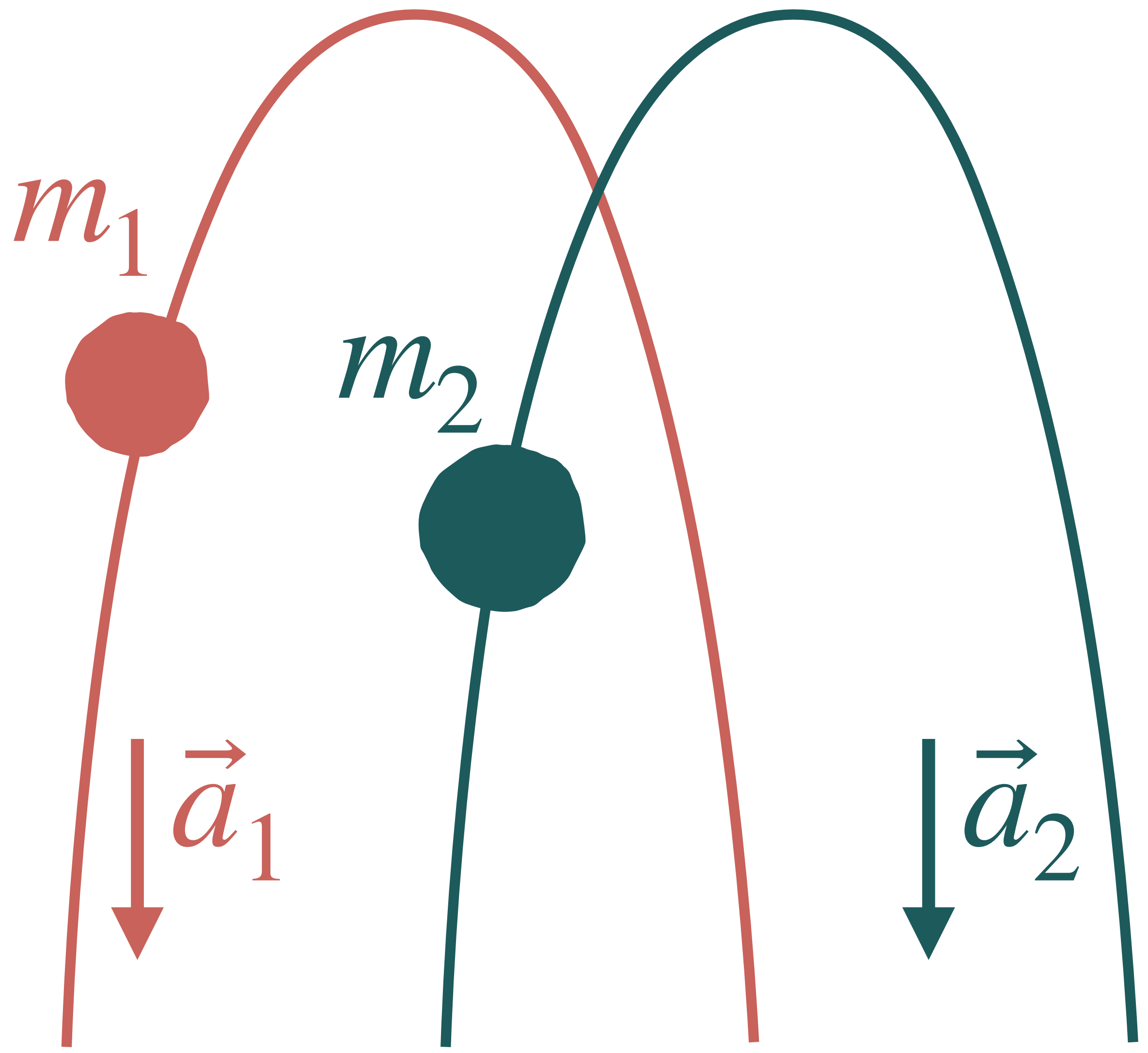




Universality of free-fall: *a constant acceleration is indistinguishable from a constant gravitational field*

C.M. Will, Theory and experiments in gravitational physics, Cambridge University Press (2018)

TEST OF THE VIOLATION OF WEP



$$m_I \neq m_g$$

Internal energy contributes differently to m_I and m_g

$$m_g = m_I + \sum_A \eta_A \frac{E_A}{c^2}$$

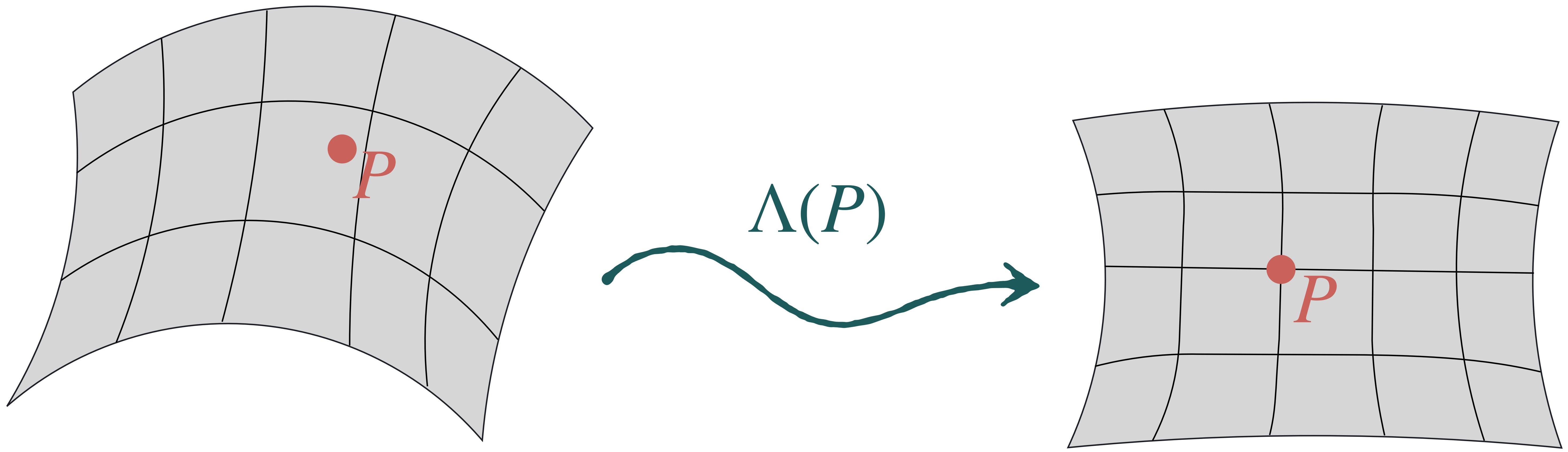
$$\eta = 2\frac{|\vec{a}_1 - \vec{a}_2|}{|\vec{a}_1 + \vec{a}_2|} = \sum_A \left| \eta_A^1 \frac{E_A^1}{m_1 c^2} - \eta_A^2 \frac{E_A^2}{m_2 c^2} \right|$$

Eötvos ratio

C.M. Will, Theory and experiments in gravitational physics, Cambridge University Press (2018)

THE EINSTEIN EQUIVALENCE PRINCIPLE (EEP)

THE EEP AND THE METRICITY OF GENERAL RELATIVITY

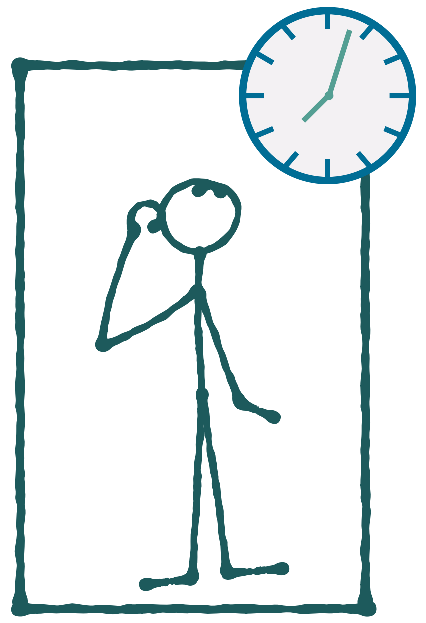


In any and every Locally Inertial Frame (LIF), anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.

Misner, Thorne, Wheeler "Gravitation"

$$\frac{d^2X^\mu}{d\tau^2} = 0$$

$$\frac{d^2x^\mu}{d\tau^2} + \tilde{\Gamma}^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$



$$\tilde{g}_{\mu\nu} = \frac{dX^\alpha}{dx^\mu}\frac{dX^\beta}{dx^\nu}\eta_{\alpha\beta}$$

Einstein Equivalence Principle

$$\tilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} \qquad \tilde{g}_{\mu\nu} = g_{\mu\nu}$$

S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity (Wiley 1972).

THE EEP AND THE METRICITY OF GENERAL RELATIVITY

- The **WEP** is valid
- **Local Lorentz Invariance (LLI):** the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed
- **Local Position Invariance (LPI):** the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.

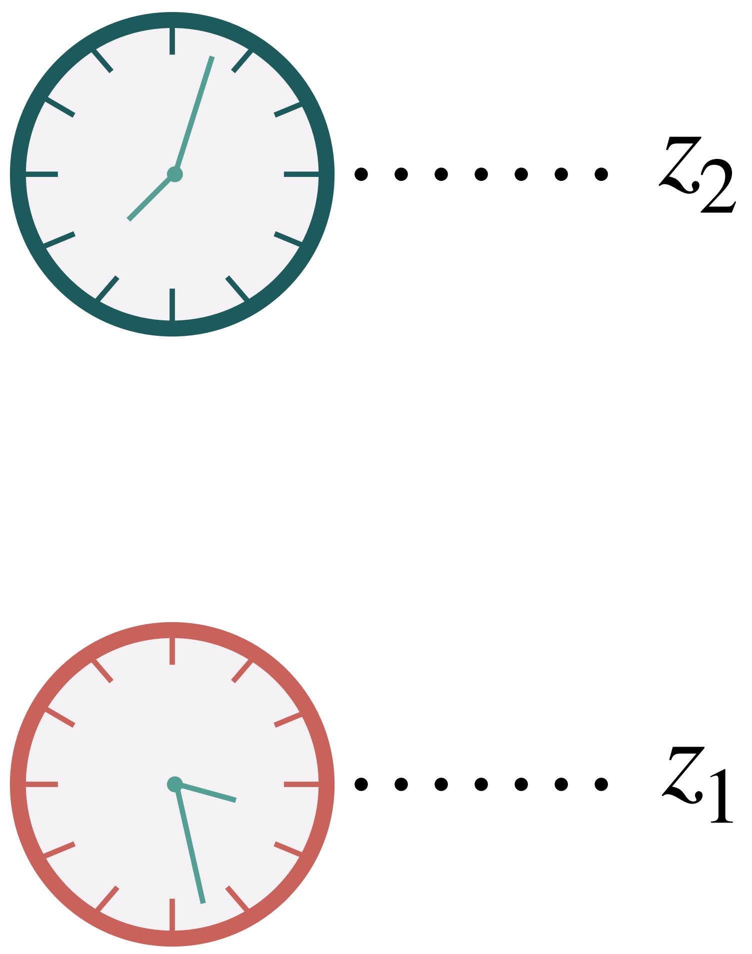
C.M. Will, Theory and experiments in gravitational physics, Cambridge University Press (2018)

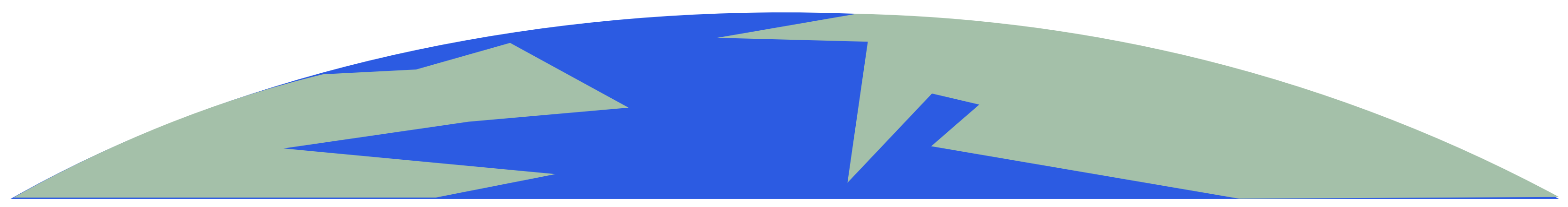
GRAVITY INFLUENCES HOW A CLOCK TELLS THE TIME

"Lower is slower"

$$\tau_2 = \Delta t(1 + V_N(z_2))$$

$$\tau_1 = \Delta t(1 + V_N(z_1))$$





Article | Published: 16 February 2022

Resolving the gravitational redshift across a millimetre-scale atomic sample

Tobias Bothwell, Colin J. Kennedy, Alexander Aeppli, Dhruv Kedar, John M. Robinson, Eric Oelker, Alexander Staron & Jun Ye

Nature **602**, 420–424 (2022) | Cite this article

Article | Open access | Published: 12 August 2023

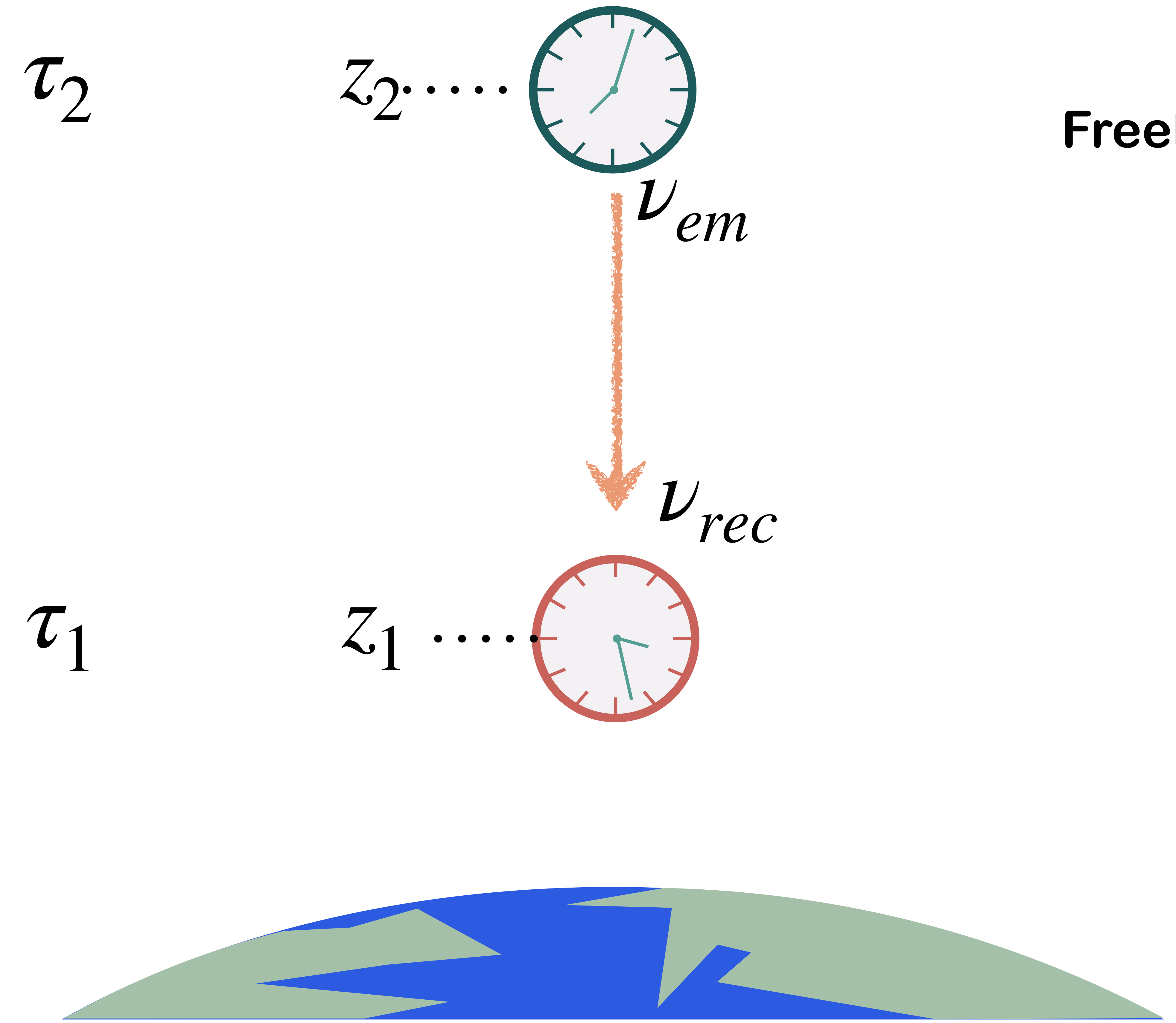
A lab-based test of the gravitational redshift with a miniature clock network

Xin Zheng, Jonathan Dolde, Matthew C. Cambria, Hong Ming Lim & Shimon Kolkowitz

Nature Communications **14**, Article number: 4886 (2023) | Cite this article

Chou, Hume, Rosenband, Wineland, Science (2010)

THE UNIVERSALITY OF GRAVITATIONAL REDSHIFT (UGR)



$$\Phi_i = g z_i + \cdots$$

Assume WEP, LLI

Freely-falling frame momentarily at rest with the clock

$$c^2 d\tau^2 = c^2 dt_F^2 - d\vec{x}_F^2$$

Violation of LPI

$$c^2 d\tau^2 = \tau^2(\Phi)(c^2 dt_F^2 - d\vec{x}_F^2)$$

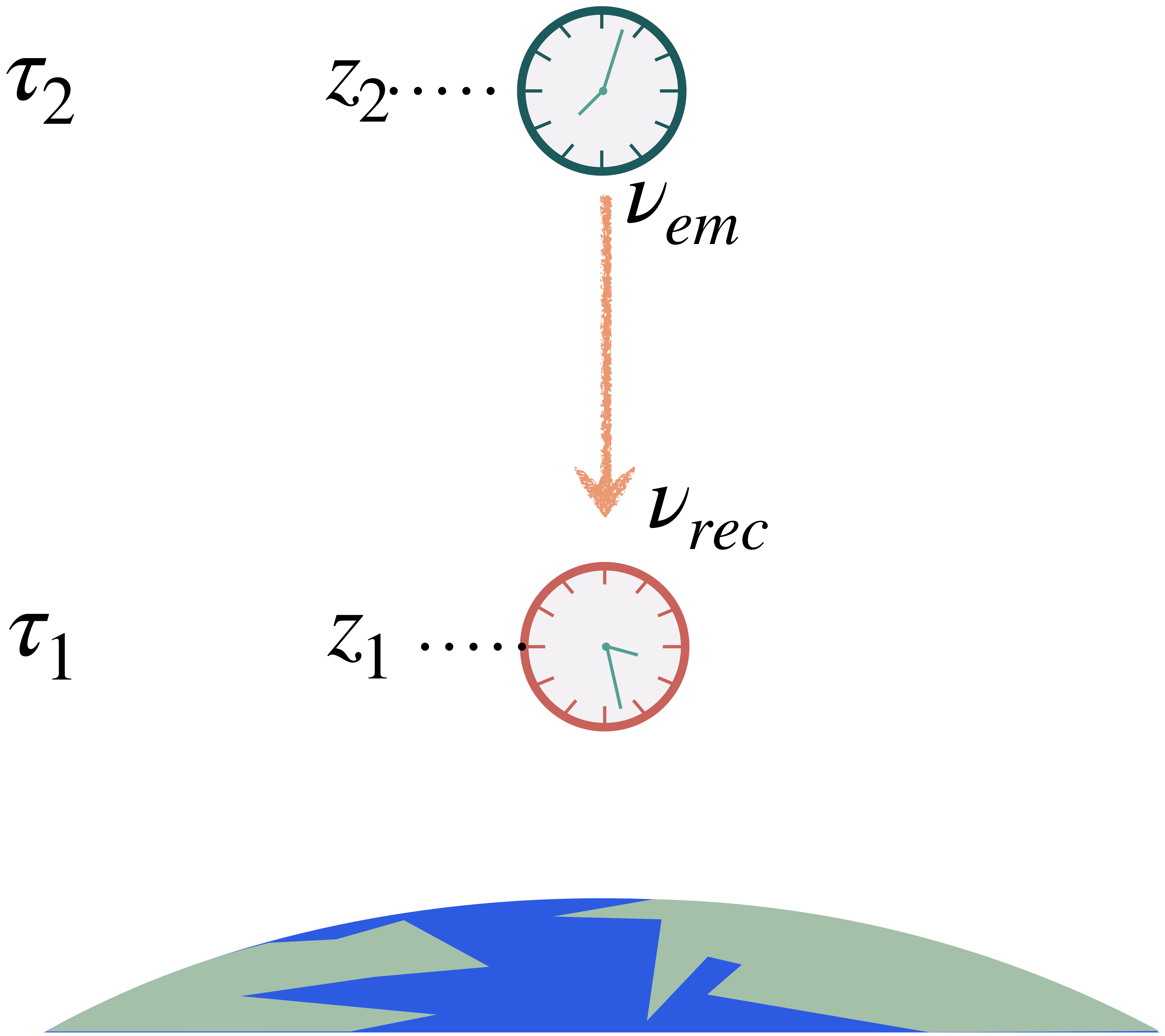
$$c^2 d\tau^2 = \tau^2(\Phi)\left[\left(1 + \frac{g z_S}{c^2}\right)^2 c^2 dt_S^2 - d\vec{x}_S^2\right]$$

$$\nu_{em} = \tau^{-1}(\Phi_{em})\left(1 + \frac{g z_2}{c^2}\right)^{-1}$$

$$\nu_{rec} = \tau^{-1}(\Phi_{rec})\left(1 + \frac{g z_1}{c^2}\right)^{-1}$$

C.M. Will, Theory and experiments in gravitational physics, Cambridge University Press (2018)

THE UNIVERSALITY OF GRAVITATIONAL REDSHIFT (UGR)



$$\Phi_i = g z_i + \cdots$$

$$\nu_{em} = \tau^{-1}(\Phi_{em})\left(1 + \frac{g z_2}{c^2}\right)^{-1}$$

$$\nu_{rec} = \tau^{-1}(\Phi_{rec})\left(1 + \frac{g z_1}{c^2}\right)^{-1}$$

$$Z = \left|\frac{\nu_{rec} - \nu_{em}}{\nu_{rec}}\right| \approx \left|1 - \frac{\tau(\Phi_{rec})}{\tau(\Phi_{em})}\left(1 + \frac{g\Delta z}{c^2}\right)\right|$$

$$\tau(\Phi_{em}) = \tau_0 \qquad \tau(\Phi_{rec}) = \tau_0\left(1 + \alpha\frac{g\Delta z}{c^2}\right)$$

$$Z \approx (1+\alpha)\frac{g\Delta z}{c^2}$$

C.M. Will, Theory and experiments in gravitational physics, Cambridge University Press (2018)

QUANTUM EQUIVALENCE PRINCIPLE(S)

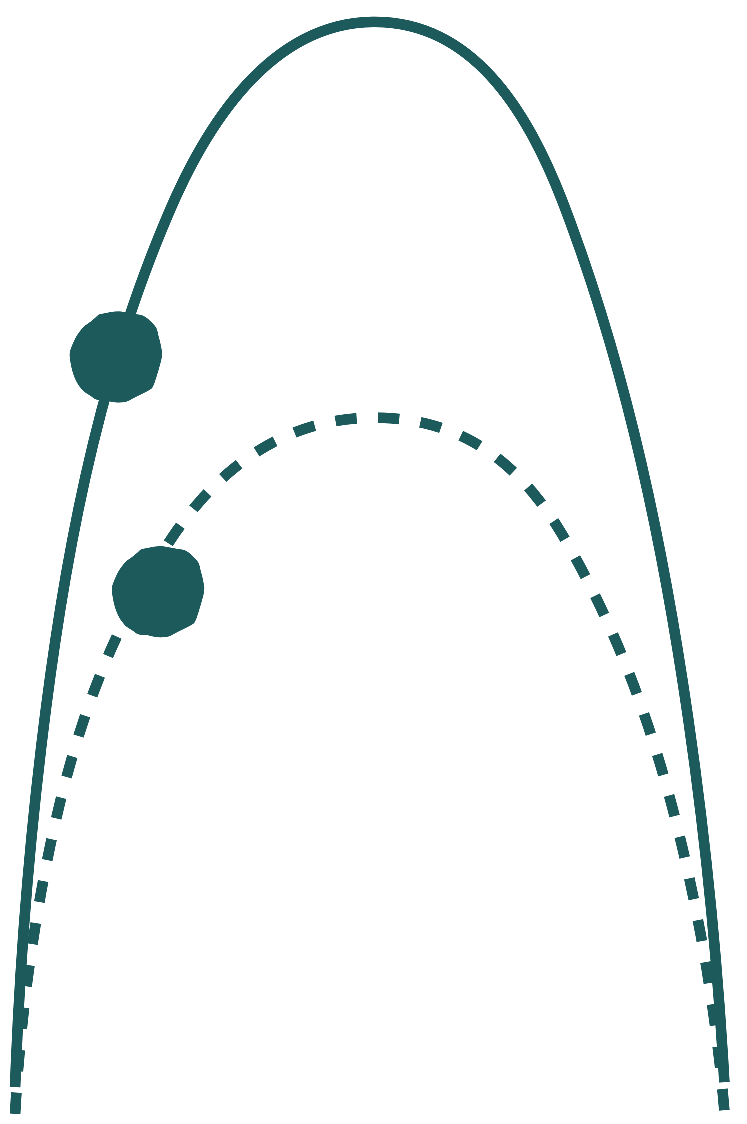
WHY A QUANTUM EQUIVALENCE PRINCIPLE

$$\psi(\vec{x}, t)$$

$$i\hbar \frac{\partial \psi(\vec{x}, t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \vec{\nabla}^2 + mgx \right) \psi(\vec{x}, t)$$

$G(\vec{x}, t, \vec{x}_0, t_0)$ **depends on the mass of the particle!**

Hu, Anastopoulos, Class. Quant. Grav. (2018)



- the trajectories of the particles that are used to test the EP may be replaced by quantum states, which can be delocalised in space
- the reference frames attached to the particles (or the clocks) become QRFs when such particles (or clocks) are in a quantum superposition state
- the internal energies may become quantum operators
- gravity is quantum!

DIFFERENT APPROACHES TO THE QUANTUM EQUIVALENCE PRINCIPLE

Quantum Aspects of the Equivalence Principle

Y. Aharonov

Department of Physics, University of South Carolina, Columbia, South

and G. Carmi[2]

Department of Physics, St. John's University, Jamaica, New York

On Gravity's Role in Quantum State Reduction

Roger Penrose[1,2]

Received August 22, 1995. Rev. version December 12, 1995

On the Equivalence Principle in Quantum Theory

Claus Lämmerzahl[1,2]

Received May 15, 1995. Rev. version September 20, 1995

Can quantum probes satisfy the weak equivalence principle?

Luigi Seveso [a], Matteo G.A. Paris [a b]

Quantum mechanics and the equivalence principle

P C W Davies

Australian Centre for Astrobiology, Macquarie University, New South Wales 2109, Australia

Equivalence principle for quantum systems: dephasing and phase shift of free-falling particles

C Anastopoulos[1] and B L Hu[2]

Article | Published: 13 August 2018

Quantum formulation of the Einstein equivalence principle

Magdalena Zych & Časlav Brukner

Nature Physics **14**, 1027–1031 (2018) | Cite this article

Implementation of the Quantum Equivalence Principle

Lucien Hardy
Perimeter Institute
31 Caroline Street North
Waterloo, Ontario N2L 2Y5, Canada

...and more!

PHYSICAL REVIEW D VOLUME 27, NUMBER 8 15 APRIL 1983

Is there a quantum equivalence principle?

P. Candelas
Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712,
Department of Astrophysics, Oxford University, England,
and International School for Advanced Studies, Trieste, Italy

D. W. Sciama
International School for Advanced Studies, Trieste, Italy,
International Center for Theoretical Physics, Trieste, Italy,
and Department of Astrophysics, Oxford University, England
(Received 12 October 1982)

VOLUME 55, NUMBER 2 15 JANUARY 1997

Testing the equivalence principle through freely falling quantum objects

Lorenza Viola and Roberto Onofrio
Dipartimento di Fisica "G. Galilei," Università di Padova, and INFN, Sezione di Padova, Via Marzolo 8, Padova, Italy 35131
(Received 16 July 1996)

RESEARCH ARTICLE | JANUARY 05 2022

Quantum superposition of spacetimes obeys Einstein's equivalence principle

Special Collection: Celebrating Sir Roger Penrose's Nobel Prize

Flaminia Giacomini; Časlav Brukner

Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames

Flaminia Giacomini[1,2,*] and Časlav Brukner[3,4]

Observation of the quantum equivalence principle for matter-waves

Or Dobkowski,[1] Barak Trok,[1] Peter Skakunenko,[1] Yonathan Japha,[1] David Groswasser,[1] Maxim Efremov,[2,3] Chiara Marletto,[4] Ivette Fuentes,[5,6] Roger Penrose,[7] Vlatko Vedral,[4] Wolfgang P. Schleich,[3,8] and Ron Folman[1]

Article | Open access | Published: 01 June 2017

Quantum test of the equivalence principle for atoms in coherent superposition of internal energy states

G. Rosi, G. D'Amico, L. Cacciapuoti, F. Sorrentino, M. Prevedelli, M. Zych, Č. Brukner & G. M. Tino

Nature Communications 8, Article number: 15529 (2017) | Cite this article

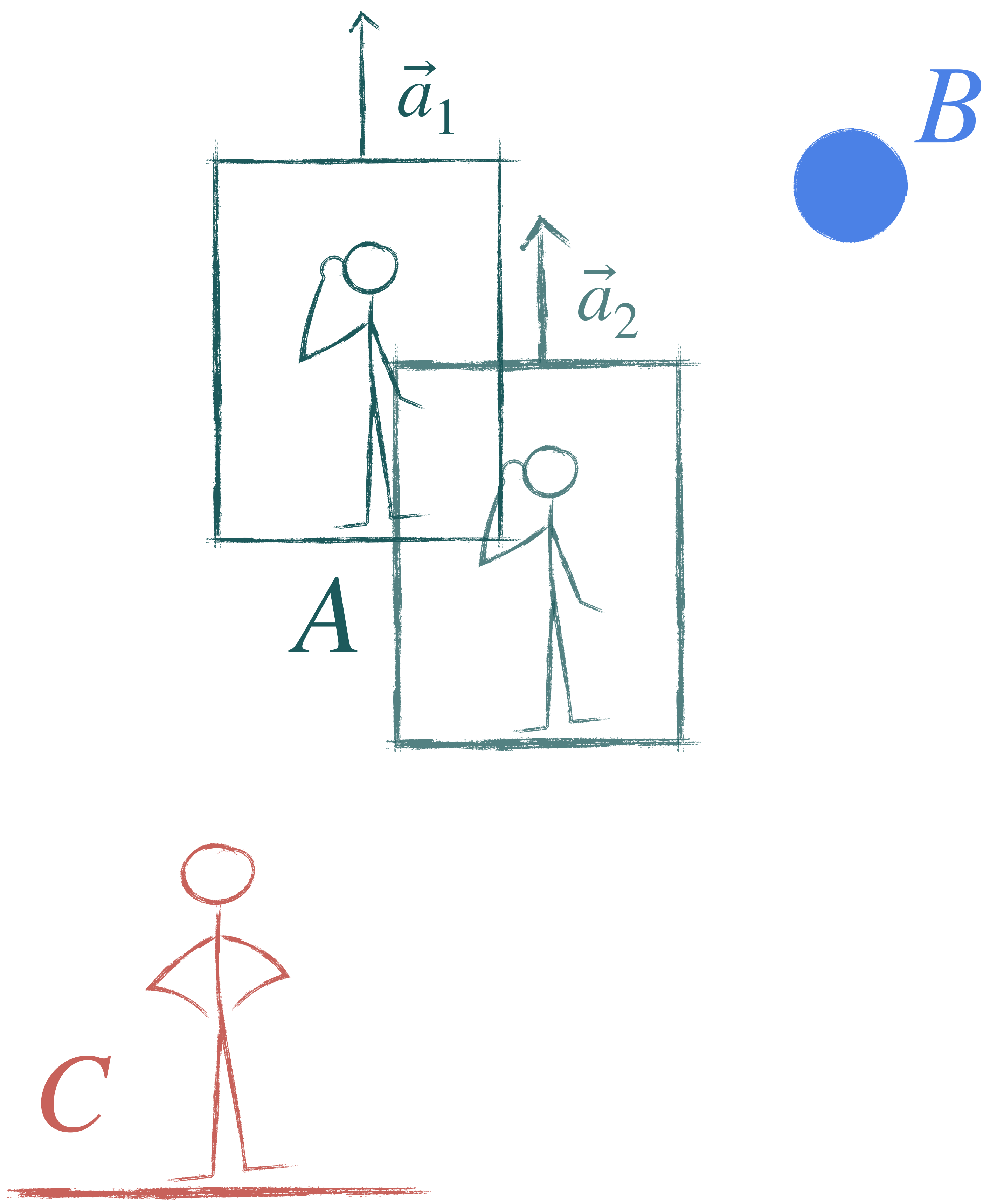
International Journal of Modern Physics D | VOL. 24, NO. 12

The equivalence principle in a quantum world

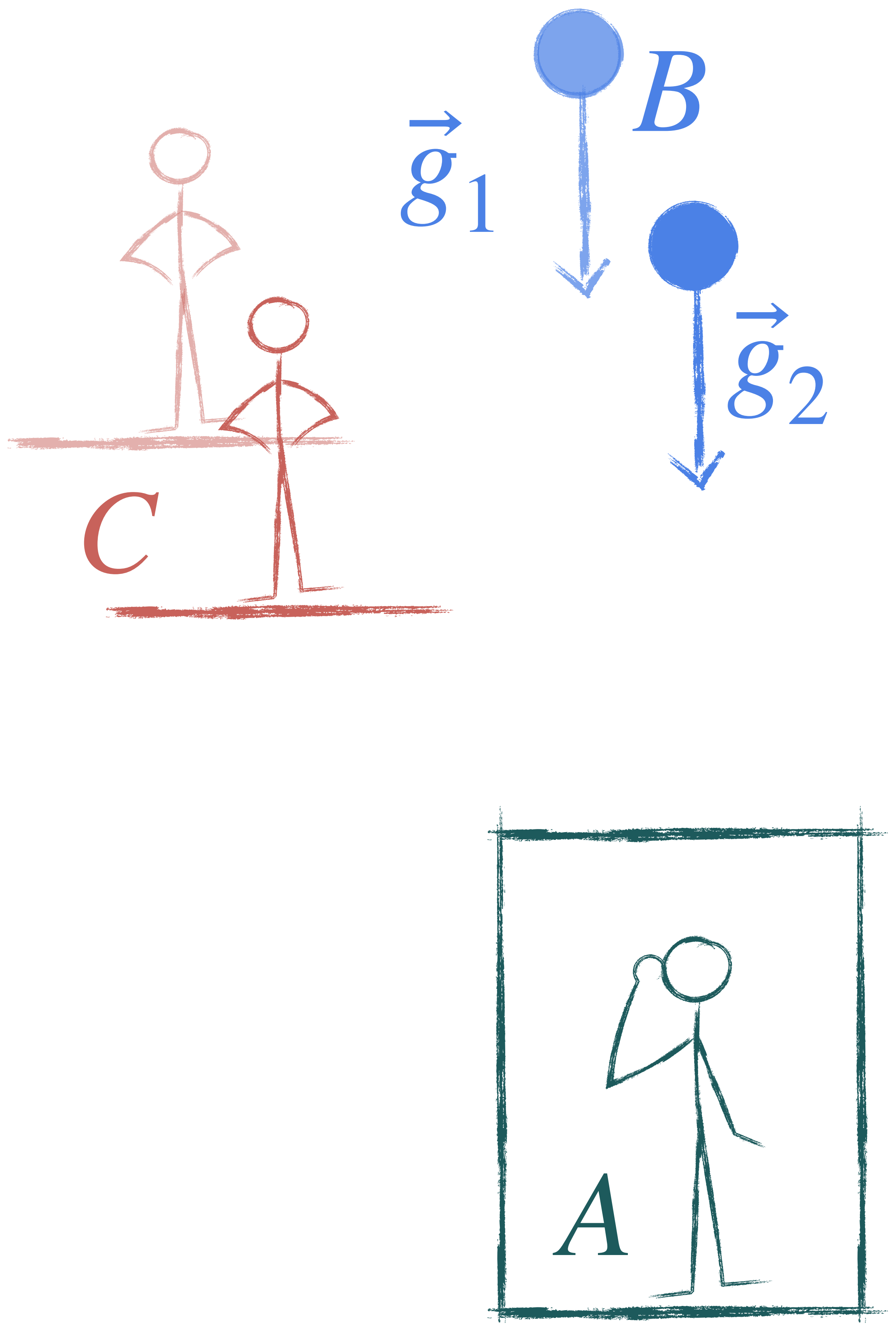
N. E. J. Bjerrum-Bohr, John F. Donoghue, Basem Kamal El-Menoufi, Barry R. Holstein, Ludovic Planté, and Pierre Vanhove

QUANTUM GENERALISATION OF THE WEP FOR QRFs

EXTENSION OF WEP TO QRFs



Equivalent
via a
QRF transformation



The equations of motion in a superposition of constant gravitational fields are indistinguishable from those in a quantum superposition of accelerations.

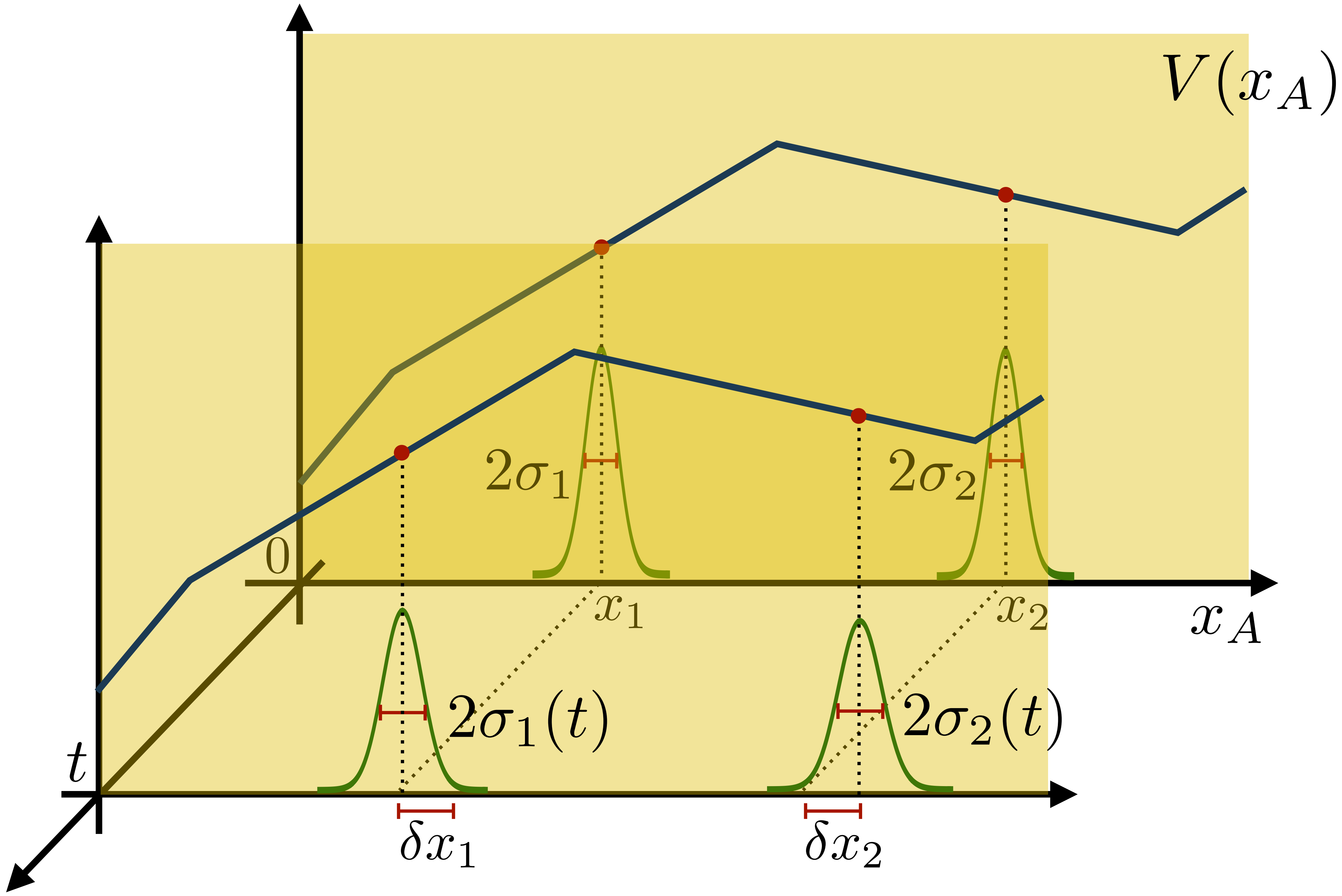
Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

EXTENSION OF WEP TO QRFs

Piecewise linear potential

$$a_1 |\psi_1(t)\rangle_A = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A} |\psi_1(t)\rangle_A$$

$$a_2 |\psi_2(t)\rangle_A = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A} |\psi_2(t)\rangle_A$$



$\hat{a}$ "Acceleration operator"

$$\frac{1}{\sqrt{2}}\left(|\psi_1(t)\rangle_A + |\psi_2(t)\rangle_A \right) |\phi\rangle_B \Rightarrow \frac{1}{\sqrt{2}}\left(|\psi_1'(t)\rangle_C \hat{U}_{g_1} |\phi\rangle_B + |\psi_2'(t)\rangle_C \hat{U}_{g_2} |\phi\rangle_B \right)$$

Standard transformation to accelerated frame

$$\hat{V}_B = -\frac{m_B}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\hat{x}_B$$

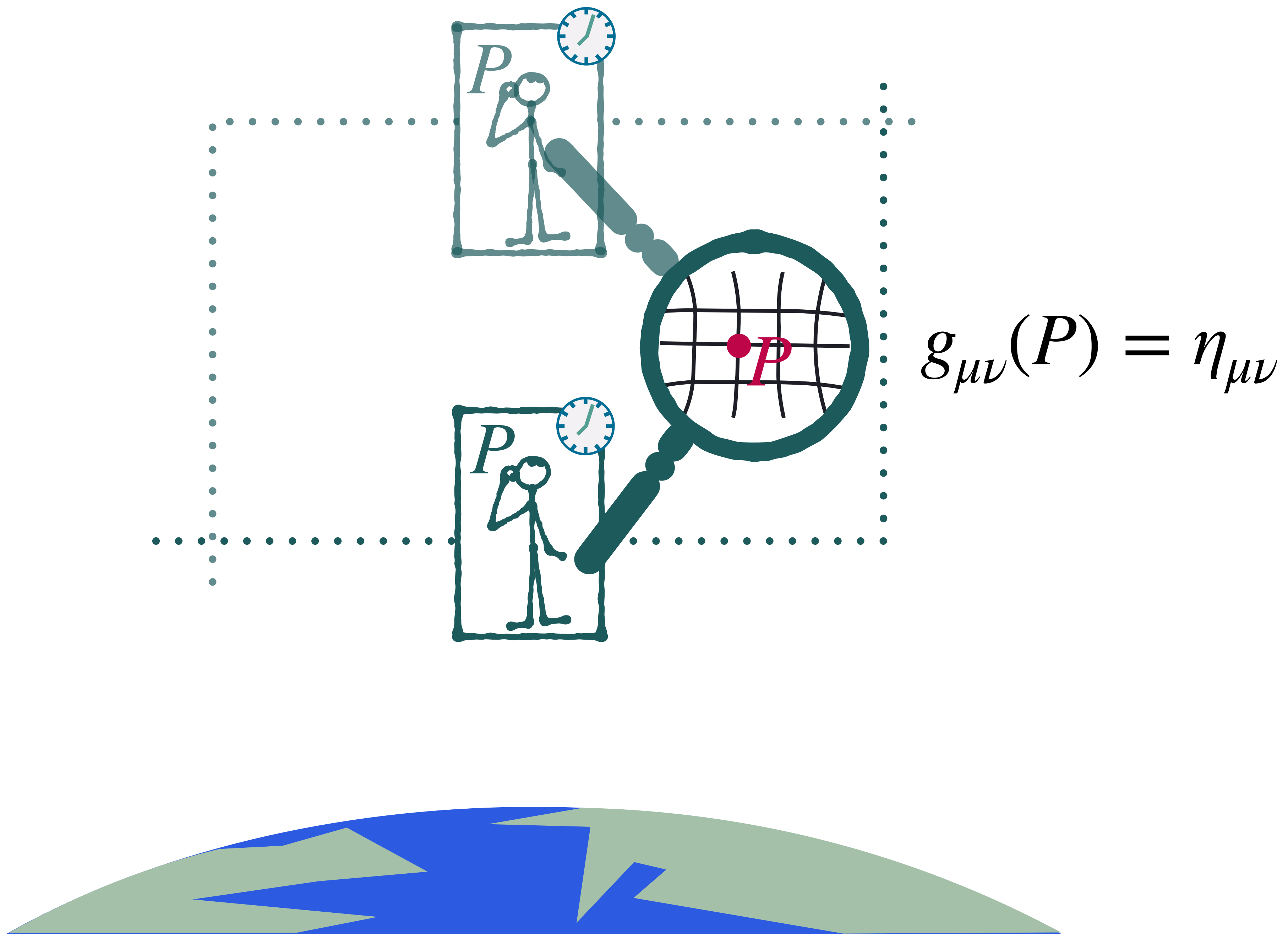
Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

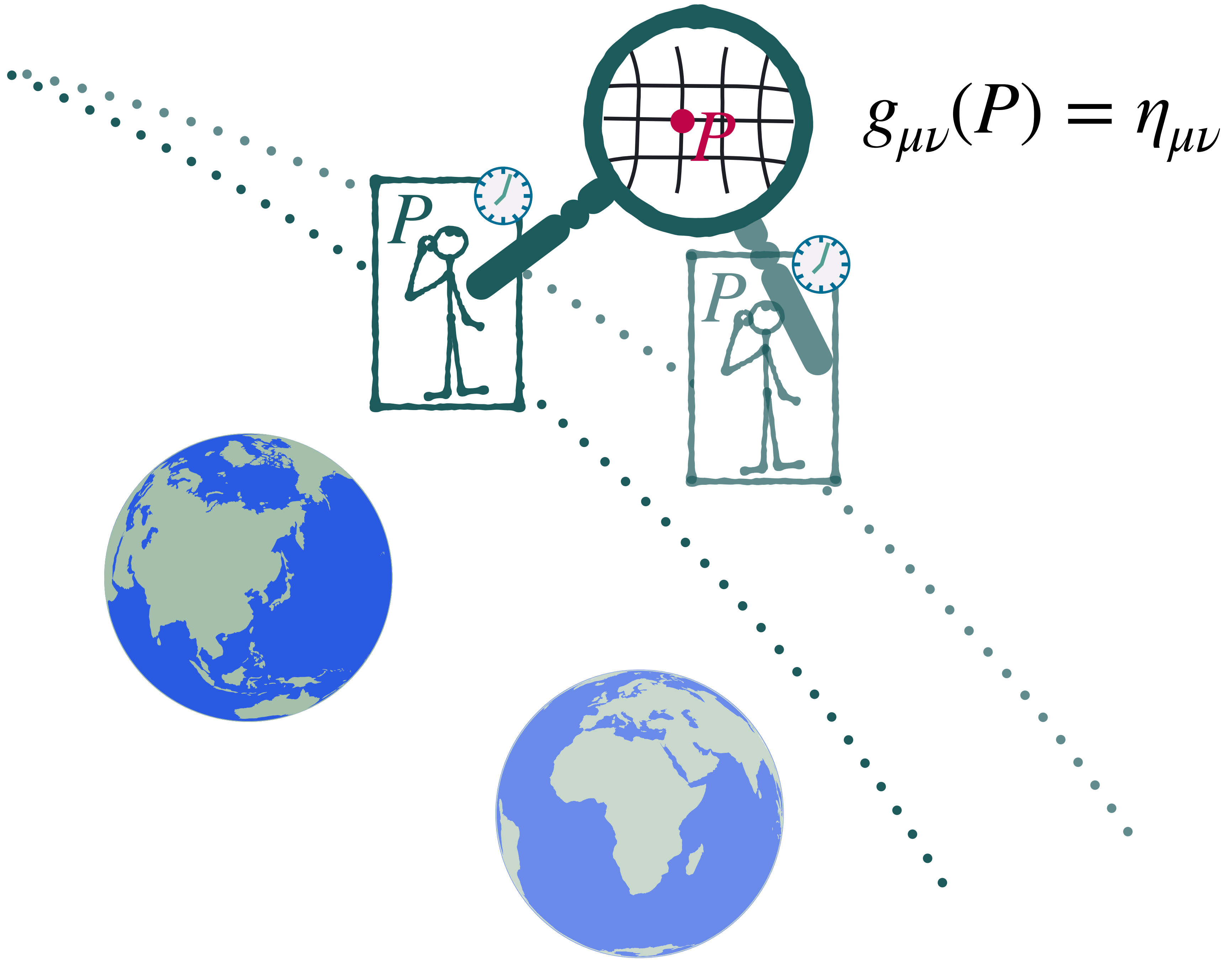
QUANTUM GENERALISATION OF THE EEP FOR QRFs

NEED FOR QUANTUM LOCALLY INERTIAL FRAMES

*In any and every **Quantum** Locally Inertial Frame (QLIF), anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar nonrelativistic form.*

WHAT IS A QLIF?

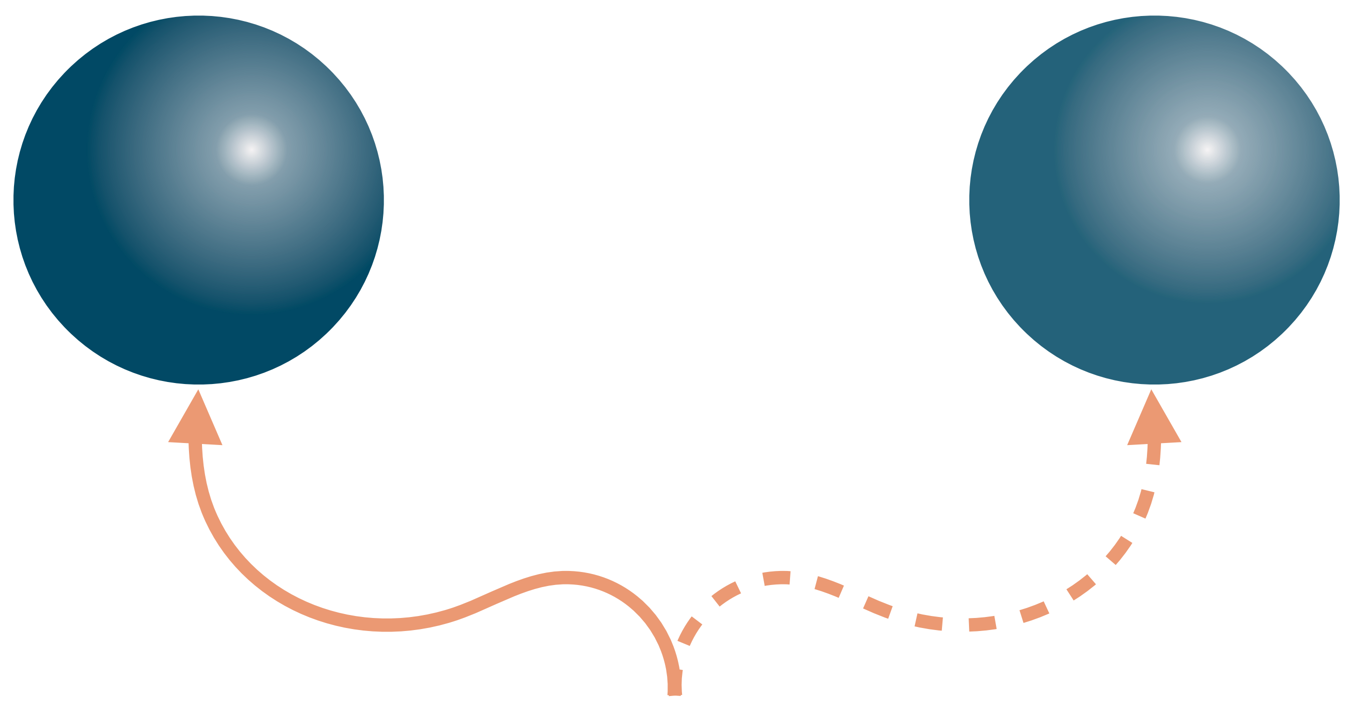




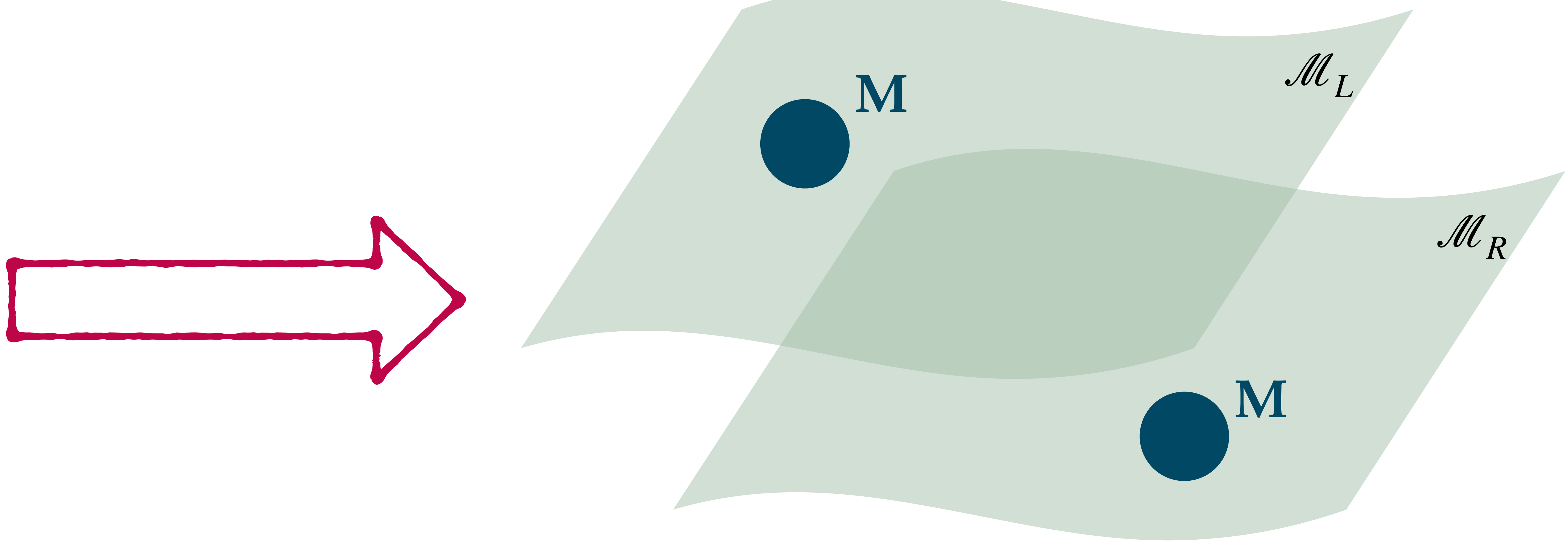
Giacomini, Brukner, 2012.13754 (2020)
Giacomini, Brukner, AVS Quantum Science (2022)
Cepollaro, Giacomini, Class. Quant. Grav. (2024)

QUANTUM SUPERPOSITIONS OF SPACETIMES

We don't really know, but…



Massive particle in a superposition



- Macroscopically distinguishable gravitational fields are assigned orthogonal quantum states. The gravitational fields are said to be distinguishable if they can be distinguished by measuring macroscopic observables
- Each well-defined gravitational field is described by general relativity
- The quantum superposition principle holds for such gravitational fields

TEST OF THE QUANTUM EEP IN QRFs

THE ELEMENTS OF THE QUANTUM EEP FOR QRFs

Cepollaro, Giacomini, Class. Quant. Grav. (2024)

- Q-WEP: The local effects of (quantum) motion in a quantum superposition of uniform gravitational fields are indistinguishable from those of an observer in flat spacetime that undergoes a quantum superposition of accelerations.

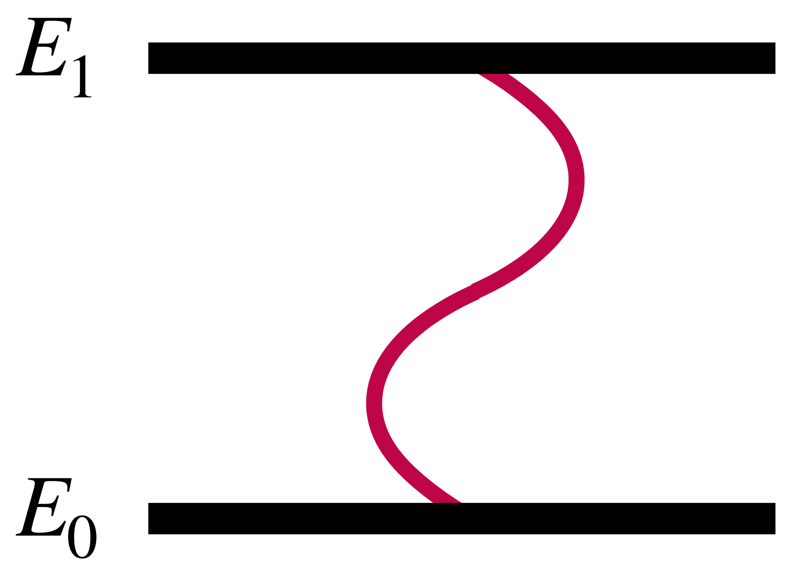
- Q-LLI: the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling quantum reference frame in which it is performed

- Q-LPI: the outcome of any local non-gravitational experiment is independent of the position of the quantum reference frame in which it is performed.

Test of Q-LPI: assume Q-WEP, Q-LLI

WHAT IS A QUANTUM CLOCK?

=



$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

$$H_C = E_0|E_0\rangle\langle E_0| + E_1|E_1\rangle\langle E_1|$$

$$|\psi_t\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + e^{\frac{i}{\hbar}(E_1-E_0)t}|E_1\rangle)$$

-1

$$\langle\psi_{t_\perp}|\psi_0\rangle = 0$$

ORTHOGONALISATION TIME

$$t_\perp = \frac{\pi\hbar}{(E_1 - E_0)}$$

WHAT IS A QUANTUM CLOCK?

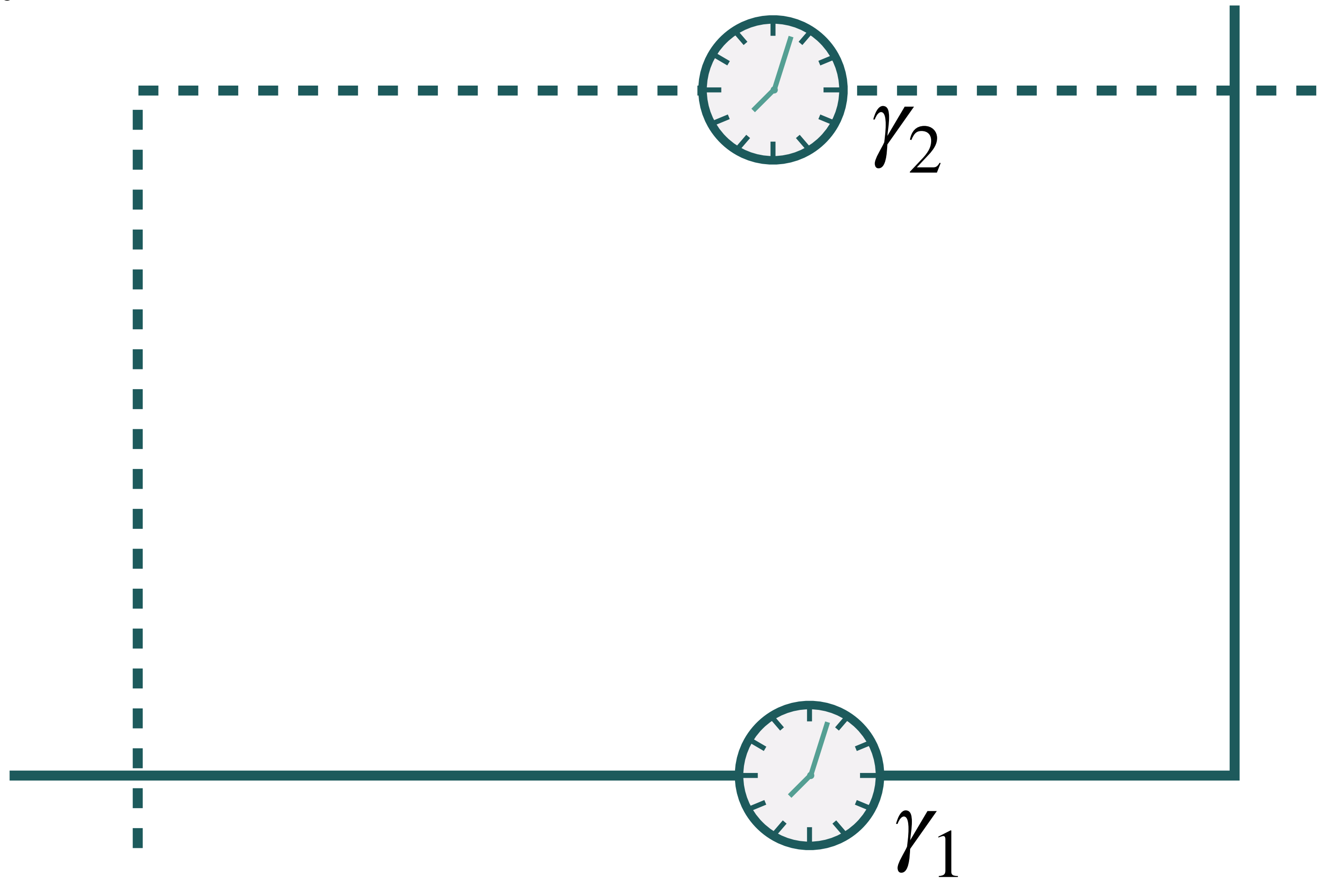
$$E = mc^2$$

$$m \to m + \frac{\hat{H}_I}{c^2}$$

THE MASS BECOMES AN OPERATOR!

QUANTUM CLOCKS AS PROBES OF SPACETIME

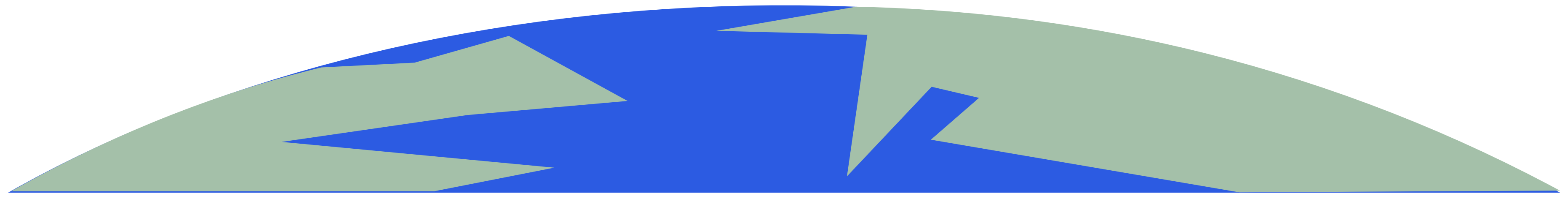
Quantum clocks test the principle of linear superposition and gravitational time dilation together.

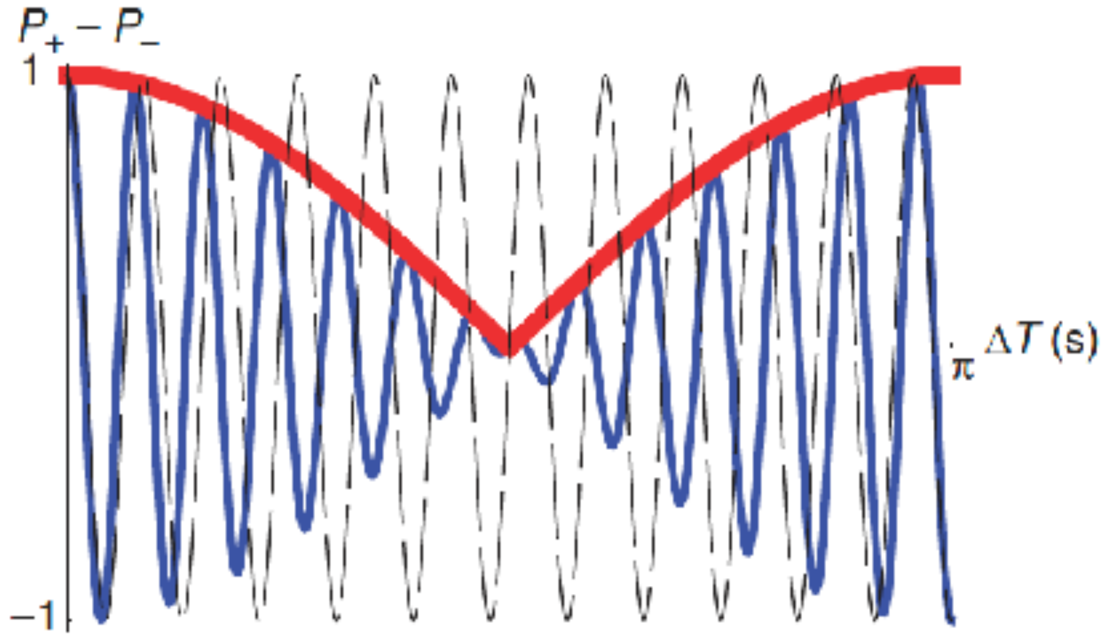


"LOWER IS SLOWER"

$$|\tau_0\rangle = \frac{1}{\sqrt{2}}\left(|E_0\rangle + |E_1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|\gamma_1\rangle|\tau_1\rangle + e^{i\Delta\phi}|\gamma_2\rangle|\tau_2\rangle\right)$$





$$\mathscr{V} = |\langle\tau_1|\tau_2\rangle|^2$$ **Visibility (interference)**

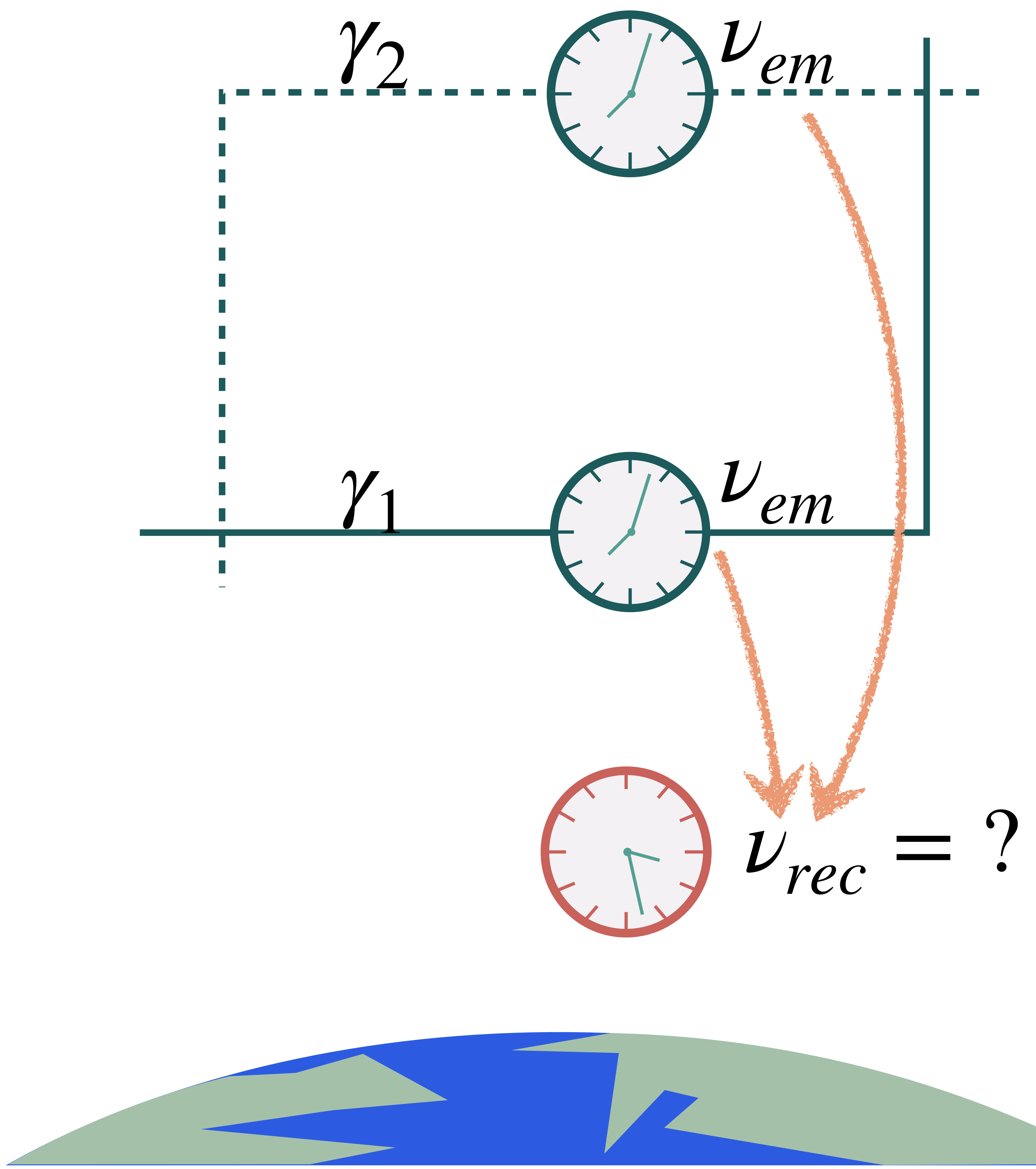
$$\mathscr{D} = 1 - |\langle\tau_1|\tau_2\rangle|^2$$ **Distinguishability (which-way information)**

The clock carries "which-way" information in its proper time.

Zych, Costa, Pikovski, Brukner, Nat. Commun. (2011)

NEED FOR QUANTUM LOCALLY INERTIAL FRAMES

Cepollaro, Giacomini, Class. Quant. Grav. (2024)



What is the "freely-falling frame momentarily at rest with the clock"?

It is a QLIF!

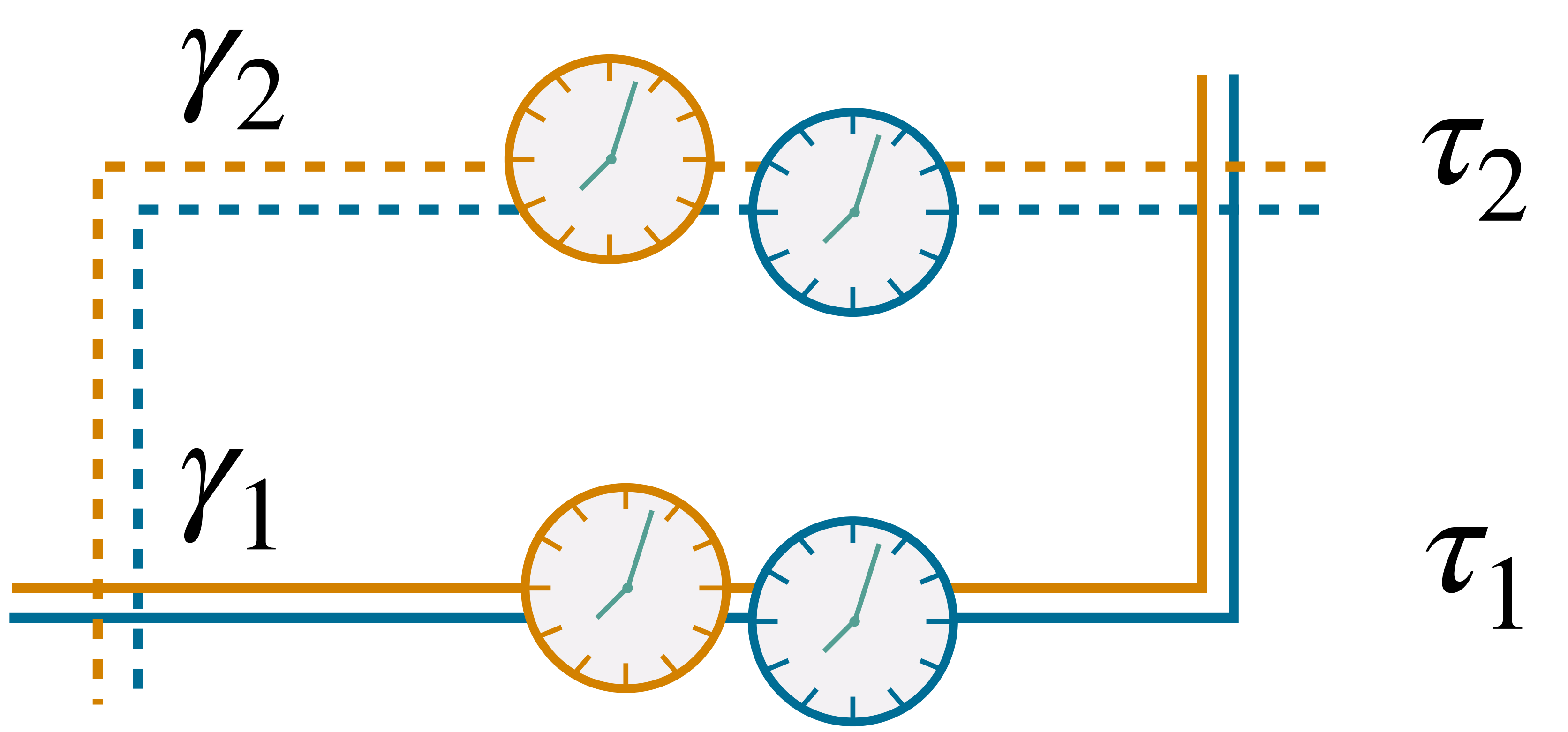
$$Z = (1 + \hat{\alpha}(\hat{x}))\frac{g\Delta U}{c^2}$$

The internal energy of the clock contributes differently to the inertial and gravitational mass operator

TEST OF THE EINSTEIN'S EQUIVALENCE PRINCIPLE FOR QRFs

Cepollaro, Giacomini, Class. Quant. Grav. (2024)

Measurement strategy:
- trace out clock degrees of freedom
- recombine paths at beam splitter



$$\langle \tau_2^{(v)} | \tau_1^{(v)} \rangle = \langle \tau_2 | \tau_1 \rangle - \frac{i}{\hbar c^2} \int_{\Delta\gamma} dx U(x_\gamma) \alpha(x_\gamma) \langle \tau_2 | \hat{H}_C | \tau_1 \rangle$$

The probabilities detected at the end of the interferometer explicitly depend on $\langle \tau_2^{(v)} | \tau_1^{(v)} \rangle$

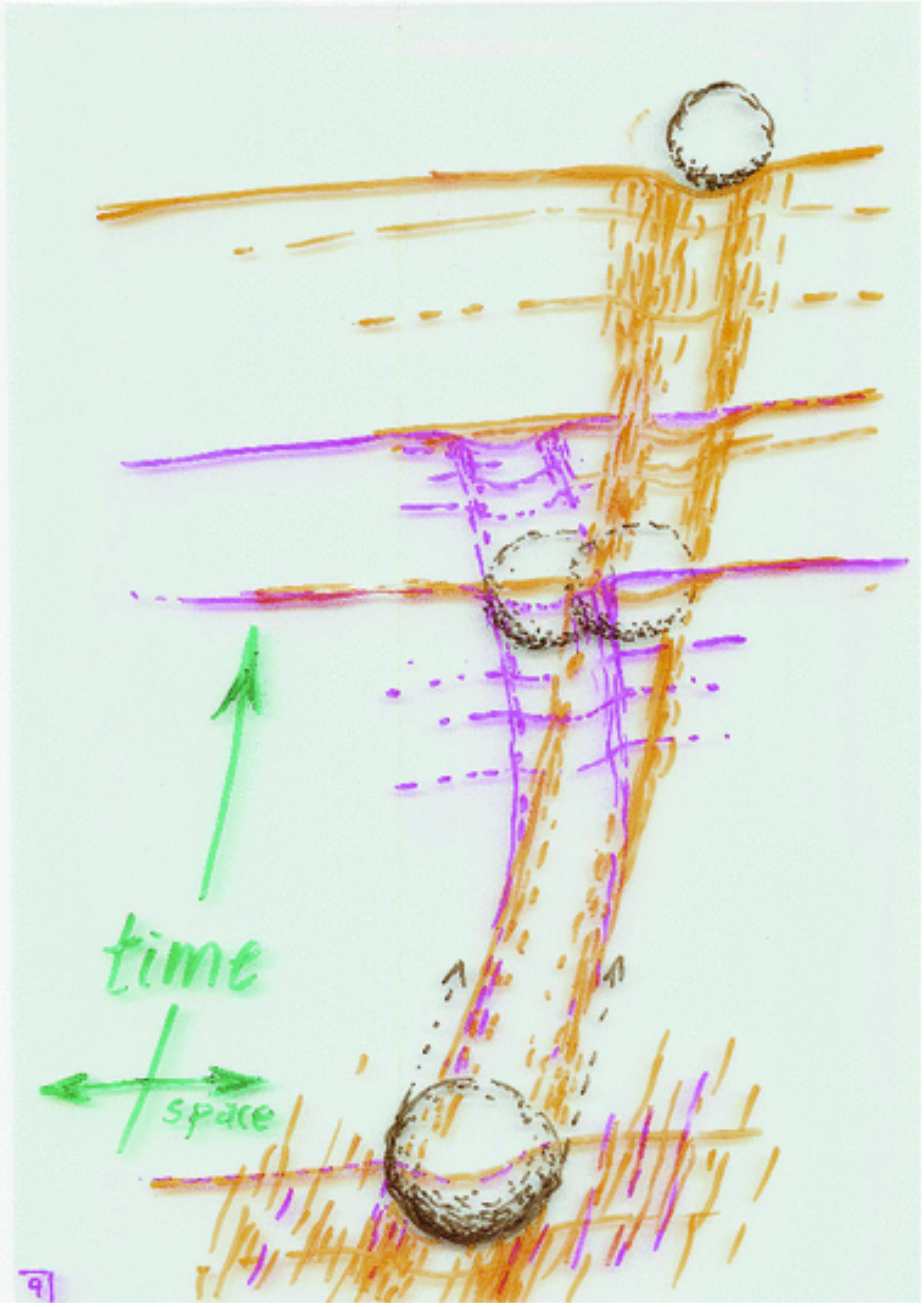
$$\Phi_i = g z_i + \cdots$$

Atomic clocks interferometers can be used to test violations of the EEP for QRFs!

SPONTANEOUS STATE REDUCTION AND EEP FOR QRFs

Incompatibility between the principle of linear superposition and the EEP.

R. Penrose, Foundations of Physics (2014)



Mass in a quantum superposition:
There is no single coordinate transformation that makes the metric locally Minkowski.

Different Killing vectors ⇒ **Global state has uncertainty in energy → unstable configuration**

Spontaneous state reduction

$$t_\Delta \sim \hbar/\Delta E_\Delta$$

QRFs reconcile the Equivalence Principle with the principle of linear superposition.

Giacomini, Brukner, AVS Quantum Science (2022)

SUMMARY

Operational and relational formalism for quantum reference frames:
associate a reference frame to a quantum system.

In quantum mechanics:
Frame-dependence of entanglement and superposition
Generalisation of covariance
Generalisation of the weak equivalence principle
Operational definition of the rest frame of a quantum system (relativistic spin)

THANK YOU!

In gravity:
Generalisation of the Einstein Equivalence Principle
Penrose decoherence

If gravity is not classical, we cannot have a classical reference frame.
Quantum reference frames can help us!

ETH zürich   PI PERIMETER INSTITUTE   QISS THE QUANTUM INFORMATION STRUCTURE OF SPACETIME   Swiss National Science Foundation